List of topics before midterm

Chapter 1
Syllabus
1/sqrt(x)
Sources of error
Alternative floats
IEEE Floats
Fund. floating point props.
Condition numbers
  Sharp (elm-wise) vs.
  Weak (norms)
Condition number of Ax=b
Overall floating point error
Variance computation

Chapter 2
Best approx prob.
Integrals, inner-products, and measures
Weierstrauss approx. thm.
Orthogonal functions
Lagrange interpolant
Chebyshev nodes
Barycentric interp.
Newton interp.
Divided differences
Hermite interpolation
Splines
Piecewise interp.
Error equation

Chapter 3
Approximating derivatives
Forward, backward diff.
Central diff.
Sensitivity of differen
Trapezoid + Simpson
Interpolatory quadrature
Degree of exactness
Orthogonal polynomials
Undetermined coefficients
Computer impl.
List of topics after midterm

Chapter 3
Gauss-Hermite quad
Richardson extrapolation

Chapter 4
Basics of nonlinear equation solving
Method of bisection
Rates of convergence
Method of false position
Secant method
Newton’s method

Chapter 5
Introduction to ODEs
Forward and Back. Euler
Methods as large linear systems
One-step methods for ODEs (in general)

Fixed point method
Newton’s method for systems
Convergence of the secant method
Midpoint method
Heun’s method
Runge-Kutta methods
Consistency
Stability
Convergence
Global error
Step length control
A-stable methods
Stiff problems
Spectral methods for ODEs
Types of problems to expect

Explain what this Julia code does

```julia
function myfunc(xx, fvals, x)
    fx = zeros(length(x))
    for i=1:length(x)
        xind = findmin(abs(xx-x[i]))
        fx[i] = fvals[xind]
    end
end
```

Grading

• Type of problem (floating point, interp., diff, quad, ...)
• Details.
Types of problems to expect

- Which code produced which output?
- Identify these concepts in an argument
- Use the idea of XXX to study ZZZ (concept generalization)
- What is the order of accuracy / scaling of the error term of this polynomial approx. / derivative approx.
By request

Convergence and divergence of bisection.

• Bisection cannot diverge on continuous functions.
• Bisection will always converge on continuous functions.
• It may be difficult to find a and b that are guaranteed to contain a root.
Convergence and divergence of false pos.

- False pos. cannot diverge on continuous functions.
- False pos. will always converge on continuous functions.
- It may be difficult to find a and b that are guaranteed to contain a root.
By request

Convergence and divergence of secant.

- Secant will converge to a simple root if $f$ is twice continuously differentiable and you start “sufficiently close”
- It will converge at a superlinear rate.
- No general divergence criteria for secant.
- It may be difficult to ensure you are sufficiently close.

Most likely question: analyze what happens for a specific function, or some “class”
By request

Convergence and divergence of Newton.

• Newton will converge to a simple root if $f$ is twice continuously differentiable and you start “sufficiently close”
• It will converge at a quadratic rate.
• No general divergence criteria for Newton.
• It may be difficult to ensure you are sufficiently close.

Most likely question: analyze what happens for a specific function, or some “class” (watch out for violations of the conditions of a theorem.)
By request

Convergence and divergence of fixed point.

- Fixed point will converge if you have a contraction map
- It will converge at a linear rate.
- No general divergence criteria for fixed point.
By request

Differences between Runge Kutta and Heun’s method.

<table>
<thead>
<tr>
<th>Method</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler</td>
<td>$y^+ = y + hf(t, y)$</td>
<td>follow current slope</td>
</tr>
<tr>
<td>Midpoint</td>
<td>$y^+ = y + hf(t + h/2, y + h/2f(t, y))$</td>
<td>follow current slope for half $h/2$ then follow new slope at $h/2$ for the full step $h$</td>
</tr>
<tr>
<td>Heun</td>
<td>$y^+ = y + h[1/2f(t, y) + 1/2(f(t + h, y + hf(t, y)))]$</td>
<td>follow current slope for $h$ then sample new slope, and average both</td>
</tr>
<tr>
<td>General</td>
<td>$y^+ = y + h[\alpha_1 k_1 + \alpha_2 k_2]$</td>
<td>Take a combination of two slopes sampled anywhere in the time-span of $h$</td>
</tr>
</tbody>
</table>

\[ k_1 = f(t, y), \quad k_2 = f(t + \mu h, y + \mu h k_1) \]
Two-stage

\[ y^+ = y + h[\alpha_1 k_1 + \alpha_2 k_2] \]
\[ k_1 = f(t, y), \]
\[ k_2 = f(t + \mu h, y + \mu h k_1) \]

r-stage

\[ y^+ = y + h[\sum_{i=1}^{r} \alpha_i k_i] \]
\[ k_1 = f(t, y), \]
\[ k_i = f(t + \mu_i h, y + h \sum_{j=1}^{i-1} \lambda_{i,j} k_j) \]
Runge-Kutta picture

\[
y^+ = y + h \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]
\]

\[
k_1 = f(t, y),
\]
\[
k_2 = f(t + \frac{1}{2}h, y + \frac{1}{2}hk_1),
\]
\[
k_3 = f(t + \frac{1}{2}h, y + \frac{1}{2}hk_2),
\]
\[
k_4 = f(t + h, y + hk_3),
\]

Do midpoint twice, then do Heun’s method on the result,
And take the weighted average of all of them.