November 18, 2016

How to debug ODE methods

In this class you should learn:

- *Two more methods: Heun's method and the RK4 method (book has Simpson's rule)*
- *What goes wrong with Forward Euler on the Spring System*
- *Understand what absolute stability is and why it is different from stability.*
- *How a small set of topics allow us to understand what is going on with an approximate solution to an ODE*

Next class

Stiff problems,
2-point BVPs & PDEs
Chapters 13, 14

Next next class

Optimization
Chapter 4

Terminology

A “solver” for an ODE is called a

- Scheme
- Method
- Integator

Somewhat interchangeably.

- Scheme – how you go from step k to k+1
- Integrator/Method – Overall approach

Notation

$\mathbf{y}^*(t)$ is exact

$\underbrace{\mathbf{y}(t)}_{\text{exact}} \approx \underbrace{\mathbf{y}(hk) = \mathbf{y}_k}_{\text{comp. approx}}$

$\mathbf{y}^*(t)$ is exact if otherwise unclear

Writing Forward Euler as a scheme

$$\mathbf{y}((k+1)h) = \mathbf{y}(kh) + h\mathbf{f}(kh, \mathbf{y}(hk))$$

$$\mathbf{y}_{k+1} = \mathbf{y}_k + h\mathbf{f}(kh, \mathbf{y}_k)$$

$$\mathbf{y}_{k+1} = \mathbf{y}_k + h\mathbf{f}(t_k, \mathbf{y}_k)$$

$$\mathbf{y}_{k+1} = \mathbf{y}_k + h\mathbf{f}(t, \mathbf{y}_k)$$

These are all meant to be equivalent ways of writing Forward Euler.

Summary of Notation

$\mathbf{y}^*(t)$ is exact

$$\underbrace{\mathbf{y}(t)}_{\text{exact}} \approx \underbrace{\mathbf{y}(hk) = \mathbf{y}_k}_{\text{comp. approx}}$$

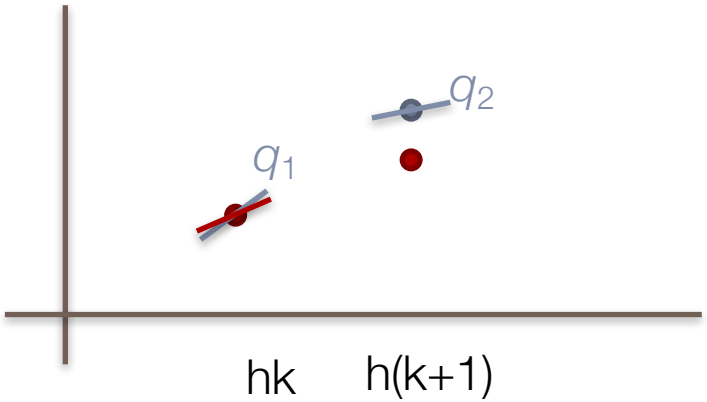
Heun's method

$$y_{k+1} = y_k + h/2[q_1 + q_2]$$
$$q_1 = f(t_k, y_k)$$
$$q_2 = f(t_k + h, y_k + hf(t_k, y_k))$$

Summary of Notation

$\mathbf{y}^*(t)$ is exact

$$\underbrace{\mathbf{y}(t)}_{\text{exact}} \approx \underbrace{\mathbf{y}(hk) = \mathbf{y}_k}_{\text{comp. approx}}$$



Runge Kutta methods

Runge-Kutta 4th order method

"Related" to a Simpson integration rule.

$$y_{k+1} = y_k + h/6[q_1 + 2q_2 + 2q_3 + q_4]$$
$$q_1 = f(t_k, y_k)$$
$$q_2 = f(t_k + h/2, y_k + h/2q_1)$$
$$q_3 = f(t_k + h/2, y_k + h/2q_2)$$
$$q_4 = f(t_k + h, y_k + q_3)$$

Just a one step method!

Julia makes it adaptive in the ode45 routine.