November 2, 2016

Numerical differentiation & Richardson extrapolation

In this class you should learn:

- *How truncation error and floating point error need to be balanced for accurate computations*
- *How to use Richardson extrapolation to improve a finite difference formula*
- *Why the derivative problem is fundamentally ill-conditioned*
- *Derivatives of polynomial approximations*

Next class

Review of Exam/Quiz

Next next class

Misc topics in numerical differ

Numerical differentiation

Key points

Numerical accuracy is tricky with regular grids

Polynomial representations make differentiation “easy” (we’ll see this today!)

There are some standard approaches to improve the accuracy of numerical derivatives on regular grids. (Richardson extrapolation)

Numerical Methods for Applied Math

Error 1/Approx 1

1. Take the continuous problem.
   e.g. integral

Error 2/Approx 2

2. Compute a discrete representation.

Error 3/Approx 3

3. Determine where to apply continuous & discrete properties to derive a *tractable* problem.
   e.g. linear system

Error 4/Approx 4

4. Solve the tractable problem.
   e.g. LU factorization

Numerical Methods for Applied Math

Compute $f'(x)$

Given $f(x)$

$$\frac{1}{h}[f(x+h) - f(x)]$$

$$[\hat{f}(x \oplus h) \ominus \hat{f}] \oslash h$$

$\hat{f}$ = floating point function f

1. Take the continuous problem.
   e.g. integral
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Demo

Forward difference

$$f'(x) = \frac{1}{h}\left[f(x+h) - f(x)\right] + O(h)$$

Central difference

$$f'(x) = \frac{1}{2h}\left[f(x+h) - f(x-h)\right] + O(h^2)$$