In this class:

- Lagrange polynomials and Lagrange interpolation
- Barycentric form of the interpolant.
- The error in the interpolant
- APPROXFUN

Various forms of the interpolating poly. & some error analysis

Next class
QUIZ
More on error & piecewise interpolants
Next next class
Derivatives
Error in polynomial interpolation

THEOREM 8.4.1

Assume that \( f \) is \( n + 1 \) times cont. diff. in a region \([a, b]\), and that \( x_0, \ldots, x_n \) are distinct points in \([a, b]\).

Let \( p(x) \) be the unique polynomial of degree \( n \) that interpolates \( f \) at \( x_0, \ldots, x_n \).

Then

\[
f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi_x) \prod_{j=0}^{n} (x - x_i)
\]

for some point \( \xi_x \) in \([a, b]\) that depends on \( x \).
Analysis

\[ f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi_x) \prod_{j=0}^{n} (x - x_i) \]

Goes down with \( n \)

Hard to control if the \( n+1 \) derivative isn’t well behaved.

A polynomial that is zero at each \( x_i \) if this is necessarily large, we’ll study this!
Interpolation at Chebyshev points

THEOREM 8.5.1

Let $f$ be a continuous function on $[-1, 1]$

$p_n$ its degree $n$ interpolant at Chebyshev points

$p_n^*$ its best approximation among $n$ degree polynomials in the uniform error

Then

uniform error in $p_n \leq (2 + \frac{2}{\pi} \log n)\text{uniform error in } p_n^*$

$p_n$ converges exponentially fast to $f$ if $f$ is smooth
Analysis

If we interpolate \( f \) at Chebyshev points, we get something close to the *best possible result*

Example. Suppose \( f \) is complicated.

and best (unknowable) degree 100 poly \( p_{100}^* \) gets

\[
\max_x |p_n^*(x) - f(x)| \leq 10^{-5}
\]

computable Chebyshev degree 100 poly \( p_{100} \) gets

\[
\max_x |p_n(x) - f(x)| \leq 5 \cdot 10^{-5} \quad (2 + 2/\pi \log(100)) \leq 5
\]

and if \( f \) doesn’t have discontinuous derivatives (e.g. \( \sin, \cos, \exp, \ldots \)), best approx gets small “very fast”
ApproxFun demo