October 26, 2016

Various forms of the interpolanting poly. & some error analysis

In this class:

- *Lagrange polynomials and Lagrange interpolation*
- *Barycentric form of the interpolant.*
- *The error in the interpolant*
- *APPROXFUN*

Next class

QUIZ
More on error & piecewise interpolants

Next next class

Derivatives

Error in polynomial interpolation

THEOREM 8.4.1

Assume

that f is $n+1$ times cont. diff. in a region $[a, b]$, and that $x_0, \dots, x_n$ are distinct points in $[a, b]$.

Let

$p(x)$ be the unique polynomial of degree n that interpolates f at $x_0, \dots, x_n$.

Then

$$f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi_x) \prod_{j=0}^{n} (x - x_i)$$

for some point ξ_x in $[a, b]$ that depends on x.

Analysis

Hard to control if the n+1 derivative isn't well behaved.

Goes down with n

$$f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi_x) \prod_{j=0}^{n} (x - x_i)$$

A polynomial that is zero at each x_i if this is necessarily large, we'll study this!

Interpolation at Chebyshev points

THEOREM 8.5.1

Let

f be a continuous function on $[-1, 1]$

p_n its degree n interpolant at Chebyshev points

p_n^* its best approximation among n degree polynomials in the uniform error

Then

uniform error in $p_n \leq (2 + \frac{2}{\pi} \log n)$uniform error in p_n^*

p_n converges exponentially fast to f if f is smooth

Analysis

If we interpolate f at Chebyshev points, we get something close to the *best possible result*

Example. Suppose f is complicated.

and best (unknowable) degree 100 poly p_{100}^* gets $$\max_x |p_n^*(x) - f(x)| \leq 10^{-5}$$

computable Chebyshev degree 100 poly p_{100} gets $$\max_x |p_n(x) - f(x)| \leq 5 \cdot 10^{-5}$$ (2 +2/pi log(100)) <= 5

and if *f* doesn't have discontinuous derivatives (e.g. sin, cos, exp, …), best approx gets small "very fast"

ApproxFun demo