October 5, 2016

Least squares & QR

In this class:

- *A geometric view on solving Least squares problems*
- *The QR factorization and Gram-Schmidt*

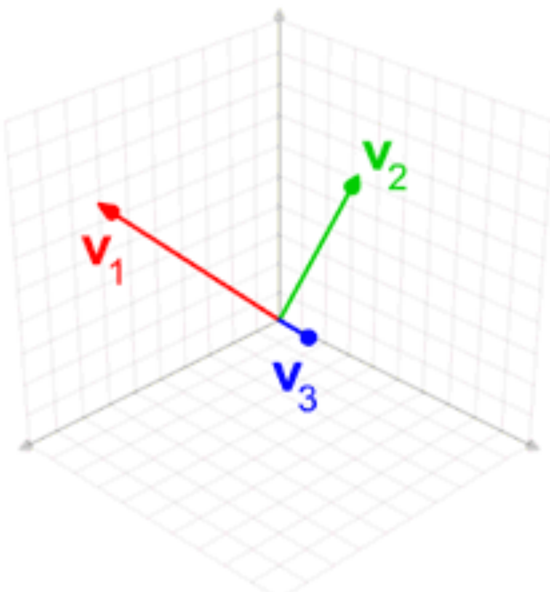
Next class

Finish up Least Squares & conditioning
G&C – Chapter 6

Next next class

Iterative methods
G&C – Chapter 7.6

QR Factorization and the Gram Schmidt process



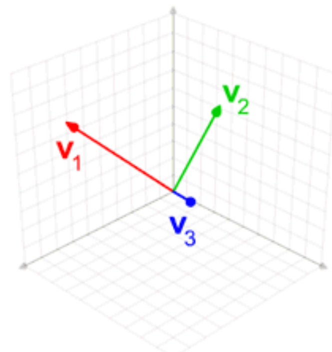
Consider a set of vectors v_1 to v_n. Set u_1 to be v_1.

Create a new vector u_2 by removing any "component" of u_1 from v_2.

Create a new vector u_3 by removing any "component" of u_1 and u_2 from v_3.

…

QR Factorization and the Gram Schmidt process



$$\mathbf{v}_1 = a_1\mathbf{u}_1$$

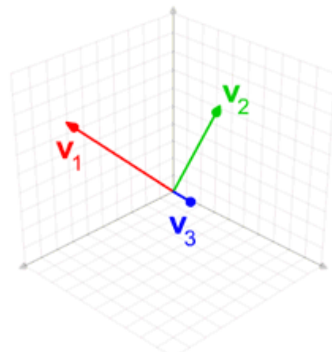
$$\mathbf{v}_2 = b_1\mathbf{u}_1 + b_2\mathbf{u}_2$$

$$\mathbf{v}_3 = c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + c_3\mathbf{u}_3$$

$$\begin{bmatrix}\mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \dots\end{bmatrix}$$

$$= \begin{bmatrix}\mathbf{u}_1 & \mathbf{u}_2 & \mathbf{v}_3 & \dots\end{bmatrix}\begin{bmatrix} a_1 & b_1 & c_1 & \dots \\ 0 & b_2 & c_2 & \dots \\ 0 & 0 & c_3 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

QR Factorization and the Gram Schmidt process



$$\mathbf{v}_1 = a_1\mathbf{u}_1$$

$$\mathbf{v}_2 = b_1\mathbf{u}_1 + b_2\mathbf{u}_2$$

$$\mathbf{v}_3 = c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + c_3\mathbf{u}_3$$

For this problem

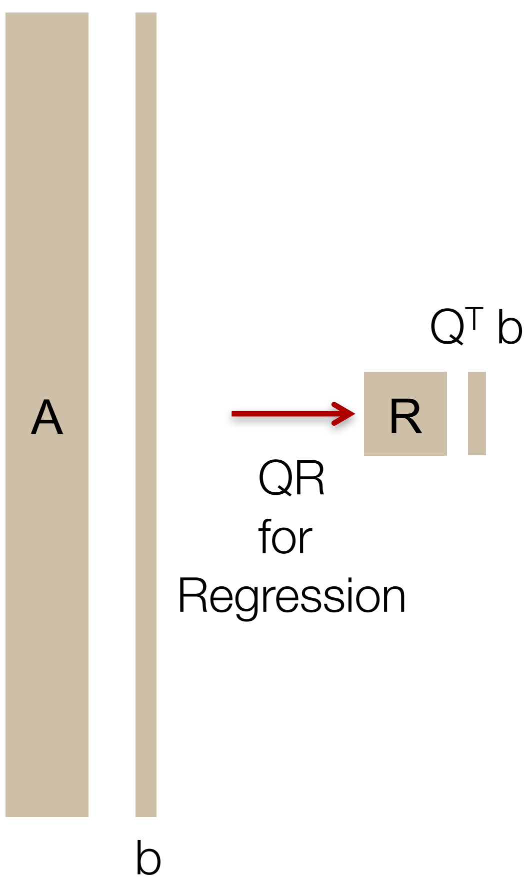
$$V = UR$$

What it's usually written as by others

$$A = QR$$

All vectors in U are at right angles, i.e. they are decoupled

More about how to compute a regression



$$\min \|\boldsymbol{Ax} - \boldsymbol{b}\|^2$$

$$= \min \|\boldsymbol{QRx} - \boldsymbol{b}\|^2$$

Orthogonal or "right angle" matrices don't change vector magnitude

$$= \min \|\boldsymbol{Q}^T\boldsymbol{QRx} - \boldsymbol{Q}^T\boldsymbol{b}\|^2$$

… what goes here?

$$= \min \|\boldsymbol{Rx} - \boldsymbol{Q}_1^T\boldsymbol{b}\|^2$$

This is a linear system!