October 3, 2016

# Operations in solving Ax=b

*In this class:*

- *An important property of pivoted LU*
- *Operations involved in solving Ax=b*
- *Review of the midterm*

*Next class*

Least squares and QR factorizations
G&C – Chapter 7.6

*Next next class*

Quiz on Linear Systems
intro to iterative methods
G&C – Chapter 7

# Recall Pivoted LU

At each step of GE

- Find the row with the largest magnitude entry
- Swap that row and the current row

Then run GE for that step as usual

This can be recorded as

Permutation matrix to record swaps $\boldsymbol{PA = LU}$ Normal LU decomp!

If $\boldsymbol{P} = \boldsymbol{I}$, then this is just an LU of A

# Example

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

Swap row 1 and 3
($|7| > |1|$, $|7| > |4|$)

$$\boldsymbol{P}^{(1)}\boldsymbol{A} = \begin{bmatrix} 7 & 8 & 0 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix} \qquad \boldsymbol{P}^{(1)} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

# But pivoted LU has one cool feature

Recall at first step of GE

$$L_{i,1} = A_{i,1} / A_{1,1}$$

With pivoting, $|A_{1,1}| \geq |A_{i,1}|$

So $|L_{i,1}| \leq 1$

And this holds at all other steps because of pivoting!

# What to remember

In a pivoted LU decomposition, the entries of L are all less than 1 in magnitude! (e.g. between -1 and 1).

# Let’s see some examples!

Juliabox!

# Supercomputer performance

The world's fastest computers are measured by their speed on the LINPACK Benchmark

- in number of floating point operations per second
- WHILE SOLVING $Ax=b$

(Floating point ops to solve Ax = b) / (Time to solve Ax=b)

Time is easy!

How many "flops" ? We'll count that next

# Your quiz

Run LINPACK on your phone (or with a partner) and report your phone's speed in Millions of Floating Point Ops per section (Mflops)

N = 256

(Use Multithreading)

(Use double if possible)