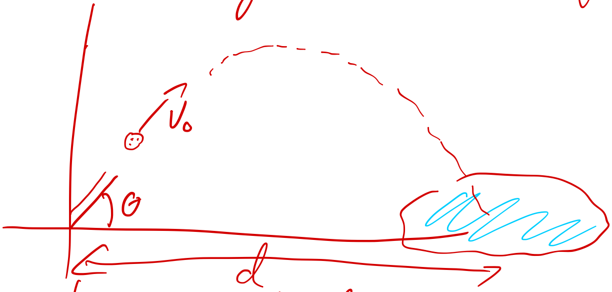
Lecture 39

Nonlinear Equations!

Artillery Problem

potato gun! (fixed velocity!)



Q: what angle to launch at to hit lake

$y(t)$ = height of potato @ time t.

$$\frac{d^2y}{dt^2} = -g \overbrace{\boxed{-K\,y'(t)}}^{\text{Air Resistance}}$$

$$y'(0) = V_0 \sin\theta$$
$$y(0) = 0$$

$$y(t) = -\frac{1}{K} e^{-Kt} V_0 \sin\theta - \frac{g}{K}\left(t + \frac{1}{K} e^{-Kt}\right) + \frac{1}{K} V_0 \sin\theta + \frac{g}{K^2}$$

How far does potato go?

$t_{impact} \iff y(t)=0,\ t>0$

$x(t_{impact}) = \ ?$

Solve for θ s.t.

$$x(t_{impact}(\theta)) = d.$$

$$x(t) = \frac{-1}{K} e^{-Kt} v_0 \cos\theta + \frac{1}{K} v_0 \cos\theta$$

function dist potato (θ)

① Solve

for t_{impact}

2. Compute $x(t_{impact})$

How to do step 1?

$$f(x) = 0$$

Example 2

$$\frac{dy}{dt} = (t+1)e^{-y}, \quad y(0) = 0$$

We want BW Euler!
for Abs. Stability Prop.

Solve for $y(h)$

$$\boxed{\frac{y(h) - y(0)}{h} = f(h, y(h))}$$

$\Rightarrow$ this becomes

$h = 0.1$

$$\frac{y(0.1) - 0}{0.1} = (0.1 + 1)e^{-y(0.1)}$$

Solve

$$\boxed{\frac{x}{0.1} = 1.1e^{-x} \text{ for } x}$$

$$\downarrow$$

$$xe^{x} = (1.1)(0.1)$$

$$f(x) = c$$

In general, we c
problem f(x)
and try and
Software/Algs/
that find x
Subroutine access

Quiz: How to us
a subroutine/Alg
find x for

onsider the

= 0

build

Black Boxes

using

to f.

e such

to

Soln:

f is arbitrary!

(non-linear!)

Let $f(x) = xe^x$

Then $\hat{f}(x) = f(x) - c$

and call ALG on

$\hat{f}(x)$

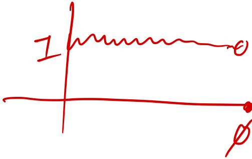
$$\hat{f}(x) = 0 \iff f(x) - c = 0$$
$$\iff f(x) = c.$$

How to Solve?

Bisection.

- Assume Subroutine
- f is continuous.

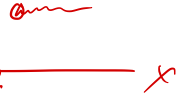
↳ Why?



$f(x) = \begin{cases} 0 \\ 1 \end{cases}$

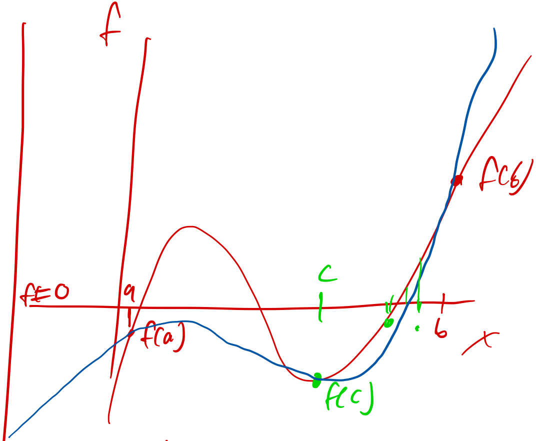
•

access to f



if $x = 0$

if $x \neq 0$.

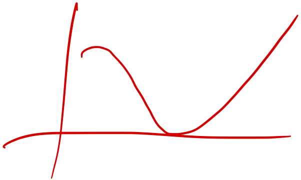


In Bisection, we assume we know a, b, where the sign of f differs.

Idea: test c in between a, b, and recurse on the interval w/ a sign change.

What value of c to use?

$$c = \frac{a+b}{2}$$



Bisection: