Lecture 37

- Absolute Stability

$\rightarrow$ the ability of a Scheme to correctly get model

$$\frac{dy}{dt} = \lambda y \quad \lambda \in \mathbb{C}$$

where $Re(\lambda) < 0$.

i.e. (correctly model)

$y_k \rightarrow 0$

$$\frac{dy}{dt} = Ay$$

$$A = V \Lambda V^T$$

$$\frac{du_1}{dt} = \lambda_1 u_1(t)$$

$$\frac{du_2}{dt} = \lambda_2 u_2(t)$$

$\vdots$

FE $\quad dy/dt = \lambda y = f(t,y)$

$$y_{k+1} = y_k + h f(t_k, y_k)$$

$$= y_k + h\lambda y_k$$

$$= (1+h\lambda)\boxed{y_k}$$

$$= (1+h\lambda)(1+h\lambda) y_{k-1}$$

$$= (1+h\lambda)^k y_0$$

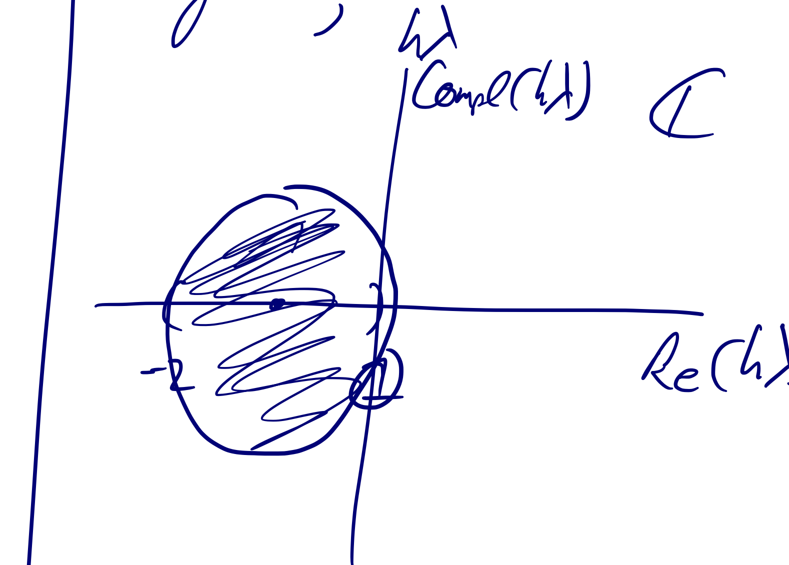
if $y_k \rightarrow 0 \iff (1+h\lambda)^k \rightarrow 0$

$\iff \rho^k \rightarrow 0$

$\rho = 1 + h\lambda$

iff $|\rho| < 1$, then $y_k \rightarrow 0$

$|1+h\lambda| < 1$ to get "correct", $h\lambda$



$Re(\lambda) < 0, h > 0$

for Heun's method!

$$\left| 1 + h\lambda + \frac{(h\lambda)^2}{2} \right|$$

Lots of Terms

- Abstract Scheme/Method

$$\boxed{y_{k+1} = y_k + h \underbrace{\text{Step}\{y_k, t, h\}}_{\text{what your comp. does!}}}$$

Ex: FE

$\text{Step}(y_k, t, h) = f(t, y_k)$

Heun ... more complicated!

- Consistency (Consistent)
- Local Truncation Error
- Stability (NOT Abs. Stability)

$\hookrightarrow$ Everything will be Stable!

- Global Error

Converged (Convergence)

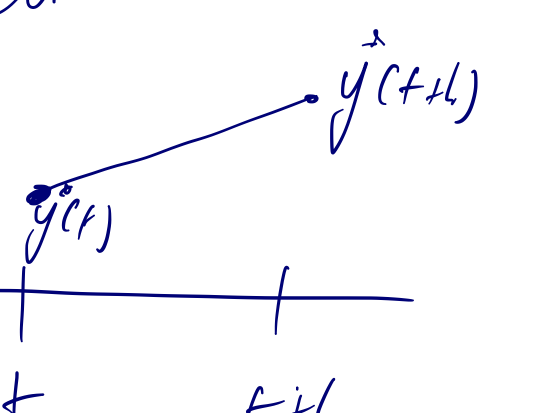
A method is convergent if as $h \rightarrow 0$ then $y_k \rightarrow y^*(kh)$

$\rightarrow$ A step is consistent if

$$\lim_{h \rightarrow 0} \text{Step}(y_k, t, h) = f(t, y_k)$$

$\rightarrow$ How good is Step vs. dy/dt

if



if

$$\frac{y^*(t+h) - y^*(t)}{h} - \text{Step}(y^*(t), t, h) = O(h)$$

then we say Step has

local truncation Error $O(h)$

if slope - Step $= O(h^2)$

$\iff$ local truncation Error $= O(h^2)$