Lecture 35

Newton-Cotes quad

: Exactly integrate the poly approx to $f(x)$ @ uniformly spread points!

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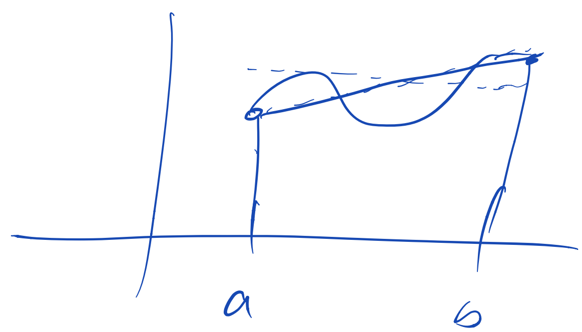
2 pts

$$\int_a^b f(x)\,dx \simeq \frac{f(a)+f(b)}{2}(b-a)$$

Trapezoid Rule



Q: How good is trapezoid rule?

~~Recall~~

A: Use fancy poly Error!

$$p(x) - f(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi(x)) \prod_{i=1}^{n+1} (x - x_i)$$

if $p(x)$ is a degree n interpolant of f @ $x_1, \ldots, x_{n+1}$

So

$$\underbrace{\text{Error} = \int_a^b p(x)\,dx - \int_a^b f(x)\,dx}_{\text{diff between approx, true}}$$

$$= \int_a^b p(x) - f(x)\,dx$$

~~Simple~~ Simple!

$$\underbrace{= \int_a^b \frac{1}{(n+1)!} f^{(n+1)}(\xi(x)) \prod_{i=1}^{n+1} (x - x_i)\,dx}_{\text{And ... ugh! What to do now?}}$$

Keep following your nose!

$$x_1 = a, \quad x_2 = b$$

$$\prod (x - x_i) = \underbrace{(x-a)}_{\text{pos. Everywhere}} \underbrace{(x-b)}_{\text{neg. Everywhere!}}$$

$$\frac{1}{2}\int_a^b \underbrace{f''(\xi(x))}_{\text{Unknown.}} \underbrace{g(x)}_{\text{neg Everywhere}} \, dx$$

$\Longrightarrow$ Use generalized MVT!

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)} \quad \text{for some } c \in [a, b]$$

$\Longleftrightarrow$ Integral form

$$\int_a^b g(x)\, w(x)\, dx = g(c) \int_a^b w(x)\, dx$$

if $w(x)$ has the same sign in the interval

$$\frac{1}{2}\int_a^b f^{(2)}(\xi(x))\,w(x)\,dx =$$

$$f^{(2)}(\eta)\underbrace{\int_a^b (x-a)(x-b)\,dx}_{\text{Back to calc!}}$$

$\boxed{\text{Your quiz!}}$

Eval $\int_a^b (x-a)(x-b)\,dx$!

Int. by parts! ahh!

$\cancel{(x-a)}$ $\quad (x-a)(x-b) = x^2 - (a+b)x + ab$

$$= \frac{x^3}{3}\Big|_a^b - (a+b)\frac{x^2}{2}\Big|_a^b + (ab)(b-a)$$

$$= \frac{b^3}{3} - \frac{a^3}{3}$$

$$= -$$

$$\int_a^b \underbrace{(x-a)}_{u(x)}\underbrace{(x-b)}_{v'(x)}dx = u(x)v(x)\Big|_a^b - \int_a^b u'(x)v(x)$$

$V(x) = \frac{x^2}{2} - bx$

$(b-a)\left(\frac{b^2}{2} - b^2\right)$

$=$

$\Rightarrow (x-a)\left(\frac{x^2}{2} - bx\right)\Big|_a^b - \underbrace{\int_a^b \left(\frac{x^2}{2} - bx\right)dx}$

$\Rightarrow \frac{(b-a)^3}{3}$