Lecture 34!

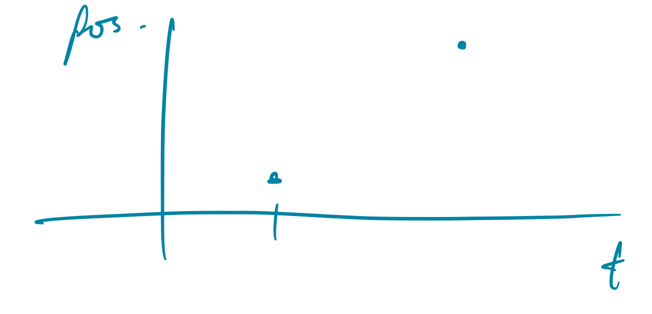
Before we begin

if $\frac{f(b)-f(a)}{b-a} = m$

then $f'(c) = m$ at

Some point $c \in [a,b]$

(if f is continuous)



MVT for integrals

$$\int_a^b f(x)\,dx = f(c)\,(b-a)$$

for some $c \in [a,b]$

If f has an anti-derivative F

then $\frac{F(b)-F(a)}{(b-a)} = F'(c)$

for $c \in [a,b]$

and that's good!

$F(b) - F(a) = \int_a^b f(x)\,dx$

$F'(c) = f(c)$

Today!

Newton-Cotes quadrature!

Recall:

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^n f(x_i) A_i$$

In Newton-Cotes

we use Equally spaced

points x_i

for Newton-Cotes,

we define A_i

s.t. we integrate

a poly approx to

f Exactly!

(deg n-1 approx)

§ Let's show this!

$$\int_a^b f(x)\,dx = \int_a^b p(x)\,dx$$

deg n-1

where $p(x)$ is the poly approx to f

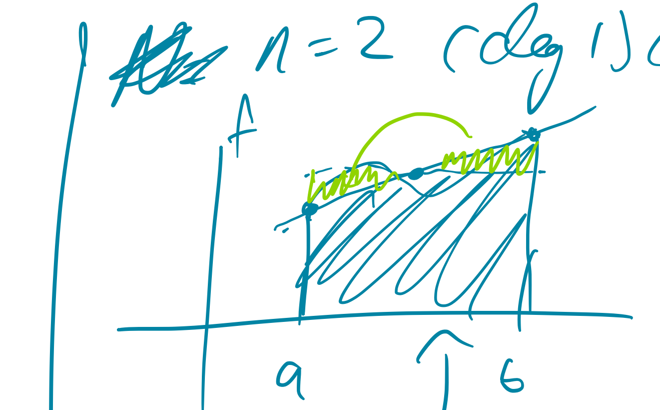
$$p(x) = \sum_{i=1}^n f(x_i) \prod_{j \neq i} \frac{(x-x_j)}{(x_i - x_j)}$$

$$\int_a^b p(x)\,dx = \int_a^b \sum f(x_i) \prod \frac{(x-x_j)}{(x_i-x_j)}\,dx$$

$$= \sum_{i=1}^n \frac{f(x_i)}{\prod_{j\neq i}(x_i - x_j)} \int_a^b \prod_{j \neq i} (x - x_j)\,dx$$

$= A_i$

$n = 2$ (deg 1) case



Area of

$$\frac{f(a)+f(b)}{2}(b-a)$$

§ $p(x) = f(a)\frac{(x-b)}{a-b} +$

$f(b)\frac{(x-a)}{(b-a)}$

$$f(a)\int_a^b \frac{x-b}{a-b}\,dx = \frac{(x-b)^2}{2(a-b)}\Big|_a^b$$

$$= 0 - \frac{(a-b)}{2} f(a)$$

$$= f(a)\frac{(b-a)}{2}$$

$$f(b)\int_a^b \frac{x-a}{a-b}\,dx = f(b)\frac{(b-a)}{2}$$

$$\int_a^b p(x) = f(a)\frac{(b-a)}{2} + f(b)\frac{(b-a)}{2}$$

$$= \left(\frac{f(a)+f(b)}{2}\right)(b-a)$$

$p(x) - f(x) =$

$$\frac{1}{(n+1)!} f^{(n+1)}(\xi(x)) \prod_{i=1}^n (x - x_i)$$

$$\int_a^b p(x) - f(x)\,dx = \text{Error!}$$

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