Applied mathematics

KEY POINT

Find a function with this property
Find a function that satisfies this equation
Find a property of this function
What is the value of the integral of a function?
What is the derivative function of a given function?
What function solves a given differential equation?

Numerical Methods for Applied Math

Take the continuous problem.
e.g. integral

Compute a discrete representation.

Determine where to apply continuous & discrete properties to derive a *tractable* problem.
e.g. linear system

Solve the tractable problem.
e.g. LU factorization

Polynomial approximation

Polynomials are one of the most useful ways of representing 1d functions on a computer.

Alternatives

Polynomial approximations

Key points

Why polynomial approximation?

Weierstrass approximation theorem

Every continuous function on an interval [a,b] can be *uniformly approximated* by as closely as desired by a polynomial

Analysis

$$f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi_x) \prod_{j=0}^{n} (x - x_i)$$

Error in polynomial interpolation

THEOREM 8.4.1

Assume
that f is $n+1$ times cont. diff. in a region $[a, b]$, and that $x_0, \dots, x_n$ are distinct points in $[a, b]$.
Let
$p(x)$ be the unique polynomial of degree n that interpolates f at $x_0, \dots, x_n$.
Then

$$f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi_x) \prod_{j=0}^{n} (x - x_i)$$

for some point ξ_x in $[a, b]$ that depends on x.

Interpolation at Chebyshev points

THEOREM 8.5.1

Let
f be a continuous function on $[-1, 1]$
p_n its degree n interpolant at Chebyshev points
p_n^* its best approximation among n degree polynomials in the uniform error
Then
uniform error in $p_n \leq (2 + \frac{2}{\pi} \log n)$uniform error in p_n^*
p_n converges exponentially fast to f if f is smooth