Lecture 23

Halfway!

$$A = \begin{bmatrix} 1 & -10 \\ -5 & -4 \end{bmatrix}$$

Let's compute Eigenvalues + vectors the 265/351 way!

$$\det(A - \lambda I) =$$

$$\begin{vmatrix} 1-\lambda & -10 \\ -5 & -4-\lambda \end{vmatrix} =$$

$$(1-\lambda)(-4-\lambda) - 50 = 0$$

find λ!

$$-4 - \lambda + 4\lambda + \lambda^2 - 50 = 0$$

$$-54 + 3\lambda + \lambda^2 = 0$$

$$\frac{-3 \pm \sqrt{9+216}}{2} = \frac{-3 \pm 15}{2}$$

$$\to \sqrt{225} = 15$$

$\lambda = 6, -9$

We still need v's!

$$\begin{bmatrix} 1-6 & -10 \\ -5 & -4-6 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -10 \\ -5 & -10 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$v_1 = 2, \; v_2 = -1$

Properties of Eigen Stuff

$A\underline{x} = \lambda \underline{x},$

$\underline{x} \neq 0$

Also,

$A(\alpha \underline{x}) = \lambda(\alpha \underline{x})$

So we usually set

$\|x\|_2 = 1$

to be more unique.

Real matrices can have Complex Eigenvalues ☹

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\lambda = \pm\sqrt{-1}$$
$$= \pm i$$

But, if A is Symmetric and real, then all Eigenvalues are real!

(and Eigenvectors too!)

Similarity

If X

Matri

X^{-1}

the Sa

So if

Q^TAQ

too

Transform

is an invertible

, then

$A X$ has

me Eigenvalues

$n \times n$

Q_n is orthog

has the Same

Eigenvalue Decomp

If A is real + Symmetric, then

$$A = Q \Lambda Q^T$$

is an Eigenvalue decomp.

Q's cols are Eigenvects

Λ is diag w/ Eigenvalues

$$A^K \to Q \Lambda^K Q'$$