Lecture 14

Notation

A : $m \times n$ (matrix) $A \in \mathbb{R}^{m \times n}$

$\underset{\sim}{x}$: n-vector (column) $x \in \mathbb{R}^{n}$

others $\vec{x}$

$n \times 1$ matrix too!

$\underline{x}^T$: $1 \times n$ row-vec.

A^T : $n \times m$ matrix!

Scalars.

$\alpha, \beta, \gamma, \phi, \xi$

greek letters

Matrix-as-Vector

A : $m \times n$

$A = [\underline{a}_1 \ \underline{a}_2 \ \ldots \ \underline{a}_n]$

$\underline{a}_i$: m-vector.

x_i = i^{th} elem of $\underline{x}$

A_{ij} = Entry in row i, col j of A

Operations

$\underline{a} + \underline{b}$ element-wise add.

$\alpha \underline{x} + \beta \underline{y}$ "axby"

$A\underline{x}$ matrix-vector product (Mat-vec)

"gemv"

AB Matrix-matrix multiplication

"gemm" (Mat-mul)

Mat-Vec

$$[A\underline{x}]_i$$

Example

$$\begin{bmatrix} 1 & 2 \\ -2 & 4 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 2 & 4 \end{bmatrix}}$$

row-vector

(not the way

$$= \sum_{j=1}^{n} A_{ij} x_j$$

$$] = \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}$$

$$\begin{matrix} 0 \\ 3 \end{matrix}] = [1 \ 6 \ 3]$$

nt-vec
in this class!)

Mat Mul

$A : m \times n$

$B : n \times k$

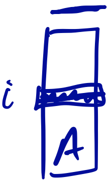
$AB \rightarrow m \times k$ matrix

$$[AB]_{ij} = \sum_{r=1}^{n} A_{ir} B_{rj}$$

if $B = [\underline{b}_1 \ \cdots \ \underline{b}_k]$

then

$$AB = [A\underline{b}_1 \ \cdots \ A\underline{b}_k]$$



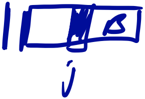
$[AB$

def: c

$x \cdot$

inner

$x \cdot y$



c_{ij} = (i^{th} row of A) (j^{th} col of B)

dot-product!

inner product!

product

$y = \sum x_i y_i$

notatn

$= \underline{x}^T y$

Mathematical Interpretation of $A\underline{x}$

A : $m \times n$

$A = [\underline{a}_1\ \underline{a}_2\ \cdots\ \underline{a}_n]$

$\underline{a}_i$: m-vector!

$$A\underline{x} = \sum_{i=1}^{n} x_i \underline{a}_i$$

← i^{th} column of A.

i^{th} element of x

$$\boxed{A\underline{x} : \text{a linear combo of } A\text{'s columns}}$$

Example 1.4 from Book

The Ducks!

live up to 3 yrs

20% survive year 1-2

50% survive year 2-3

0% survive year 3-4

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a 2yo duck → 3 offspring

← year-old

a 3yo " → 6 offspring

$\mathbf{p}$: 3x1 vector

p_1 - # of 1 yo ducks

p_2 - # of 2 yo ducks

p_3 - # of 3 yo ducks

$$A\mathbf{p} = \sum_{i=1}^{3} p_i \underline{a}_i$$

goal

$A\mathbf{p}$ = # of ducks next year!

$$A\underline{p} = p_1 \underline{a}_1 + p_2 \underline{a}_2 + p_3 a_3$$

1 yo old duck gives next year

1 2yo duck makes

3yo makes

$$\begin{bmatrix} 0 \\ .2 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 0 \\ .5 \end{bmatrix} \quad \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 3 & 6 \\ .2 & 0 & 0 \\ 0 & .5 & 0 \end{bmatrix}$$

Ap = # of duck next year;

$A^2 p = A \cdot Ap =$

# of duck in

$A^k \underline{p}$ = # of du

in k yea

2 years

rds

s!

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