1 WHAT ARE EIGENVALUES

\[ Ax = \lambda x \]
\[ \det(A - \lambda I) = 0 \]

2 WHY DO WE CARE ABOUT THEM?

Take good notes here! You'll be surprised when you need this information!

3 THE OLD WAY

\[ A = \begin{bmatrix} 1 & -10 \\ -5 & -4 \end{bmatrix} \]

We'll soon see a much better way to compute these eigenvalues and vectors
Real-valued matrices can have If $A$ is symmetric

5 THE POWER METHOD

6 THE POWER METHOD IN PRACTICE

$y \approx x = \text{rand}$

for $i=1$ to ...

$y =$

$x =$
7 SMALL EXAMPLES

\[ A = \frac{1}{3} \begin{bmatrix} 2.8 & 0.6 \\ 0.6 & 1.2 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -10 \\ -5 & -4 \end{bmatrix} \]

\[ A^k = \]
\[ \lambda = \]
\[ x = \]

8 CONVERGENCE OF THE POWER METHOD

Assumption 1 \( A \) is diagonalizable.

Assumption 2 There is one eigenvalue \( \lambda_1 \) with a magnitude larger than every other eigenvalue. Let \( v_1 \) be the eigenvector.

Assumption 3 If we write the starting vector \( x \) in terms of eigenvectors of \( A \), then the component associated with \( v_1 \) is non-zero.

Assumption 3 implies:

Assumption 1 implies:

The rate of convergence for the power method is determined by the second largest magnitude eigenvalue of \( A \).

|\( \lambda_1 \)| > |\( \lambda_2 \)| ≥ |\( \lambda_3 \)| ≥ ... ≥ |\( \lambda_n \)|

Then if |\( \lambda_2 \)/\( \lambda_1 \)| is nearly 1, it'll converge slowly. And if |\( \lambda_2 \)/\( \lambda_1 \)| is \( \ll 1 \), it'll converge quickly.
BEYOND A SINGLE VECTOR

How do we get multiple eigenvectors?

Property 1 The eigenvectors of a symmetric matrix are orthogonal.

Idea 1 If we find $x_1, \lambda_1$, then we can project on the orthogonal subspace.

“we can get-rid-of $x_1$ in running the power method”

This gives rise to a process called deflation!

Instead, let’s just do them all at once.

Let $X$ be a block of orthogonal vectors

Multiply $Y = AX$

Normalize