QUESTIONS ON BOUNDARY VALUE PROBLEMS

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1. In what sense are the following two systems of linear equations equivalent?

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 100 & -200 & 100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & -200 & 100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & -200 & 100 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & -200 & 100 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & -200 & 100 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 & -200 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 & -200 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & -200 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & -200 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \\ u_{10} \\ u_{11} \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -2 \\ -2 \\ -2 \\ -2 \\ -2 \\ -2 \\ -2 \\ -2 \\ 1/2 \end{bmatrix}.$$

$$\begin{bmatrix} -200 & 100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 100 & -200 & 100 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & -200 & 100 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & -200 & 100 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & -200 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & -200 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 & -200 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 & -200 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & -200 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \\ u_{10} \end{bmatrix} = \begin{bmatrix} -2-100 \\ -2 \\ -2 \\ -2 \\ -2 \\ -2 \\ -2 \\ -2 \\ -2-50 \end{bmatrix}.$$

2. How does this linear system change if you alter h?

3. Suppose that we have the BVP

$$u''(x) = -\pi^2 \sin(\pi x), u(0) = 1, u(1) = 1.$$

What does the linear system of equations look like to solve this if we set $h = 1/N = 1/4$

4. How would you alter the Julia code below to solve the BVP with $N = 20$?

```julia
N = 10
xgrid = collect(0:1/N:1)

alpha = 1
beta = 0.5
f = x -> -2.0
h = 1/N

A = zeros(N+1,N+1)
b = zeros(N+1,1)
for i=1:N+1
  if i == 1
    A[i,i] = 1
    b[i] = alpha
  elseif i==N+1
    A[i,i] = 1
    b[i] = beta
  else
    A[i,i-1] = 1/h^2
    A[i,i] = (-2)/h^2
    A[i,i+1] = 1/h^2
    b[i] = f(xgrid[i])
  end
end
u = A\b
plot(xgrid, u, marker=:circle)
```

5. Consider the BVP

$$\frac{\partial}{\partial x}\left[p(x)\frac{\partial u}{\partial x}\right] = -10.$$

Let $p(x) = 1 - 4(x - 1/2)^2$, $u(0) = -1$, $u(1) = 1$. Write down the linear system for this function if $N = 1/4$.