1. In what sense are the following two systems of linear equations equivalent?

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
100 & -200 & 100 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 100 & -200 & 100 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 100 & -200 & 100 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 100 & -200 & 100 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 100 & -200 & 100 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 100 & -200 & 100 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 100 & -200 & 100 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & -200 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2 \\
u_3 \\
u_4 \\
u_5 \\
u_6 \\
u_7 \\
u_8 \\
u_9 \\
u_{10}
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
-2 \\
-2 \\
-2 \\
-2 \\
-2 \\
-2 \\
-2 \\
-2 \\
1/2
\end{bmatrix}
\]

\[
\begin{bmatrix}
-200 & 100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
100 & -200 & 100 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 100 & -200 & 100 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 100 & -200 & 100 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 100 & -200 & 100 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 100 & -200 & 100 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 100 & -200 & 100 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 100 & -200 & 100 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & -200 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100
\end{bmatrix}
\begin{bmatrix}
u_2 \\
u_3 \\
u_4 \\
u_5 \\
u_6 \\
u_7 \\
u_8 \\
u_9 \\
u_{10}
\end{bmatrix}
= 
\begin{bmatrix}
-2 - 100 \\
-2 \\
-2 \\
-2 \\
-2 \\
-2 \\
-2 \\
-2 \\
-2 \\
-2
\end{bmatrix}
\]

2. How does this linear system change if you alter \( h \)?

3. Suppose that we have the BVP

\[u''(x) = -\pi^2 \sin(\pi x), \ u(0) = 1, \ u(1) = 1.\]

What does the linear system of equations look like to solve this if we set \( h = 1/N = 1/4 \)
4. How would you alter the Julia code below to solve the BVP with \( N = 20 \)?

```julia
N = 10
xgrid = collect(0:1/N:1)
alpha = 1
beta = 0.5
f = x -> -2.0
h = 1/N
A = zeros(N+1,N+1)
b = zeros(N+1,1)
for i=1:N+1
    if i == 1
        A[i,i] = 1
        b[i] = alpha
    elseif i==N+1
        A[i,i] = 1
        b[i] = beta
    else
        A[i,i-1] = 1/h^2
        A[i,i] = (-2)/h^2
        A[i,i+1] = 1/h^2
        b[i] = f(xgrid[i])
    end
end
u = A\b
plot(xgrid, u, marker=:circle)
```

5. Consider the BVP

\[
\frac{\partial}{\partial x} \left[ p(x) \frac{\partial u}{\partial x} \right] = -10.
\]

Let \( p(x) = 1 - 4(x - 1/2)^2 \), \( u(0) = -1 \), \( u(1) = 1 \). Write down the linear system for this function if \( N = 1/4 \).