Homework 5

Please answer the following questions in complete sentences in a typed manuscript and submit the solution to me in class on February 21th, 2012.

The previous homework was focused heavily on implementations and studying these methods in a practical setting. In this homework, we will return our focus back to theory.

Problem 0: List your collaborators.

Please identify anyone, whether or not they are in the class, with whom you discussed your homework.

Problem 1: Solving the trust region problem (Griva, Nash, and Sofer, 11.6.6)

In class, your professor mentioned that any solution to the trust region subproblem,

$$\begin{array}{lll} \text{minimize}_{\mathbf{p}} & f(\mathbf{x}_k) + g(\mathbf{x}_k)^T\mathbf{p} + 1/2\mathbf{p}^T\boldsymbol{H}(\mathbf{x}_k)\mathbf{p} \quad = \quad m_k(\mathbf{p}) \\ \text{subject to} & \|\mathbf{p}\| \le \Delta, \end{array}$$

could be characterized as follows:

- $(\boldsymbol{H}(\mathbf{x}_k) + \lambda\boldsymbol{I})\mathbf{p}^* = -\mathbf{g}$
- $\lambda(\|\mathbf{p}^*\| - \Delta) = 0$
- $\boldsymbol{H}(\mathbf{x}_k) + \lambda\boldsymbol{I} \succeq 0$ (it's positive semi-definite).

Use these conditions to show that, if $\lambda \neq 0$, then $\mathbf{p}$ solves:

$$\begin{array}{ll} \text{minimize}_{\mathbf{p}} & f(\mathbf{x}_k) + g(\mathbf{x}_k)^T\mathbf{p} + 1/2\mathbf{p}^T\boldsymbol{H}(\mathbf{x}_k)\mathbf{p} \\ \text{subject to} & \|\mathbf{p}\| = \Delta. \end{array}$$

Hint First show that $m_k(\mathbf{p}^*) \le m_k(\mathbf{p}) + 1/2\lambda(\mathbf{p}^T\mathbf{p} - \mathbf{p}^{*T}\mathbf{p}^*)$ for any $\mathbf{p}$.

Problem 2: Advantages and disadvantages

Is a dog-leg method or a subspace minimization method better when using a trust region method on a convex optimization problem? Discuss and explain why.

Problem 3: Read and prove

Read the proof of Lemma 4.2. There is one missing step. Let $\boldsymbol{B}$ be symmetric positive definite. Prove the inequality:

$$\frac{(\mathbf{g}^T\mathbf{g})^2}{\mathbf{g}^T\boldsymbol{B}\mathbf{g}\mathbf{g}^T\boldsymbol{B}^{-1}\mathbf{g}} \le 1.$$

(Hint: Use Cauchy-Schwarz!) Can you remove any of the conditions from this proof? When do you have equality?