HOMEWORK
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Homework 3

Please answer the following questions in complete sentences in a typed manuscript and submit the solution to me in class on February 7th, 2012.

Problem 1: Checking Hessians and matrix calculus

In class, we mentioned that it was vital to double-check gradient calculations, and we then used the `gradientcheck.m` function to do this. Of course, if we want to utilize Newton's method to solve a problem, then we need to check the Hessian matrix as well.

1. Implement `hessiancheck.m` to verify that the Hessian is correct. You may reuse the function `gradientcheck.m`
2. Determine *and verify* the Hessian of $f(\mathbf{x}) = e^{-\mathbf{x}^T \boldsymbol{A} \mathbf{x}}$ where $\boldsymbol{A}$ is not necessarily symmetric or positive definite. Show the result of `hessiancheck` for a set of test matrices $\boldsymbol{A}$ and test points $\mathbf{x}$. Be adversarial in your choice. Analyze any erratic behavior your find.
3. Determine *and verify* the Hessian of $f(\mathbf{x}) = \mathbf{e}^T \cos(\mathbf{x})$. Show the result of `hessiancheck` for a set of test matrices test points $\mathbf{x}$. Be adversarial in your choice. Analyze any erratic behavior your find.
4. Determine *and verify* the Hessian of $f(\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}) = \text{trace}(\begin{bmatrix} \mathbf{x} & \mathbf{y} \end{bmatrix} \begin{bmatrix} \mathbf{x}^T \\ \mathbf{y}^T \end{bmatrix})$. Show the result of `hessiancheck` for a set of test matrices test points $\mathbf{x}$. Be adversarial in your choice. Analyze any erratic behavior your find.

Problem 2: Accuracy of finite differences

In class, your professor mentioned that using a step size of the square root of machine precision ($\sqrt{2^{-52}} \approx 10^{-8}$) was a reasonable compromise when computing finite differences. The tension is between the error in the function evaluation and the error in the finite difference approximation.

1. For the function $f(x) = \frac{1}{2}\mathbf{x}^T\mathbf{x}$, show (i.e. use matlab to compute) how the error varies in a forward finite difference approximation of the *gradient* when $\mathbf{x} \in \mathbb{R}^{10}$ as the step-length approximation changes from $h \approx 2^{-52}$ to $h \approx 2^{-8}$. What do you observe?
2. Does this change if you switch to a centered finite difference formula?

Problem 3: Steepest descent

(Nocedal and Wright, Exercise 3.6) Let's conclude with a quick problem to show that steepest descent can converge very rapidly! Consider the steepest descent method with exact line search for the function $f(\mathbf{x}) = (1/2)\mathbf{x}^T\boldsymbol{Q}\mathbf{x} - \mathbf{x}^T\mathbf{b}$. Suppose that we know $\mathbf{x}_0 - \mathbf{x}^*$ is parallel to an eigenvector of $\boldsymbol{Q}$. Show that the method will converge in a single iteration.