Computational Geometry
CS 531

Plane Subdivision
- Edge & edge use
- Faces
- Winged edge & Brep
- Quad edge structure

Doubly-Connected Edge List
Boundary Representation
- Represent a planar subdivision by finite-length edges that are line segments
- Represent the surface of solid objects
Example

Composed of vertices, edges and faces.

- Vertex list with coordinates
- Faces as cyclic lists of vertex names
- Edge direction?
- Face adjacency?

Graphics .poly

Vertex list: coordinates (xy or xyz, possibly a normal)
Face polygons: incident vertices, ccw as seen from the “outside”

```
7
-1.2 0.334
0 1
1.19 0.34
0 0
-0.25 -0.8
0.287 -0.666
0.11 -1.34
5
1 4 2
2 4 3
6 5 7
6 4 5
5 4 1
```
Logically we might want to…

• Represent edge use, half edges, in addition to, or rather than, edges
• Find opposite half edge
• Link edge cycles around face
• Find face from half edge
• Find face adjacent from a (half) edge
• Iterate over edges incident to a vertex

Operational Needs

Incremental:
1. point in triangle, on edge
2. point in circum circle
3. edge added or deleted
4. triangle adjacent by edge
5. diagonal flip

Divide & Conquer:
1. point in circum circle
2. edge added or deleted
3. next incident edge cw, ccw
4. other vertex of edge
Winged Edge Representation

Common in early CAD systems, appropriate for (closed) manifold surfaces of any genus.

Per edge:
- Vertices, defining default orientation
- Left and right face
- Cw pred and succ
- Ccw pred and succ

Example

<table>
<thead>
<tr>
<th>edge</th>
<th>from vertex</th>
<th>to vertex</th>
<th>left face 1</th>
<th>right face 2</th>
<th>left pred f</th>
<th>right succ g</th>
<th>right pred k</th>
<th>right succ h</th>
</tr>
</thead>
</table>

Diagram:
- Pred left face
- Succ left face
- Pred right face
- Succ right face
Text

See p.32 for an example.

- Vertex: coordinates, name
- Half edge:
  - Origin vertex
  - Face delimited
  - Link to partner (twin)
  - Next and previous half edge in cycle
- Face:
  - Outer loop
  - Inner loops

All needed for (which) Delaunay?

Quad-Edge Data Structure

Appropriate for manifold structures only!

- Vertex: coordinates, one incident edge
- Edge: topology diagram
- Face: dual edge may be used
Example: Edge $1 \to 2$

- ccw around vertex 1
- ccw around vertex 2
- ccw around right face
- ccw around left face

Edge orientation arbitrary
Faces could be dual graph edges

Triangle Data Structure

Specifically for triangle meshes:
Triangle Data Structure

Specifically for triangle meshes:

Sources etc.

Search terms:
- quad edge cmu
- quad edge data structure

1) https://www.cs.cmu.edu/~quake/tripaper/triangle2.html
2) http://cstheory.stackexchange.com/questions/4746/the-quad-edge-data-structure-delaunay-voronoi
3) http://www.cs.rpi.edu/~cutler/classes/advancedgraphics/F05/lectures/03_Adjacency_Data_Structures.pdf

Check also the triangle data structure in