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CS54701: Information Retrieval

Link Analysis

16 February 2016

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Ad-Hoc Retrieval: Beyond the Words

- Web is a graph
  - Each web site correspond to a node
  - A link from one site to another site forms a directed edge

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Citation Analysis

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Ad-Hoc Retrieval: Beyond the Words

- Web is a graph
  - Each web site correspond to a node
  - A link from one site to another site forms a directed edge
- What does it look like?
  - Web is small world
  - The diameter of the web is 19
    - e.g. the average number of clicks from one web site to another is 19

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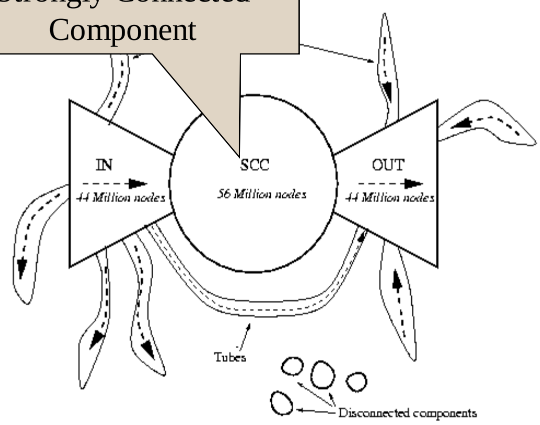
Bowtie Structure

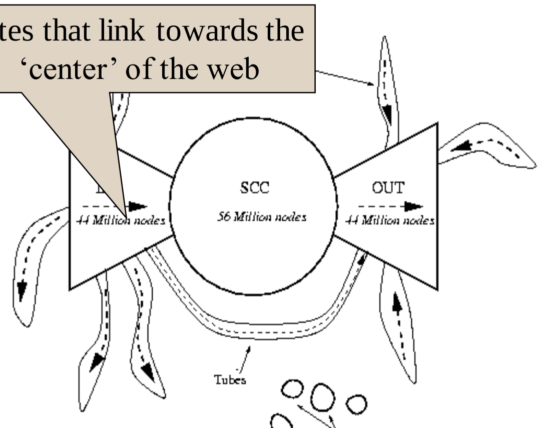
Strongly Connected Component



Broder et al., 2001

Bowtie Structure

Sites that link towards the 'center' of the web



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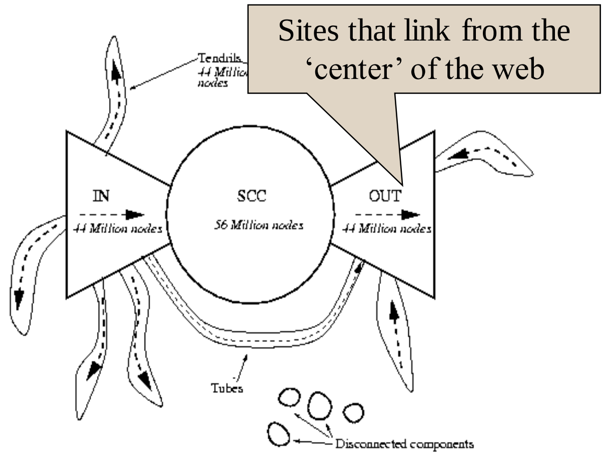
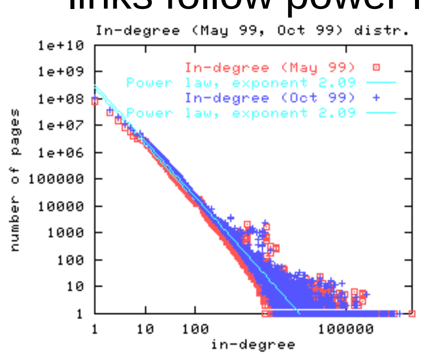
Bowtie Structure

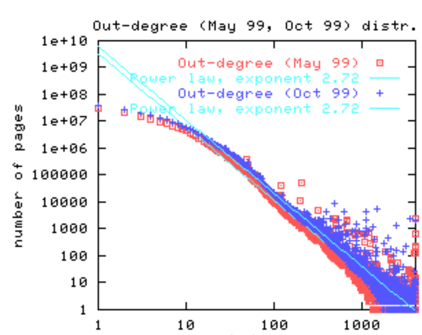


Broder et al., 2001

Inlinks and Outlinks

- Both degrees of incoming and outgoing links follow power law





Broder et al., 2001

Early Approaches

Basic Assumptions

- Hyperlinks contain information about the human judgment of a site
- The more incoming links to a site, the more it is judged important

Bray 1996

- The visibility of a site is measured by the number of other sites pointing to it
- The luminosity of a site is measured by the number of other sites to which it points

➢ Limitation: failure to capture the relative importance of different parents (children) sites

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HITS - Kleinberg's Algorithm

- HITS – Hypertext Induced Topic Selection
- For each vertex v ϵ V in a subgraph of interest:
  - a(v) - the authority of v
  - h(v) - the hubness of v
- A site is very authoritative if it receives many citations.
  - Citation from important sites weight more than citations from less-important sites
- Hubness shows the importance of a site.
  - A good hub is a site that links to many authoritative sites

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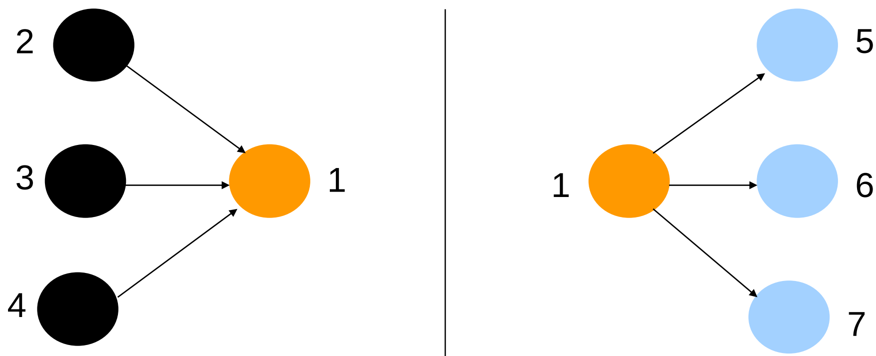
Authority and Hubness



$a(1) = h(2) + h(3) + h(4)$

$h(1) = a(5) + a(6) + a(7)$

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Authority and Hubness: Version 1

Recursive dependency

$$a(v) = \sum_{w \in pa[v]} h(w)$$

$$h(v) = \sum_{w \in ch[v]} a(w)$$

HubsAuthorities(G)

$\mathbf{1} \leftarrow [1,\ldots,1] \in R^{|V|}$

$a_0 \leftarrow h_0 \leftarrow \mathbf{1}$

$t \leftarrow 1$

repeat

for each v in V

do $a_t(v) \leftarrow \sum_{w \in pa[v]} h_{t-1}(w)$

$h_t(v) \leftarrow \sum_{w \in pa[v]} a_{t-1}(w)$

$t \leftarrow t + 1$

until $\|a_t - a_{t-1}\| + \|h_t - h_{t-1}\| < \varepsilon$

return (a_t, h_t)

Problems ?

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Authority and Hubness: Version 2

Recursive dependency

$$a(v) = \sum_{w \in pa[v]} h(w)$$

$$h(v) = \sum_{w \in ch[v]} a(w)$$

+ Normalization

$$a(v) = \frac{a(v)}{\sum_w a(w)}$$

$$h(v) = \frac{h(v)}{\sum_w h(w)}$$

HubsAuthorities(G)

```
1 ← [1,…,1] Є R^|V|
a_0 ← h_0 ← 1
t ← 1
repeat
    for each v in V
        do  a_t (v) ← Σ_{w Є pa[v]} h_{t-1} (w)
            h_t (v) ← Σ_{w Є pa[v]} a_{t-1} (w)
        a_t ← a_t / || a ||
        h_t ← h_t / || h ||
        t ← t + 1
until || a_t – a_{t-1} || + || h_t – h_{t-1} || < ε
return (a_t , h_t)
```

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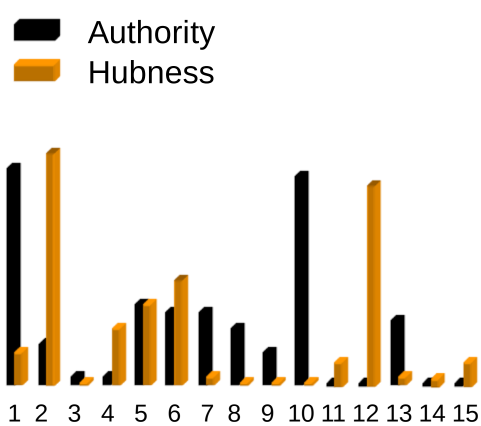
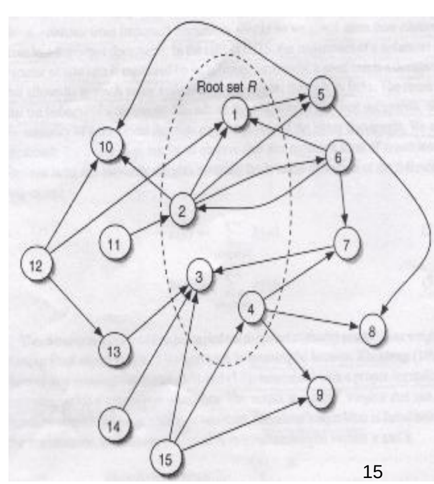
HITS Example Results



Authority and hubness weights



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Authority and Hubness

- Authority score
  - Not only depends on the number of incoming links
  - But also the ‘quality’ (e.g., hubness) of the incoming links
- Hubness score
  - Not only depends on the number of outgoing links
  - But also the ‘quality’ (e.g., hubness) of the outgoing links

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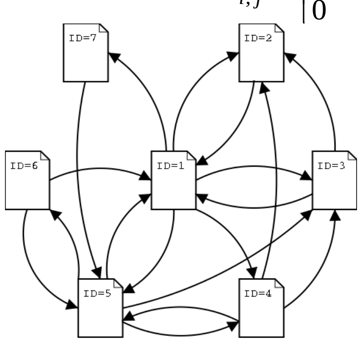
Authority and Hub

- Column vector ***a***: a_i is the authority score for the i-th site
- Column vector ***h***: h_i is the hub score for the i-th site

- Matrix **M**:

$$\mathbf{M}_{i,j} = \begin{cases} 1 & \text{the } i\text{th site points to the } j\text{th site} \\ 0 & \text{otherwise} \end{cases}$$



$$\mathbf{M} = \begin{pmatrix} 0&1&1&1&1&0&1 \\ 1&0&0&0&0&0&0 \\ 1&1&0&0&0&0&0 \\ 0&1&1&0&1&0&0 \\ 1&0&1&1&0&1&0 \\ 1&0&0&0&1&0&0 \\ 0&0&0&0&1&0&0 \end{pmatrix}$$

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Authority and Hub

- Vector ***a***: a_i is the authority score for the i-th site
- Vector ***h***: h_i is the hub score for the i-th site
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- Recursive dependency:

$$a(v) \leftarrow \Sigma_{w \in pa[v]} h(w)$$

$$h(v) \leftarrow \Sigma_{w \in ch[v]} a(w)$$

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$$h(v) \leftarrow \Sigma_{w \in ch[v]} a(w)$$

$$\Rightarrow \quad \mathbf{a} = \mathbf{M}^T\mathbf{h}, \quad \mathbf{h} = \mathbf{M}\mathbf{a}$$

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Authority and Hub

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- Recursive dependency:

a(v) ← Σ w Є pa[v] h(w)

h(v) ← Σ w Є ch[v] a(w)

Normalization Procedure

$$\mathbf{a}_t = \boxed{\alpha_t} \mathbf{M}^T \mathbf{h}_t$$
$$\mathbf{h}_t = \boxed{\beta_t} \mathbf{M}\mathbf{a}_t$$

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Authority and Hub

$$\left.\begin{aligned} \mathbf{a}_t &= \alpha_t \mathbf{M}^T \mathbf{h}_t \\ \mathbf{h}_t &= \beta_t \mathbf{M}\mathbf{a}_t \end{aligned}\right\} \rightarrow \begin{aligned} \mathbf{a}_t &= \alpha_t \beta_t \mathbf{M}^T \mathbf{M}\mathbf{a}_t \\ \mathbf{h}_t &= \alpha_t \beta_t \mathbf{M}\mathbf{M}^T \mathbf{h}_t \end{aligned}$$

- Apply SVD to matrix **M**

$$\mathbf{M} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T = \sum_i \lambda_i \mathbf{u}_i \mathbf{v}_i^T \longrightarrow \mathbf{a} = \mathbf{u}_1, \mathbf{h} = \mathbf{v}_1$$

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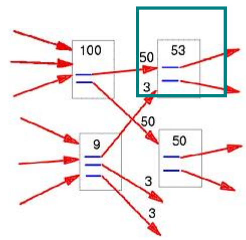
PageRank

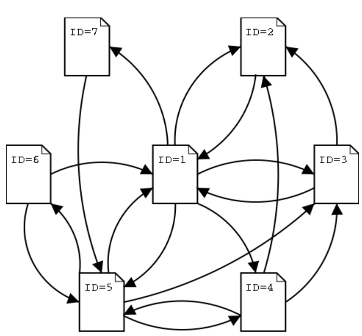
- Introduced by Page et al (1998)
  - The weight is assigned by the rank of parents
- Difference with HITS
  - HITS takes Hubness & Authority weights
  - The page rank is proportional to its parents' rank, but inversely proportional to its parents' outdegree



Matrix Notation

$$\mathbf{M}_{i,j} = \begin{cases} 1 & \text{the } i\text{th site points to the } j\text{th site} \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{B}_{i,j} = \begin{cases} \dfrac{1}{\sum_j \mathbf{M}_{i,j}} & \sum_j \mathbf{M}_{i,j} > 0 \\ 0 & \text{otherwise} \end{cases}$$



$$\mathbf{M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 1/5 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 1/3 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 0 & 1/4 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

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Matrix Notation

$\mathbf{r} : \mathbf{r}_i$ represents the rank score for the i-thweb page

$$r(v) = \alpha \sum_{w \in \mathrm{pa}[v]} \frac{r(w)}{|\mathrm{ch}[w]|'} \longrightarrow$$

r = α **B**ᵀ **r**

α : eigenvalue

r : eigenvector of **B**

Finding Pagerank

→ finding principle eigenvector of B

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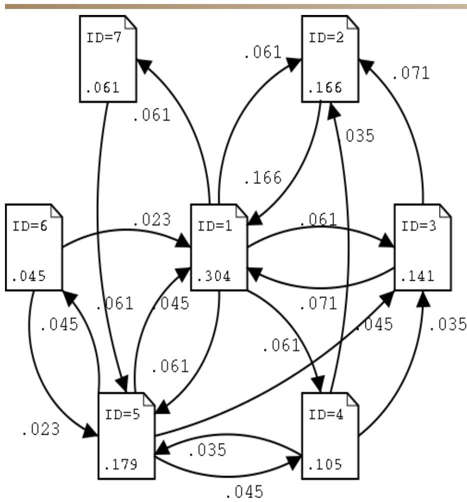
Matrix Notation



PR	ID	OutLink	InLink
0.304	1	2,3,4,5,7	2,3,5,6
0.179	5	1,3,4,6	1,4,6,7
0.166	2	1	1,3,4
0.141	3	1,2	1,4,5
0.105	4	2,3,5	1,5
0.061	7	5	1
0.045	6	1,5	5

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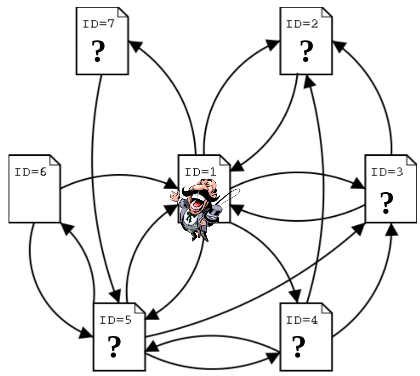
Random Walk Model

- Consider a random walk through the Web graph



$$B = \begin{pmatrix} 0 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 1/5 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 1/3 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 0 & 1/4 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

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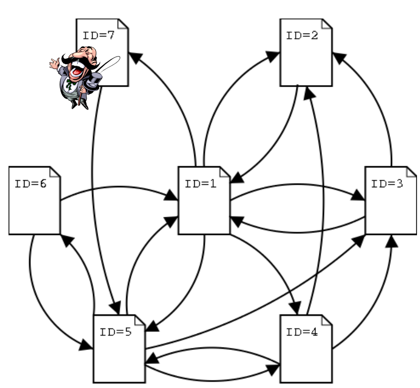

$$B = \begin{pmatrix} 0 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 1/5 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 1/3 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 0 & 1/4 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

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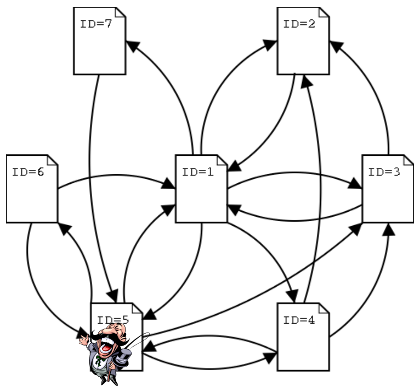

$$\mathbf{B} = \begin{pmatrix} 0 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 1/5 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 1/3 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 0 & 1/4 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

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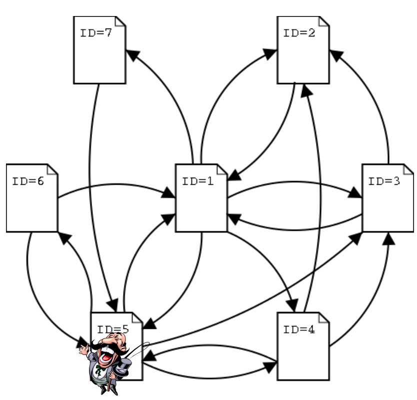
Random Walk Model

- Consider a random walk through the Web graph



$$\mathbf{B} = \begin{pmatrix} 0 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 1/5 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 1/3 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 0 & 1/4 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$T \rightarrow \infty$, what is portion of time that the surfer will spend time on each site?

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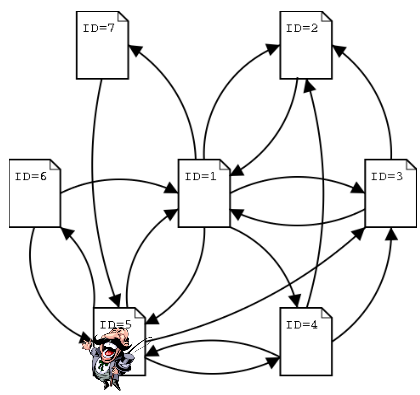
Random Walk Model

- Consider a random walk through the Web graph



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$p(k)$: percentage of time that the surfer will stay at the i-th site

$$p(k) = \sum_i p(i)\mathbf{B}_{i,k}$$

$$\mathbf{p} = \mathbf{B}^T \mathbf{p}$$

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Link Analysis

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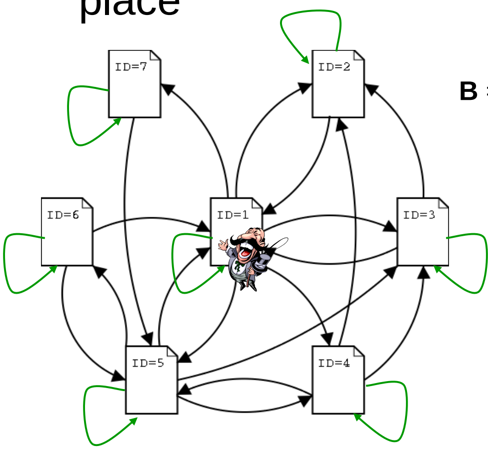
Adding Self Loops

- Allow surfer to decide to stay on the same place



$$\mathbf{B} = \begin{pmatrix} 0 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 1/5 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 1/3 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 0 & 1/4 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\mathbf{B}' = \alpha\mathbf{B} + (1-\alpha)\mathbf{I}$$

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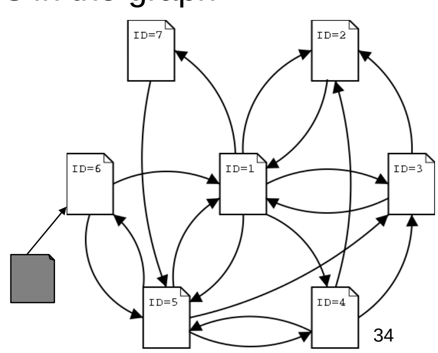
Problem

- "Rank Sink" Problem
  - In general, many Web pages have no inlinks/outlinks
  - Results in dangling edges in the graph

$$\mathbf{B} = \begin{array}{ccccccc|c} 0 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 1/5 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 1/3 & 0 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 0 & 1/4 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}$$



r(new page) = 0

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Problem

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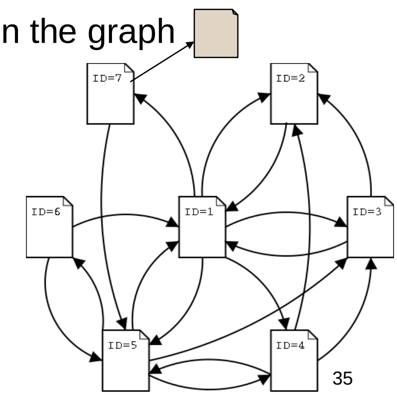
  - In general, many Web pages have no inlinks/outlinks
  - Results in dangling edges in the graph



$$\mathbf{B} = \begin{pmatrix} 0 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 1/5 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 1/3 & 0 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 0 & 1/4 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

r(new page) = 1

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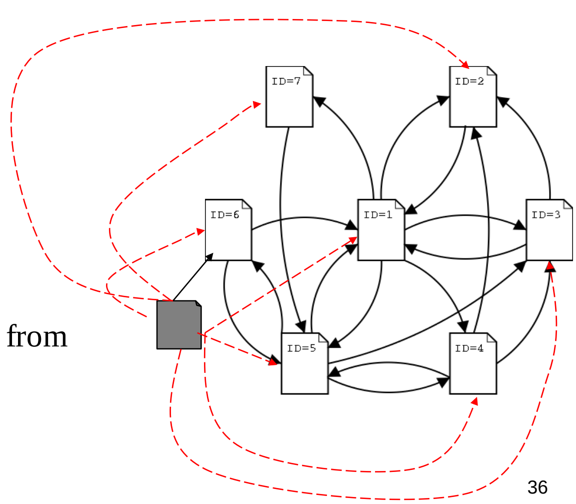
Distribution of the Mixture Model

$$\mathbf{H}_{i,j} = 1/n$$

$$\mathbf{B}' = \varepsilon\mathbf{H} + (1-\varepsilon)\mathbf{B}$$

$$\downarrow$$

$$\mathbf{r} = \mathbf{B}'^{T}\,\mathbf{r}$$



Prevents the page ranks from being 0 or 1

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Stability

- Are link analysis algorithms based on eigenvectors stable?
  - Will small changes in graph result in major changes in outcomes?
- What if the connectivity of a portion of the graph is changed arbitrarily?
  - How will this affect the results of algorithms?

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Stability of HITS

Ng et al (2001)

• A bound on the number of hyperlinks k that can be added or deleted from one page without affecting the authority or hubness weights

• It is possible to perturb a symmetric matrix by a quantity that grows as δ that produces a constant perturbation of the dominant eigenvector

$$k \le \left(\sqrt{d + \frac{\alpha\delta}{4 + \sqrt{2}\alpha}} - \sqrt{d} \right)^2$$

$$\|\boldsymbol{a} - \tilde{\boldsymbol{a}}\|_2 \le \alpha$$

δ: eigengap $\lambda_1 - \lambda_2$

d: maximum outdegree of G

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Stability of PageRank

$$\|\tilde{r} - r\| \le \frac{2\sum_{j\in \mathrm{V}} r(j)}{\epsilon}$$ Ng et al (2001)

V: the set of vertices touched by the perturbation

- The parameter **ε** of the mixture model has a stabilization role
- If the set of pages affected by the perturbation have a small rank, the overall change will also be small

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