Homework 3
CS526
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Problem 1

\( T = \{ doctor, nurse, patient, healthcarerecord \} \)
\( TS = \{ doctor, nurse, patient \} \)
\( TO = \{ healthcarerecord \} \)
\( RI = \{ r:c, w:c, publish:x \} \)
\( RC = \emptyset \)

\( link_1( doctor, nurse ) = \text{true} \)
\( f_1( doctor, nurse ) = \{ healthcarerecord \} \times \{ r:c, w:x \} \)
\( link_2( patient, nurse ) = \text{true} \)
\( f_2( patient, nurse ) = \{ healthcarerecord \} \times \{ publish \} \)
\( link_3( patient, doctor ) = \text{true} \)
\( f_3( patient, nurse ) = \{ healthcarerecord \} \times \{ publish \} \)

A ticket \( X/r : c \in dom(Y) \) can be copied from \( dom(Y) \) to \( dom(Z) \) iff, for some \( i \), the following are true:

1. \( X/r c \in dom(Y) \)
2. \( link_i(Y, Z) \)
3. \( \tau(X)/r : c \in f_i(\tau(Y), \tau(Z)) \)

Here, we can see that the subjects are \( TS \). There are no restrictions on when record transfers are allowed, or needing a specific right to do so, other than the implicit condition that we have the right to begin with. As such all of the link functions are inherently true. We then simply use the filter function to strip down which rights are allowed, and over which entities (healthcarerecord).

Problem 2

I denote the beginning and ending time of the right with \( X/z_{b,e} \), where \( b \) represents the beginning time, and \( e \) represents the ending time for the right \( z \) over \( X \), where \( b \leq e \) and \( 0 \) and \( \infty \) are acceptable values.

Link Function

From definition 3-13, we know that a link predicate \( link_i(X, Y) \) is a conjunction or disjunction of the following rules \( r \), for any \( z \in RC : \)

1. \( X/z \in dom(X) \)
2. \( X/z \in dom(Y) \)
3. \( Y/z \in dom(X) \)
4. \( Y/z \in \text{dom}(Y) \)

5. \textbf{true}

In order to extend this, first, I expand the function inputs to include a time input, \( \text{line}_{t_i}(X, Y, T) \). \( T \) represents the precise timing of the link predicate. I then modify the rules \( r \) to create \( r_{t_i} \) such that \( r_{t_i} = r \land b \leq C \leq e \). The new rules \( r_{t_i} \), for a \( z \in RC \) and given time \( C \) are:

1. \( X/z^c_b \in \text{dom}(X) \land b \leq C \leq e \)
2. \( X/z^c_b \in \text{dom}(Y) \land b \leq C \leq e \)
3. \( Y/z^c_b \in \text{dom}(X) \land b \leq C \leq e \)
4. \( Y/z^c_b \in \text{dom}(Y) \land b \leq C \leq e \)
5. \( b \leq T \leq e \)
6. \textbf{true}

Note the \( b \) and \( e \) found in rule 5. This is similar to rule 6, with the addition of being able to restrict the linkages to solely during specified times, without requiring an explicit right. Rule 6, as with the original \( r \), simply allows linkage between two subjects.

**Filter Function**

A filter function is a function \( f_{t_i}: TS \times TS \rightarrow 2^{T \times R} \), which has as its range the set of copyable tickets. With the time addition, our function will now take an additional parameter \( C \), while giving out additional results, the start and end times of rights that have permissions where \( b \leq C \leq e \). As such, the new filter function becomes \( f_{t_i}: TS \times TS \times C \rightarrow 2^{T \times R \times B \times E} \).

Previously, a filter function was defined in terms of \( \tilde{T} \) and \( \tilde{R} \), where \( \tilde{T} \) represented the \textit{types} over which the rights \( \tilde{R} \) were allowed to be transferred, per the link function \( \text{link}_{t_i} \). This is represented via the notation \( f_{t_i}(\tau(Y), \tau(Z)) = \tilde{T} \times \tilde{R} \). Now, however, \( \tilde{T} \times \tilde{R} \) has the additional \textit{time} constraint. We represent this as \( [\tilde{T} \times \tilde{R}]_{b}^{e} \), or the tickets for the rights \( \tilde{R} \) over the types \( \tilde{T} \) that begin at time \( b \) and end at time \( e \). Our filter function can now use these values and incorporate a constraint based on time \( C \):

\[
f_{t_i}(\tau(Y), \tau(Z), C) = \left\{ [\tilde{T} \times \tilde{R}]_{b}^{e} : b \leq C \leq e \right\}
\]

We can see that this formalism allows for choosing types \( \tilde{T} \) and rights \( \tilde{R} \), then constraining them so they are solely allowed if the time \( T_{t_i} \) is between the start and end time of the ticket.

**Copying Tickets**

From pg. 68, we know that SPM only allows transference of a ticket \( X/r:c \) from \( \text{dom}(Y) \) and \( \text{dom}(Z) \) if three conditions are met:

1. \( X/rc \in \text{dom}(Y) \)
2. \( \text{link}_{t_i}(Y, Z) \)
3. \( \tau(X)/r:c \in f_{t_i}(\tau(Y), \tau(Z)) \)

Now, we want to copy a right \( X/r:c \) from \( \text{dom}(Y) \) to \( \text{dom}(Z) \). Additionally, the time \( Y \) is able to have a right is not necessarily the time \( Z \) should be allowed to have access. For this, I denote the times that \( Y \) has access to a right \( b_{y} \) and \( e_{y} \), while \( Z \) wants access to the right at from time \( b_{z} \) to time \( e_{z} \). Here, I make an additional restriction - \( Y \) \textit{cannot} assign a right to \( Z \) for times which \( Y \) does not have the access. If this was not enforced, then \( Y \) could get timed access to a right that was not intended, through using \( Z \).
We can now tie this with the link function and filter function definitions. We say a ticket \( X/r:e_{by}^x \) that \( Y \) holds between \( b_y \) and \( e_y \) can be copied from \( \text{dom}(Y) \) to \( \text{dom}(Z) \) where it can be accessed by \( Z \) between times \( b_z \) and \( e_z \) (represented by \( X/r:e_{bz}^z \)) during a time \( C \) if, for some \( i \), the following are true:

1. \( X/r:e_{by}^x \in \text{dom}(Y) \land b_y \leq b_z \land e_y \geq e_z \)

2. \( \text{link}_{f_i}(Y, Z, C) \)

3. \( \tau(X)/r:e_{by}^x \in f_i(\tau(Y), \tau(Z), C) \)

Here, we have stated that a) we have the ticket, and the time we are transferring for is during the time we are allowed the ticket, b) we are allowed to link from \( Y \) to \( Z \) at time \( C \), and c) the right we are trying to copy is allowed to be copied at this specific instance in time. As such, we have effectively constrained SPM to only allow the transfer of tickets during allowed times.

**Creation**

Now, we need to be able to allow people to create items. In the original SPM, we simply declared:

\[
cc(a, b)
\]

and that implicitly allowed subjects of type \( a \) to create an entity of type \( b \). With the time-based model, we might want to only allow the creation of a new entity at a specific time. To deal with this, I use the approach the linkage uses, by defining rules which can be combined via conjunction or disjunction. So, for a function \( cc(a, b, C) \), where \( C \) is the time, we only have two rules:

1. \( b \leq C \leq e \)

2. \( \text{true} \)

\( b \) and \( e \) are times which can be set. Note that this allows us to join multiple times together, such as \( b_1 \leq C \leq e_1 \lor b_2 \leq C \leq e_2 \), meaning we can create during one time, or another.

Lastly, we have the rules for what permissions are added to both the parent and child when the child is created. In the non-temporal SPM, we would list a set of tickets that each would get via

\[
\begin{align*}
\text{cr}_p(a, b) &= \{ \tilde{R}a \} \\
\text{cr}_c(a, b) &= \{ \tilde{R}b \}
\end{align*}
\]

Now, however, we need to specify the times which rules are allowed. This results in the addition of the usual \( b \) and \( e \), which need to be specified by the person who writes the rules. The syntax for this would be:

\[
\begin{align*}
\text{cr}_p(a, b, C) &= \{ \tilde{R}a_b^e \} \\
\text{cr}_c(a, b, C) &= \{ \tilde{R}b_b^e \}
\end{align*}
\]

This specifies, for a particular time \( C \), what rules would we like to apply, meaning we have the flexibility to have different rule sets created at different times.

**Problem 3**

For this problem, I choose to use the HRU model. This involves the use of people being the subjects, and the individual problems, group problems, individual results and group results all being objects. In order to distinguish between the types of objects, the objects are given a right corresponding to its type. For instance if we take the Individual Problem 1, this corresponds to the column \( IP_1 \) in our ACM, and the right \( ip_1 \) is then present in every row under that column for the initial matrix \( M_0 \). In addition to the Individual Problem type,
we can also have Individual Response (for the student’s answer to individual problems), Group Problem and Group Response, with the corresponding rights ir, gp, and gr (with a subscript indicating which problem it is associated with).

In addition to the rights used to explicitly define available problems and solutions, we also have the rights t for talk, r for read and w for write. I allow the students to converse with the professor, but not with each other. Additionally, the professor is allowed to read and write each of the problems, but students are only allowed to read the problems. This is so we don’t have a student modifying what the question is. In order to get the students’ responses to the questions, we use the ir and gr rights, creating new objects (which contain the students answers). Overall, our set of rights is:

$$Rights = \{t, r, w, ip_1, ip_2, ip_3, ip_4, ip_5, gp_1, gp_2, ir_1, ir_2, ir_3, ir_4, ir_5, gp_1, gp_2, gr_1, gr_2\}$$

We can now look at Table 1, which is our initial access control matrix. We see that the the instructor is allowed to speak with any student, as they are allowed to speak with the instructor. Additionally, the instructor and read and write each of the problems, while the students are only given read access. Finally, we have given an ‘indicator’ right to each problem for each student, so we can be aware of which problem they are doing.

Next, we need to allow students to answer the individual problems. For this, we create 5 similar functions, which ensure the student is a) answering the correct problem and b) has not answered the problem previously. We do not want a student to be able to submit many answers to a single problem.

// Answering individual problem 1, we take the student, instructor, problem, and response
Command Answer_Ind_Problem_One(subject1, subject2, object1, object2)
if ir1 in a[subject1, subject2] // Have we already answered problem 1?
and t in a[subject1, subject2] // Is the other person the instructor?
and ip1 in a[subject1, object1] // Are we answering the right problem?
and r in a[subject1, object1] // Can we read the test?
then
  create object object2; // Create the answer
  enter r into a[subject1, object2]; // Give student permissions to the answer
  enter w into a[subject1, object2];
  enter r into a[subject2, object2]; // Give instructor permission to read the answer
  enter ir1 into a[subject1, object2]; // ‘Label’ the answer
  enter ir1 into a[subject2, object2]; // ‘Label’ the answer
  delete ir1 from a[subject1, subject1]; // Remove student’s permission to create a new answer
end

// Answering individual problem 2, we take the student, instructor, problem, and response
Command Answer_Ind_Problem_Two(subject1, subject2, object1, object2)
if ir₂ in a[subject₁, subject₁] // Have we already answered problem 2?
and t in a[subject₁, subject₂] // Is the other person the instructor?
and ip₂ in a[subject₁, object₁] // Are we answering the right problem?
and r in a[subject₁, object₁] // Can we read the test?
then
    create object object₂; // Create the answer
    enter r into a[subject₁, object₂]; // Give student permissions to the answer
    enter w into a[subject₁, object₂];
    enter r into a[subject₂, object₂]; // Give instructor permission to read the answer
    enter ir₂ into a[subject₁, object₂]; // ‘Label’ the answer
    enter ir₂ into a[subject₂, object₂]; // ‘Label’ the answer
    delete ir₂ from a[subject₁, subject₁]; // Remove student’s permission to create a new answer
end

// Answering individual problem 3, we take the student, instructor, problem, and response
Command Answer_Ind_Problem_Three(subject₁, subject₂, object₁, object₂)
if ir₃ in a[subject₁, subject₁] // Have we already answered problem 3?
and t in a[subject₁, subject₂] // Is the other person the instructor?
and ip₃ in a[subject₁, object₁] // Are we answering the right problem?
and r in a[subject₁, object₁] // Can we read the test?
then
    create object object₂; // Create the answer
    enter r into a[subject₁, object₂]; // Give student permissions to the answer
    enter w into a[subject₁, object₂];
    enter r into a[subject₂, object₂]; // Give instructor permission to read the answer
    enter ir₃ into a[subject₁, object₂]; // ‘Label’ the answer
    enter ir₃ into a[subject₂, object₂]; // ‘Label’ the answer
    delete ir₃ from a[subject₁, subject₁]; // Remove student’s permission to create a new answer
end

// Answering individual problem 4, we take the student, instructor, problem, and response
Command Answer_Ind_Problem_Four(subject₁, subject₂, object₁, object₂)
if ir₄ in a[subject₁, subject₁] // Have we already answered problem 4?
and t in a[subject₁, subject₂] // Is the other person the instructor?
and ip₄ in a[subject₁, object₁] // Are we answering the right problem?
and r in a[subject₁, object₁] // Can we read the test?
then
    create object object₂; // Create the answer
    enter r into a[subject₁, object₂]; // Give student permissions to the answer
    enter w into a[subject₁, object₂];
    enter r into a[subject₂, object₂]; // Give instructor permission to read the answer
    enter ir₄ into a[subject₁, object₂]; // ‘Label’ the answer
    enter ir₄ into a[subject₂, object₂]; // ‘Label’ the answer
    delete ir₄ from a[subject₁, subject₁]; // Remove student’s permission to create a new answer
end

// Answering individual problem 5, we take the student, instructor, problem, and response
Command Answer_Ind_Problem_Four(subject₁, subject₂, object₁, object₂)
if ir₅ in a[subject₁, subject₁] // Have we already answered problem 5?
and t in a[subject₁, subject₂] // Is the other person the instructor?
and ip₅ in a[subject₁, object₁] // Are we answering the right problem?
and r in a[subject_1, object_1] // Can we read the test?

then

create object object_2; // Create the answer
enter r into a[subject_1, object_2]; // Give student permissions to the answer
enter w into a[subject_1, object_2];
enter r into a[subject_2, object_2]; // Give instructor permission to read the answer
enter ir_5 into a[subject_1, object_2]; // 'Label' the answer
enter ir_5 into a[subject_2, object_2]; // 'Label' the answer
delete ir_5 from a[subject_1, subject_1]; // Remove student's permission to create a new answer

end

Walking through each of the individual functions, we can see that four comparisons happen. First, we check whether or not we have already answered the problem. Second, we check whether the other subject passed in was the instructor, leveraging the fact that only the instructor can talk to the student. Next, we check whether we are answering the correct problem (which won't affect the problem), and finally, we check whether or not we can actually read the problem (maybe the test has ended).

After these checks are done, the creation is straightforward, we just create the object, giving the student r/w permissions, as well as giving the instructor read permissions (for grading). We then remove the student's ability to create a new answer. This means that should we ever call this function again with the same parameters, it will fail, because the student has already answered the question. Note that the student is still free to modify the actual answer, they just cannot create a new one.

For the next section, we use similar logic to extend how many students are allowed to edit a problem.

// Answering group problem 1, we take the student, instructor, problem, and response
Command Answer_Grp_Problem_One(subject_1, subject_2, subject_3, object_1, object_2)
if gr_1 in a[subject_1, subject_1] // Has student 1 already answered group problem
and gr_1 in a[subject_2, subject_2] // Has student 2 already answered group problem
and t in a[subject_1, subject_3] // Is the other person the instructor
and t in a[subject_2, subject_3] // Is the other person the instructor
and gp_1 in a[subject_1, object_1] // Are we answering the right problem
and gp_1 in a[subject_2, object_1] // Are we answering the right problem
and r in a[subject_1, object_1] // Can student 1 read the test?
and r in a[subject_2, object_1] // Can student 2 read the test?
then

create object object_2; // Create the answer
enter r into a[subject_1, object_2]; // Give student 1 permissions to the answer
enter w into a[subject_1, object_2];
enter r into a[subject_2, object_2]; // Give student 2 permissions to the answer
enter w into a[subject_2, object_2];
enter r into a[subject_2, object_2]; // Give instructor permission to read the answer
enter gr_1 into a[subject_1, object_2]; // 'Label' the answer
enter gr_1 into a[subject_2, object_2]; // 'Label' the answer
enter gr_1 into a[subject_3, object_2]; // 'Label' the answer
delete gr_1 from a[subject_1, subject_1]; // Remove student 1's permission to create a new group answer
delete gr_1 from a[subject_1, subject_1]; // Remove student 1's permission to create a new answer (for group problem 2)
delete gr_1 from a[subject_2, subject_2]; // Remove student 2's permission to create a new group answer
delete gr_2 from a[subject_2, subject_2]; // Remove student 2's permission to create a new answer (for group problem 2)
end

// Answering group problem 2, we take the student, instructor, problem, and response
Two problems after this (either 1 or 2). So, if they are ever called to pair with another student, it will be
possible for them to create one result for each individual problem.

Problem creation, so only one will ever be created. r/w rights on the answer are given to the student, while
creation prevents removes the right that allowed the creation, so only one will ever be created. r/w rights on the answer are given to the student, while

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Table 2: Our ACM after student 1 has created an answer for problem 1, and after student 1 and student 2

Command: `Answer_Grp_Problem_Two(subject₁, subject₂, subject₃, object₁, object₂)`
if gr₂ in [subject₁, subject₂] // Has student 1 already answered group problem
and gr₂ in [subject₂, subject₂] // Has student 2 already answered group problem
and t in [subject₁, subject₃] // Is the other person the instructor
and t in [subject₂, subject₃] // Is the other person the instructor
and gp₂ in [subject₁, object₁] // Are we answering the right problem
and gp₂ in [subject₂, object₁] // Are we answering the right problem
and r in [subject₁, object₁] // Can student 1 read the test?
and r in [subject₂, object₁] // Can student 2 read the test?
then
create object object₂; // Create the answer
enter r into [subject₁, object₂]; // Give student 1 permissions to the answer
enter w into [subject₁, object₂];
enter r into [subject₂, object₂]; // Give student 2 permissions to the answer
enter w into [subject₂, object₂];
enter r into [subject₂, object₂]; // Give instructor permission to read the answer
enter gr₂ into [subject₁, object₂]; // ‘Label’ the answer
enter gr₂ into [subject₂, object₂]; // ‘Label’ the answer
enter gr₂ into [subject₃, object₂]; // ‘Label’ the answer
delete gr₁ from [subject₁, subject₁]; // Remove student 1’s permission to create a new answer (for
problem 1)
delete gr₂ from [subject₁, subject₁]; // Remove student 1’s permission to create a new answer gr₂
delete gr₁ from [subject₂, subject₂]; // Remove student 2’s permission to create a new answer (for
problem 1)
delete gr₂ from [subject₂, subject₂]; // Remove student 2’s permission to create a new answer gr₂
end

Looking at these group problems, we can see that (again) there is a list of requirements. Both students
must have not previously answered a group problem, both students must have the instructor available for
communication, both students must be answering the correct problem, and both students must be able to
read the test. After these requirements are met, the answer is created, with both students being able to r/w
the answer, while the instructor can read it. Additionally, the students are not allowed to answer any more
group problems after this (either 1 or 2). So, if they are ever called to pair with another student, it will be
denied.

We can see that the usage of these rules, along with the initial starting matrix, allows each student to
create one result for each individual problem. Creation of an answer prevents removes the right that allowed
the creation, so only one will ever be created. r/w rights on the answer are given to the student, while
the instructor gets read rights. Additionally, for the group problem, a student can participate in one group problem, which results in permissions being taken away from both, disallowing them to work with each another student. We note that at no point does $t$ change, so we know that no one can cheat by speaking with another person.

In Table 2, we see the effects after student 1 creates an assignment response for problem 1, then teams with student 2 to answer group problem 2. We see that student 1 has lost $ir_1$ in $a[S_1, S_1]$, while both have lost $gr_2$. Two additional objects have been created, which is their group answer.

I chose the HRU model for its flexibility in removing a right. Specifically, I was able to define a function which deleted the right as part of the function. This restricted any student from creating multiple copies, and from working with multiple students. The SPM does not allow this deletion – once a right is given, it is assumed to be there forever. SPM also requires the link function to be in terms of concatenation of rights present via conjunction or disjunction, not is explicitly forbidden, so we can not add a right that restricts the actions. The Extended SPM allows for mutual creation, but HRU also allows this. Additionally, TG does not have the kind of restrictive capabilities necessary – once a grant is given, it cannot be taken away, and students can grant permission to read their problems to anyone else. While it is true that TG can have the ‘overload’, or instructor, grant permissions to students to share a homework (see the buffer example) this implies that the instructor chooses who is working with who, and on what project. With HRU, students answering the group problems lies in their domain, rather than being granted access.