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CS47300: Web Information Search and Management

Using Graph Structure for Retrieval

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Material adapted from slides created by Dr. Rong Jin (formerly Michigan State, now at Alibaba)

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Ad-Hoc Retrieval: Beyond the Words

- Web is a graph
  - Each web site correspond to a node
  - A link from one site to another site forms a directed edge

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Citation Analysis

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Ad-Hoc Retrieval: Beyond the Words

- Web is a graph
  - Each web site correspond to a node
  - A link from one site to another site forms a directed edge
- What does it look like?
  - Web is small world
  - The diameter of the web is 19
    - e.g. the average number of clicks from one web site to another is 19

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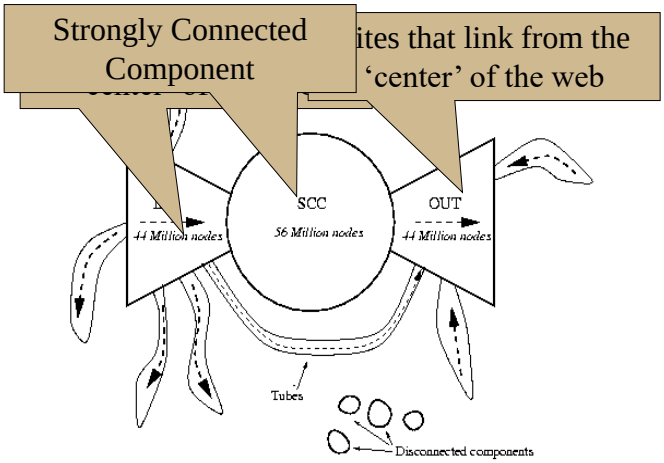
Bowtie Structure



Broder et al., 2001

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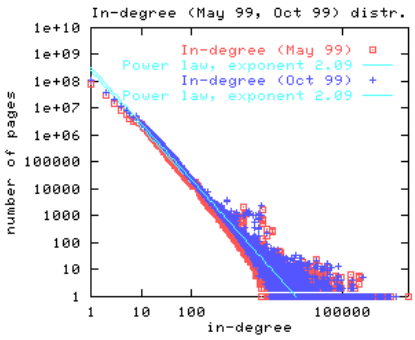
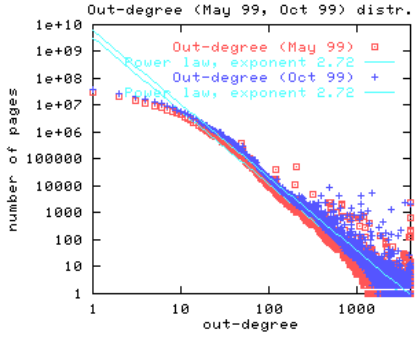
Inlinks and Outlinks

- Both degrees of incoming and outgoing links follow power law





Broder et al., 2001

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So what good is Link Structure?

- When you can't find something, do you:
  - Keep looking in the same place?
  - Look somewhere else?
  - Give up?
  - Ask for help?
- *Other people may already know the answer!*
  - Links: Reflect human judgement

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Early Approaches

- Basic Assumptions
  - Hyperlinks contain information about the human judgment of a site
  - The more incoming links to a site, the more it is judged important
- Bray 1996
  - The visibility of a site is measured by the number of other sites pointing to it
  - The luminosity of a site is measured by the number of other sites to which it points
  - Limitation: failure to capture the relative importance of different parents (children) sites

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HITS - Kleinberg's Algorithm

- HITS – Hypertext Induced Topic Selection
- For each vertex v ϵ V in a subgraph of interest:
  - a(v) - the authority of v
  - h(v) - the hubness of v
- A site is very authoritative if it receives many citations.
  - Citation from important sites weight more than citations from less-important sites
- Hubness shows the importance of a site.
  - A good hub is a site that links to many authoritative sites

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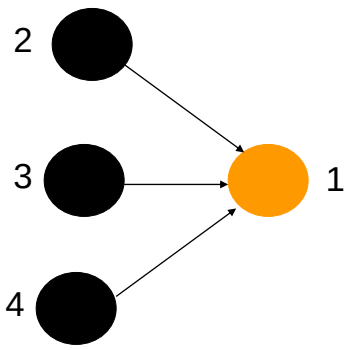
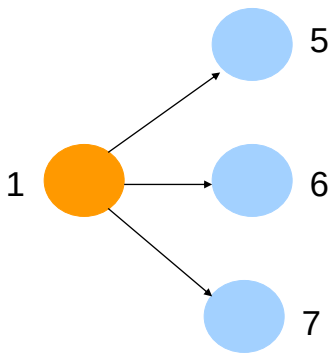
Authority and Hubness



a(1) = h(2) + h(3) + h(4)



h(1) = a(5) + a(6) + a(7)

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Authority and Hubness: Version 1

Recursive dependency

$$a(v) = \sum_{w \in pa[v]} h(w)$$

$$h(v) = \sum_{w \in ch[v]} a(w)$$

HubsAuthorities(G)

1 **1** ← [1,…,1] Є $R^{|V|}$
2 a_0 ← h_0 ← **1**
3 t ← 1
4 repeat
5 for each v in V
6 do $a_t(v)$ ← $\Sigma_{w \in pa[v]} h_{t-1}(w)$
7 $h_t(v)$ ← $\Sigma_{w \in pa[v]} a_{t-1}(w)$
8 t ← t + 1
9 until $\| a_t - a_{t-1} \| + \| h_t - h_{t-1} \| < \varepsilon$
10 return (a_t, h_t)

Problems ?

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Authority and Hubness: Version 2

Recursive dependency

$$a(v) = \sum_{w \in pa[v]} h(w)$$

$$h(v) = \sum_{w \in ch[v]} a(w)$$

+ Normalization

$$a(v) = \frac{a(v)}{\sum_w a(w)}$$

$$h(v) = \frac{h(v)}{\sum_w h(w)}$$

HubsAuthorities(G)

1 **1** ← [1,…,1] Є $R^{|V|}$
2 a_0 ← h_0 ← **1**
3 t ← 1
4 repeat
5 for each v in V
6 do $a_t(v)$ ← $\Sigma_{w \in pa[v]} h_{t-1}(w)$
7 $h_t(v)$ ← $\Sigma_{w \in pa[v]} a_{t-1}(w)$
8 a_t ← a_t / || a ||
9 h_t ← h_t / || h ||
10 t ← t + 1
11 until $\| a_t - a_{t-1} \| + \| h_t - h_{t-1} \| < \varepsilon$
12 return (a_t, h_t)

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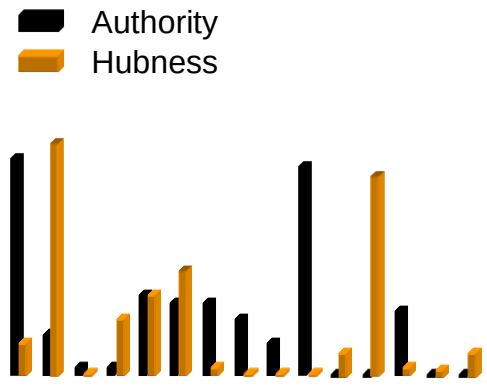
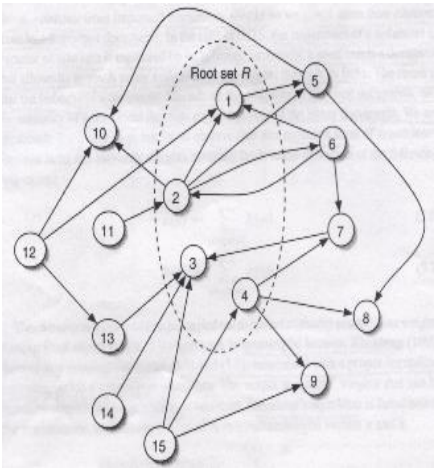
HITS Example Results



Authority and hubness weights



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Authority and Hubness

- Authority score
  - Not only depends on the number of incoming links
  - But also the 'quality' (e.g., hubness) of the incoming links
- Hubness score
  - Not only depends on the number of outgoing links
  - But also the 'quality' (e.g., hubness) of the outgoing links

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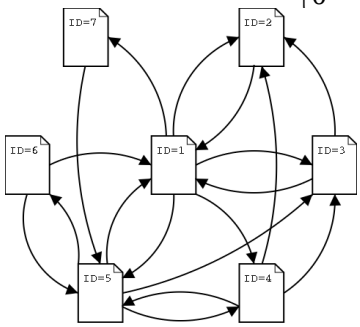
Authority and Hub

- Column vector ***a***: a_i is the authority score for the i-th site
- Column vector ***h***: h_i is the hub score for the i-th site

- Matrix **M**:

$$\mathbf{M}_{i,j} = \begin{cases} 1 & \text{the } i\text{th site points to the } j\text{th site} \\ 0 & \text{otherwise} \end{cases}$$



$$\mathbf{M} = \begin{pmatrix} 0&1&1&1&1&0&1 \\ 1&0&0&0&0&0&0 \\ 1&1&0&0&0&0&0 \\ 0&1&1&0&1&0&0 \\ 1&0&1&1&0&1&0 \\ 1&0&0&0&1&0&0 \\ 0&0&0&0&1&0&0 \end{pmatrix}$$

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$$\Rightarrow \quad \mathbf{a} = \mathbf{M}^T\mathbf{h}, \quad \mathbf{h} = \mathbf{M}\mathbf{a}$$

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Authority and Hub

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$$\Rightarrow \quad \mathbf{a}_t = \alpha_t \mathbf{M}^T\mathbf{h}_t, \quad \mathbf{h}_t = \beta_t \mathbf{M}\mathbf{a}_t$$

Normalization Procedure

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Authority and Hub

$$\left.\begin{aligned}\mathbf{a}_t &= \alpha_t \mathbf{M}^T \mathbf{h}_t \\ \mathbf{h}_t &= \beta_t \mathbf{M}\mathbf{a}_t\end{aligned}\right\} \rightarrow \begin{aligned}\mathbf{a}_t &= \alpha_t \beta_t \mathbf{M}^T \mathbf{M}\mathbf{a}_t \\ \mathbf{h}_t &= \alpha_t \beta_t \mathbf{M}\mathbf{M}^T \mathbf{h}_t\end{aligned}$$

- Apply SVD to matrix **M**

$$\mathbf{M} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T = \sum_i \lambda_i \mathbf{u}_i \mathbf{v}_i^T \longrightarrow \mathbf{a} = \mathbf{u}_1, \mathbf{h} = \mathbf{v}_1$$

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