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CS47300: Web Information Search and Management

Using Graph Structure for Retrieval

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Material adapted from slides created by Dr. Rong Jin (formerly Michigan State, now at Alibaba)

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Ad-Hoc Retrieval: Beyond the Words

- Web is a graph
  - Each web site correspond to a node
  - A link from one site to another site forms a directed edge

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Citation Analysis

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Ad-Hoc Retrieval: Beyond the Words

- Web is a graph
  - Each web site correspond to a node
  - A link from one site to another site forms a directed edge
- What does it look like?
  - Web is small world
  - The diameter of the web is 19
    - e.g. the average number of clicks from one web site to another is 19

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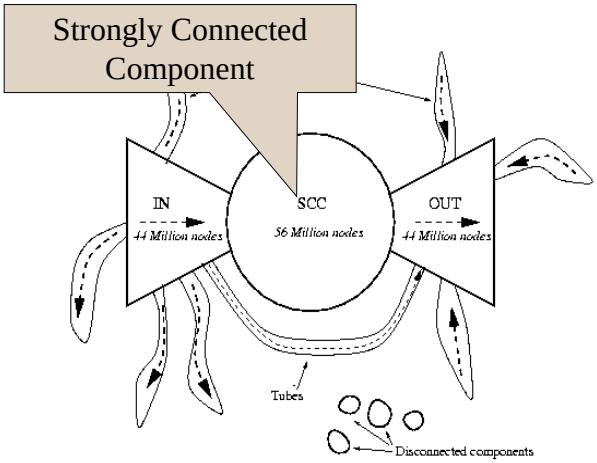
Bowtie Structure



Broder et al., 2001

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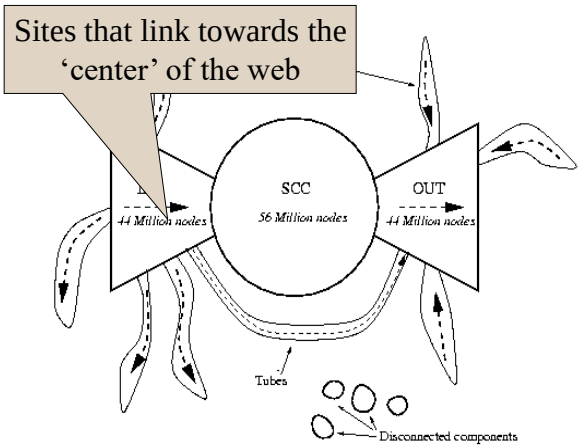
Bowtie Structure



Broder et al., 2001

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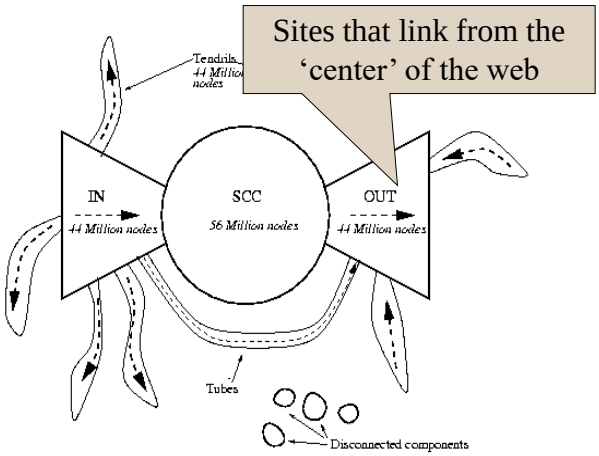
Bowtie Structure



Broder et al., 2001

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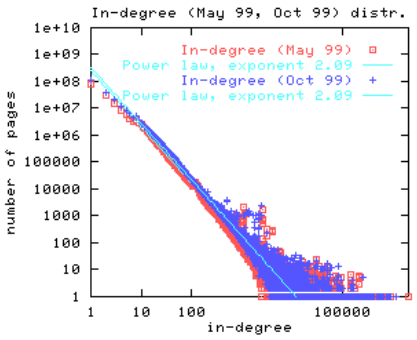
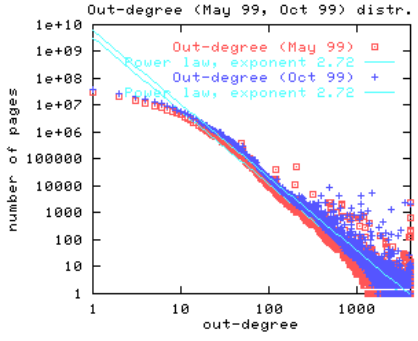
Inlinks and Outlinks

- Both degrees of incoming and outgoing links follow power law





Broder et al., 2001

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So what good is Link Structure?

- When you can't find something, do you:
  - Keep looking in the same place?
  - Look somewhere else?
  - Give up?
  - Ask for help?
- *Other people may already know the answer!*
  - Links: Reflect human judgement

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Early Approaches

- Basic Assumptions
  - Hyperlinks contain information about the human judgment of a site
  - The more incoming links to a site, the more it is judged important
- Bray 1996
  - The visibility of a site is measured by the number of other sites pointing to it
  - The luminosity of a site is measured by the number of other sites to which it points
  - Limitation: failure to capture the relative importance of different parents (children) sites

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HITS - Kleinberg's Algorithm

- HITS – Hypertext Induced Topic Selection
- For each vertex v ϵ V in a subgraph of interest:
  - a(v) - the authority of v
  - h(v) - the hubness of v
- A site is very authoritative if it receives many citations.
  - Citation from important sites weight more than citations from less-important sites
- Hubness shows the importance of a site.
  - A good hub is a site that links to many authoritative sites

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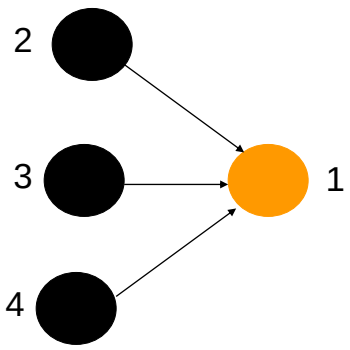
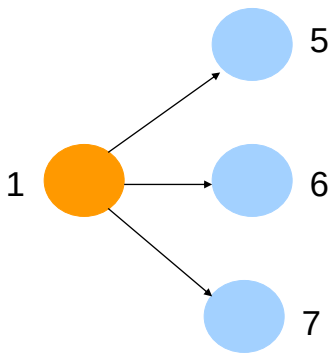
Authority and Hubness



a(1) = h(2) + h(3) + h(4)



h(1) = a(5) + a(6) + a(7)

Authority and Hubness: Version 1

Recursive dependency

$$a(v) = \sum_{w \in pa[v]} h(w)$$

$$h(v) = \sum_{w \in ch[v]} a(w)$$

HubsAuthorities(G)

```
1   1  ← [1,…,1] Є R^|V|
2   a_0 ← h_0 ← 1
3   t  ← 1
4   repeat
5         for each v in V
6         do a_t (v) ←   Σ_{w Є pa[v]} h_{t-1} (w)
7            h_t (v) ←   Σ_{w Є pa[v]} a_{t-1} (w)
8   t  ← t + 1
9   until || a_t – a_{t-1} || + || h_t – h_{t-1} || < ε
10  return (a_t , h_t)
```

Problems ?

Authority and Hubness: Version 2

Recursive dependency

$$a(v) = \sum_{w \in pa[v]} h(w)$$

$$h(v) = \sum_{w \in ch[v]} a(w)$$

+ Normalization

$$a(v) = \frac{a(v)}{\sum_w a(w)}$$

$$h(v) = \frac{h(v)}{\sum_w h(w)}$$

HubsAuthorities(G)

```
1   1  ← [1,…,1] Є R^|V|
2   a_0 ← h_0 ← 1
3   t  ← 1
4   repeat
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8         a_t ← a_t / || a ||
9         h_t ← h_t / || h ||
10        t  ← t + 1
11  until || a_t – a_{t-1} || + || h_t – h_{t-1} || < ε
12  return (a_t , h_t)
```

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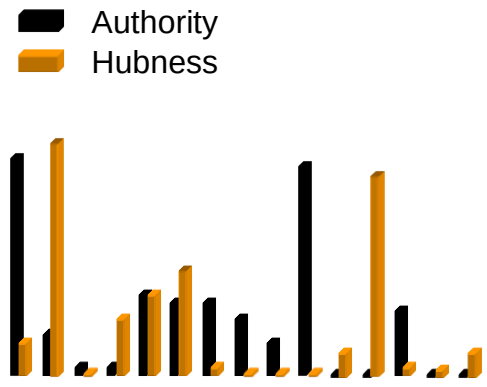
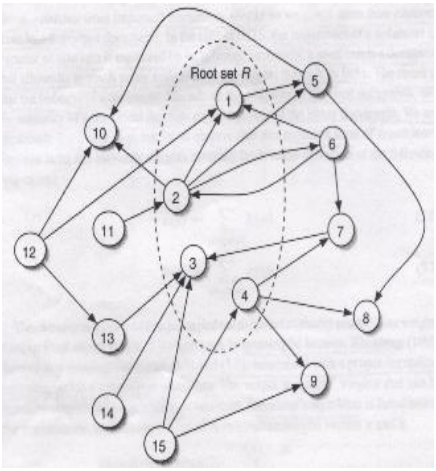
HITS Example Results



Authority and hubness weights



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Authority and Hubness

- Authority score
  - Not only depends on the number of incoming links
  - But also the 'quality' (e.g., hubness) of the incoming links
- Hubness score
  - Not only depends on the number of outgoing links
  - But also the 'quality' (e.g., hubness) of the outgoing links

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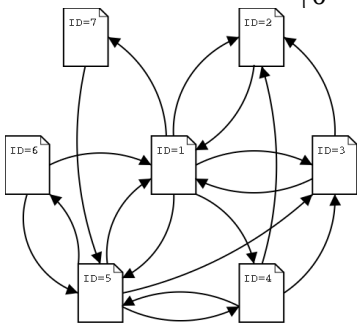
Authority and Hub

- Column vector ***a***: a_i is the authority score for the i-th site
- Column vector ***h***: h_i is the hub score for the i-th site

- Matrix **M**:

$$\mathbf{M}_{i,j} = \begin{cases} 1 & \text{the } i\text{th site points to the } j\text{th site} \\ 0 & \text{otherwise} \end{cases}$$



$$\mathbf{M} = \begin{pmatrix} 0&1&1&1&1&0&1 \\ 1&0&0&0&0&0&0 \\ 1&1&0&0&0&0&0 \\ 0&1&1&0&1&0&0 \\ 1&0&1&1&0&1&0 \\ 1&0&0&0&1&0&0 \\ 0&0&0&0&1&0&0 \end{pmatrix}$$

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- Vector ***a***: a_i is the authority score for the i-th site
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- Recursive dependency:

$$a(v) \leftarrow \Sigma_{w \in pa[v]}\ h(w)$$

$$h(v) \leftarrow \Sigma_{w \in ch[v]}\ a(w)$$

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$$\Rightarrow \quad \mathbf{a} = \mathbf{M}^T\mathbf{h}, \quad \mathbf{h} = \mathbf{M}\mathbf{a}$$

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Authority and Hub

- Column vector ***a***: a_i is the authority score for the i-th site
- Column vector ***h***: h_i is the hub score for the i-th site

- Matrix **M**:

$$\mathbf{M}_{i,j} = \begin{cases} 1 & \text{the } i\text{th site points to the } j\text{th site} \\ 0 & \text{otherwise} \end{cases}$$

Normalization Procedure

- Recursive dependency:

$$a(v) \leftarrow \Sigma_{w \in pa[v]}\ h(w)$$
$$h(v) \leftarrow \Sigma_{w \in ch[v]}\ a(w)$$

$$\Rightarrow \quad \mathbf{a}_t = \alpha_t \mathbf{M}^T\mathbf{h}_t, \quad \mathbf{h}_t = \beta_t \mathbf{M}\mathbf{a}_t$$

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Authority and Hub

$$\left.\begin{aligned}\mathbf{a}_t &= \alpha_t \mathbf{M}^T \mathbf{h}_t \\ \mathbf{h}_t &= \beta_t \mathbf{M} \mathbf{a}_t\end{aligned}\right\} \rightarrow \begin{aligned}\mathbf{a}_t &= \alpha_t \beta_t \mathbf{M}^T \mathbf{M} \mathbf{a}_t \\ \mathbf{h}_t &= \alpha_t \beta_t \mathbf{M} \mathbf{M}^T \mathbf{h}_t\end{aligned}$$

- Apply SVD to matrix **M**

$$\mathbf{M} = \mathbf{U}\Sigma\mathbf{V}^T = \sum_i \lambda_i \mathbf{u}_i \mathbf{v}_i^T \longrightarrow \mathbf{a} = \mathbf{u}_1, \mathbf{h} = \mathbf{v}_1$$

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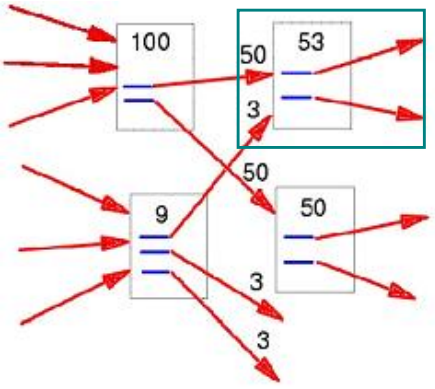
PageRank

- Introduced by Page et al. (1998)
  - The weight is assigned by the rank of parents
- Difference from HITS
  - HITS separates Hubness & Authority weights
  - Page rank is proportional to its parents' rank, but inversely proportional to its parents' outdegree



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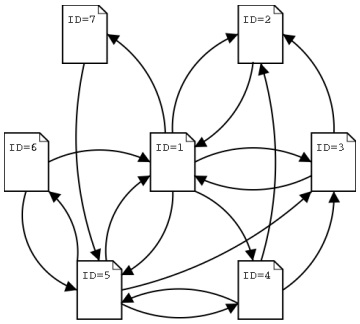
Matrix Notation

$$\mathbf{M}_{i,j} = \begin{cases} 1 & \text{the } i\text{th site points to the } j\text{th site} \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{B}_{i,j} = \begin{cases} \dfrac{1}{\sum_j \mathbf{M}_{i,j}} & \sum_j \mathbf{M}_{i,j} > 0 \\ 0 & \text{otherwise} \end{cases}$$



$$\mathbf{M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 1/5 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 1/3 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 0 & 1/4 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

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Matrix Notation

$\mathbf{r} : \mathbf{r}_i$ represents the rank score for the i-th web page

$$r(v) = \alpha \sum_{w \in \mathrm{pa}[v]} \frac{r(w)}{|\mathrm{ch}[w]|'}$$

$\mathbf{r} = \alpha\, \mathbf{B}^{\mathsf{T}}\, \mathbf{r}$

α : eigenvalue

r : eigenvector of **B**

Finding Pagerank

→ find principle eigenvector of B

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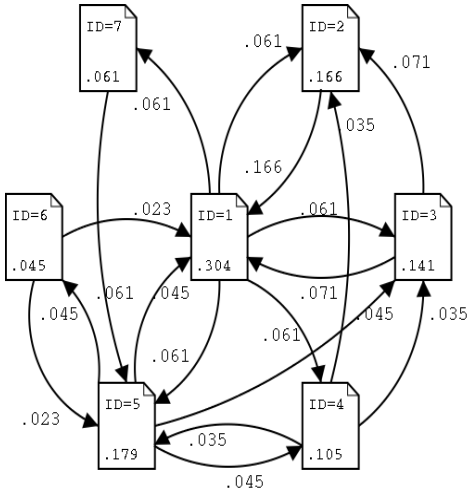
Matrix Notation



PR	ID	OutLink	InLink
0.304	1	2,3,4,5,7	2,3,5,6
0.179	5	1,3,4,6	1,4,6,7
0.166	2	1	1,3,4
0.141	3	1,2	1,4,5
0.105	4	2,3,5	1,5
0.061	7	5	1
0.045	6	1,5	5

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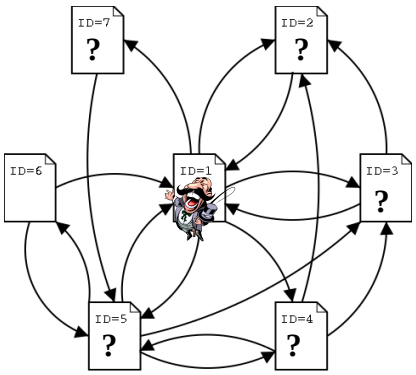
Random Walk Model

- Consider a random walk through the Web graph



$$B = \begin{pmatrix} 0 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 1/5 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 1/3 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 0 & 1/4 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

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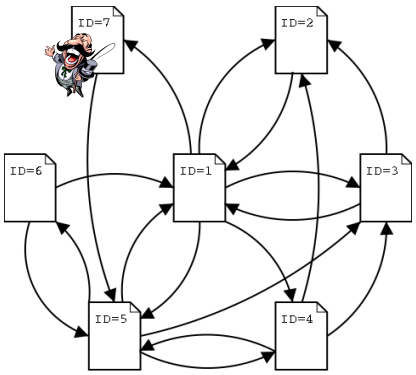
Random Walk Model

- Consider a random walk through the Web graph



B =

$$\begin{pmatrix} 0 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 1/5 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 1/3 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 0 & 1/4 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

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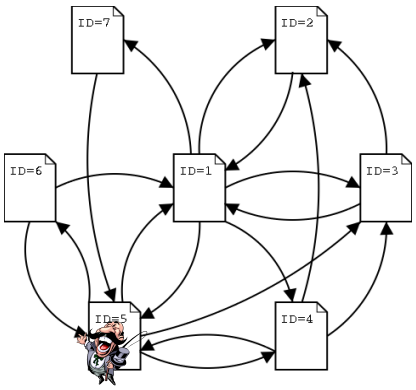
- Consider a random walk through the Web graph



B =

$$\begin{pmatrix} 0 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 1/5 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 1/3 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 0 & 1/4 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

As T→∞, what is portion of time that the web surfer will spend at each page?

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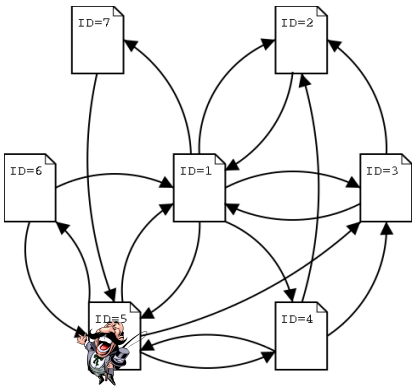
Random Walk Model

- Consider a random walk through the Web graph



$$\mathbf{B} = \begin{pmatrix} 0 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 1/5 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 1/3 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 0 & 1/4 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$p(k)$: percentage of time that the surfer will stay at the i-th site

$$p(k) = \sum_i p(i)\mathbf{B}_{i,k}$$

$$\mathbf{p} = \mathbf{B}^T\mathbf{p}$$

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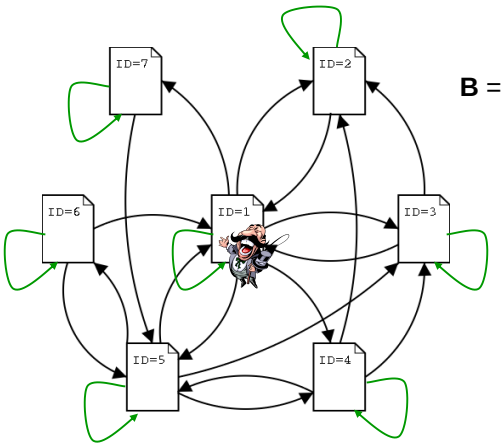
Adding Self Loop

- Allow surfer to decide to stay on the same place



$$\mathbf{B} = \begin{pmatrix} 0 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 1/5 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 1/3 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 0 & 1/4 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

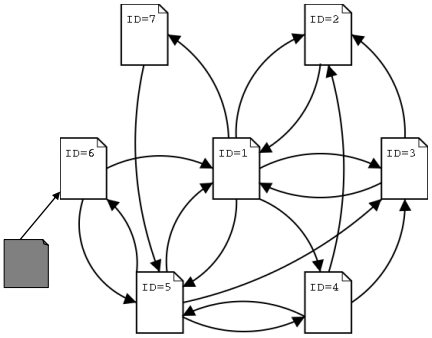
$$\mathbf{B}' = \alpha\mathbf{B} + (1-\alpha)\mathbf{I}$$

Problem

- "Rank Sink" Problem
  - Many Web pages have no inlinks
  - Results in dangling edges in the graph

B =

0	1/5	1/5	1/5	1/5	0	1/5	0
1	0	0	0	0	0	0	0
1/2	1/2	0	0	0	0	0	0
0	1/3	1/3	0	1/3	0	0	0
1/4	0	1/4	1/4	0	1/4	0	0
1/2	0	0	0	1/2	0	0	0
0	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0



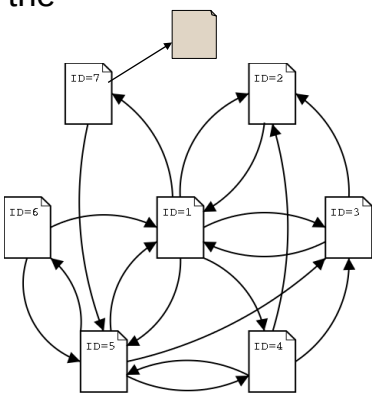
r(new page) = 0

Problem

- "Rank Sink" Problem
  - Many Web pages have no outlinks
  - Results in dangling edges in the graph

B =

0	1/5	1/5	1/5	1/5	0	1/5	0
1	0	0	0	0	0	0	0
1/2	1/2	0	0	0	0	0	0
0	1/3	1/3	0	1/3	0	0	0
1/4	0	1/4	1/4	0	1/4	0	0
1/2	0	0	0	1/2	0	0	0
0	0	0	0	1	0	0	1
0	0	0	0	0	0	0	0



r(new page) = 1

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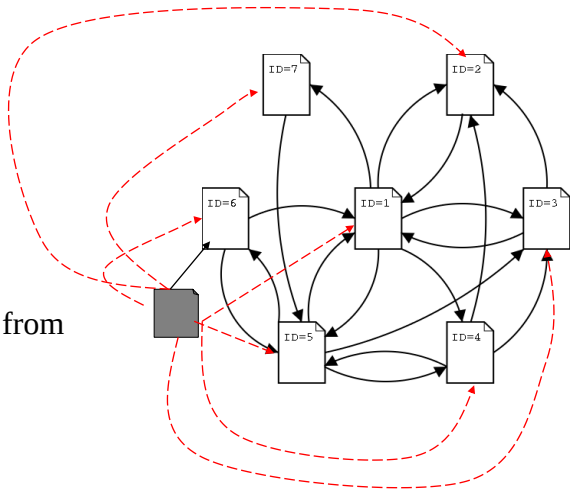
Distribution of the Mixture Model

$$\mathbf{H}_{i,j} = 1/n$$

$$\mathbf{B}' = \varepsilon\mathbf{H} + (1-\varepsilon)\mathbf{B}$$

$$\mathbf{r} = \mathbf{B}'^{T}\,\mathbf{r}$$

Prevents the page ranks from being 0 or 1



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Stability

- Are link analysis algorithms based on eigenvectors stable?
  - Will small changes in graph result in major changes in outcomes?
- What if the connectivity of a portion of the graph is changed arbitrarily?
  - How will this affect the results of algorithms?

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Stability of HITS

Ng et al (2001)

• A bound on the number of hyperlinks k that can be added or deleted from one page without affecting the authority or hubness weights

• It is possible to perturb a symmetric matrix by a quantity that grows as δ that produces a constant perturbation of the dominant eigenvector

$$k \leq \left(\sqrt{d + \frac{\alpha\delta}{4 + \sqrt{2}\alpha}} - \sqrt{d} \right)^2$$

$$\|\boldsymbol{a} - \tilde{\boldsymbol{a}}\|_2 \leq \alpha$$

δ: eigengap λ1 – λ2

d: maximum outdegree of G

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Stability of PageRank

$$\|\tilde{r} - r\| \leq \frac{2\sum_{j \in V} r(j)}{\epsilon}$$ Ng et al (2001)

V: the set of vertices touched by the perturbation

- The parameter ε of the mixture model has a stabilization role
- If the set of pages affected by the perturbation have a small rank, the overall change will also be small