Query Optimization
Prof. Chris Clifton
26 October 2021

Outline - Query Processing

• Relational algebra level
  – transformations
  – good transformations
• Detailed query plan level
  – estimate costs
  – generate and compare plans
Estimating result size

- Keep statistics for relation R
  - \( T(R) \) : # tuples in R
  - \( S(R) \) : # of bytes in each R tuple
  - \( B(R) \) : # of blocks to hold all R tuples
  - \( V(R, A) \) : # distinct values in R for attribute A

Example Table Statistics

<table>
<thead>
<tr>
<th>Example</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>cat</td>
<td>1</td>
<td>10</td>
<td>a</td>
</tr>
<tr>
<td></td>
<td>cat</td>
<td>1</td>
<td>20</td>
<td>b</td>
</tr>
<tr>
<td></td>
<td>dog</td>
<td>1</td>
<td>30</td>
<td>a</td>
</tr>
<tr>
<td></td>
<td>dog</td>
<td>1</td>
<td>40</td>
<td>c</td>
</tr>
<tr>
<td></td>
<td>bat</td>
<td>1</td>
<td>50</td>
<td>d</td>
</tr>
</tbody>
</table>

- \( T(R) = 5 \)
- \( S(R) = 37 \)
- \( V(R, A) = 3 \)
- \( V(R, C) = 5 \)
- \( V(R, B) = 1 \)
- \( V(R, D) = 4 \)

A: 20 byte string
B: 4 byte integer
C: 8 byte date
D: 5 byte string
Size estimates for $W = R_1 \times R_2$

- $T(W) = T(R_1) \times T(R_2)$
- $S(W) = S(R_1) + S(R_2)$

Size estimate for $W = \sigma$

- $S(W) = S(R)$
- $T(W) = ?$
Example: Select

Example

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>V(R,A)</th>
<th>V(R,B)</th>
<th>V(R,C)</th>
<th>V(R,D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cat</td>
<td>1</td>
<td>10</td>
<td>a</td>
<td></td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>cat</td>
<td>1</td>
<td>20</td>
<td>b</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

\[ W = \sigma_{z=\text{val}(R)} \quad T(W) = \frac{T(R)}{V(R,Z)} \]

Assumption:

- Values in select expression \( Z = \text{val} \) are uniformly distributed over possible \( V(R,Z) \) values?
- Values in select expression \( Z = \text{val} \) are uniformly distributed over domain with \( \text{DOM}(R,Z) \) values?
Example

Alternate assumption

\begin{align*}
V(R,A) &= 3 \quad \text{DOM}(R,A) = 10 \\
V(R,B) &= 1 \quad \text{DOM}(R,B) = 10 \\
V(R,C) &= 5 \quad \text{DOM}(R,C) = 10 \\
V(R,D) &= 4 \quad \text{DOM}(R,D) = 10 
\end{align*}

\[
W = \sigma_{z=\text{val}(R)} T(W) = ?
\]

\[
C=\text{val} \implies T(W) = (1/10)1 + (1/10)1 + ... = (5/10) = 0.5
\]

\[
B=\text{val} \implies T(W) = (1/10)5 + 0 + 0 = 0.5
\]

\[
A=\text{val} \implies T(W) = (1/10)2 + (1/10)2 + (1/10)1 = 0.5
\]
Alternate assumption

\[
V(R,A) = 3 \quad \text{DOM}(R,A) = 10 \\
V(R,B) = 1 \quad \text{DOM}(R,B) = 10 \\
V(R,C) = 5 \quad \text{DOM}(R,C) = 10 \\
V(R,D) = 4 \quad \text{DOM}(R,D) = 10
\]

\[
W = \sigma_{z=\text{val}(R)}(R) \quad T(W) = \frac{T(R)}{\text{DOM}(R,Z)}
\]

Selection cardinality

\[
\text{SC}(R,A) = \text{average \# records that satisfy equality condition on } R.A
\]

\[
\text{SC}(R,A) = \begin{cases} 
V(R,A) & T(R) \\
\frac{T(R)}{\text{DOM}(R,A)} & \text{DOM}(R,A)
\end{cases}
\]
What about $W = \sigma_{z \geq \text{val}}(R)$?

$T(W) = ?$

- Solution #1:
  
  $T(W) = T(R)/2$

- Solution #2:
  
  $T(W) = T(R)/3$

Selection: Range Estimates

- Solution #3: Estimate values in range

Example

<table>
<thead>
<tr>
<th>Z</th>
<th>V(R,Z) = 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min=1</td>
<td></td>
</tr>
<tr>
<td>Max=20</td>
<td></td>
</tr>
</tbody>
</table>

$f = \frac{20 - 15 + 1}{20 - 1 + 1} = \frac{6}{20} = 0.3$ (fraction of range)

$T(W) = f \times T(R)$
Equivalently:
\[ f \times V(R,Z) = \text{fraction of distinct values} \]
\[ T(W) = \left[ f \times V(Z,R) \right] \times T(R) = \frac{f \times T(R)}{V(Z,R)} \]

Estimating result size

- Keep statistics for relation R
  - \( T(R) \): # tuples in R
  - \( S(R) \): # of bytes in each R tuple
  - \( B(R) \): # of blocks to hold all R tuples
  - \( V(R, A) \): # distinct values in R for attribute A
Size estimate for \( W = R_1 \bowtie R_2 \)

- Let \( x \) = attributes of \( R_1 \)
- \( y \) = attributes of \( R_2 \)

**Case 1**

\[ X \cap Y = \emptyset \]

\( S(W) \) same as \( R_1 \times R_2 \)

\( T(W) ? \)

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**Case 2**

\[ W = R_1 \bowtie R_2 \quad X \cap Y = A \]

<table>
<thead>
<tr>
<th>R1</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
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<tr>
<td></td>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>A</td>
<td>D</td>
<td></td>
</tr>
</tbody>
</table>

**Assumption:**

\( V(R_1,A) \leq V(R_2,A) \) ⇒ Every A value in \( R_1 \) is in \( R_2 \)
\( V(R_2,A) \leq V(R_1,A) \) ⇒ Every A value in \( R_2 \) is in \( R_1 \)

“containment of value sets”
Computing $T(W)$ when $V(R1,A) \leq V(R2,A)$

1 tuple matches with $\frac{T(R2)}{V(R2,A)}$ tuples...

so $T(W) = \frac{T(R2) \times T(R1)}{V(R2, A)}$

$V(R1,A) \leq V(R2,A)$  $T(W) = \frac{T(R2) \times T(R1)}{V(R2,A)}$

$V(R2,A) \leq V(R1,A)$  $T(W) = \frac{T(R2) \times T(R1)}{V(R1,A)}$

[A is common attribute]
In general \[ W = R_1 \Join R_2 \]

\[
T(W) = \frac{T(R_2) \cdot T(R_1)}{\max\{ V(R_1,A), V(R_2,A) \}}
\]

Case 2 with alternate assumption

Values uniformly distributed over domain

This tuple matches \( T(R_2)/\text{DOM}(R_2,A) \) so

\[
T(W) = \frac{T(R_2) \cdot T(R_1)}{\text{DOM}(R_2, A)} \cdot \frac{T(R_2) \cdot T(R_1)}{\text{DOM}(R_1, A)}
\]

Assume the same
Join: Tuple width

- In all cases:

\[ S(W) = S(R1) + S(R2) - S(A) \]

size of attribute A

---

Size Estimation for Other Operations

- Projection: estimated size of \( \Pi_A(r) = V(A, r) \)
- Aggregation: estimated size of \( g^\gamma_A(r) = V(G, r) \)
- Set operations
  - For unions/intersections of selections on the same relation: rewrite and use size estimate for selections
    - E.g., \( \sigma_{01}(r) \cup \sigma_{02}(r) \) can be rewritten as \( \sigma_{01 \text{ or } 02}(r) \)
  - For operations on different relations:
    - estimated size of \( r \cup s = \text{size of } r + \text{size of } s \)
    - estimated size of \( r \cap s = \text{minimum size of } r \text{ and size of } s \)
    - estimated size of \( r - s = r \)
  - All the three estimates may be quite inaccurate, but provide upper bounds on the sizes.
Note: for complex expressions, need intermediate T,S,V results.

E.g. \( W = [\sigma_{A=a} (R1)] \bowtie R2 \)

Treat as relation U

\( T(U) = T(R1)/V(R1,A) \quad S(U) = S(R1) \)

Also need \( V(U, *) \) !!

To estimate \( V_s \)

E.g., \( U = \sigma_{A=a} (R1) \)

Say R1 has attrs A,B,C,D

\( V(U, A) = \)
\( V(U, B) = \)
\( V(U, C) = \)
\( V(U, D) = \)
Example

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<td>50</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

\[ V(R1,A) = 3 \]
\[ V(R1,B) = 1 \]
\[ V(R1,C) = 5 \]
\[ V(R1,D) = 3 \]

\[ U = \sigma_{A=a}(R1) \]

\[ V(U,A) = 1 \quad V(U,B) = 1 \quad V(U,C) = \frac{T(R1)}{V(R1,A)} \]

\[ V(D,U) \ldots \text{somewhere in between} \]

Possible Guess \( U = \sigma_{A=a}(R) \)

\[ V(U,A) = 1 \]
\[ V(U,B) = V(R,B) \]
For Joins \[ U = R_1(A,B) \bowtie R_2(A,C) \]

\[
\begin{align*}
V(U,A) &= \min\{V(R_1, A), V(R_2, A)\} \\
V(U,B) &= V(R_1, B) \\
V(U,C) &= V(R_2, C)
\end{align*}
\]

Example:

\[ Z = R_1(A,B) \bowtie R_2(B,C) \bowtie R_3(C,D) \]

| \( R_1 \)   | \( T(R_1) = 1000 \) | \( V(R_1,A)=50 \) | \( V(R_1,B)=100 \) |
| \( R_2 \)   | \( T(R_2) = 2000 \) | \( V(R_2,B)=200 \) | \( V(R_2,C)=300 \) |
| \( R_3 \)   | \( T(R_3) = 3000 \) | \( V(R_3,C)=90 \)  | \( V(R_3,D)=500 \) |
Partial Result: \( U = R \Join S \)

\[
\begin{align*}
T(U) &= \frac{1000 \times 2000}{200} = 50 \\
V(U, A) &= 50 \\
V(U, B) &= 100 \\
V(U, C) &= 300
\end{align*}
\]

\[
\begin{align*}
T(Z) &= \frac{1000 \times 2000 \times 3000}{200 \times 300} = 50 \\
V(Z, A) &= 50 \\
V(Z, B) &= 100 \\
V(Z, C) &= 90 \\
V(Z, D) &= 500
\end{align*}
\]

\( Z = U \Join R3 \)
Estimation of Number of Distinct Values

Selections: $\sigma_\theta (r)$
- If $\theta$ forces $A$ to take a specified value: $V(A, \sigma_\theta (r)) = 1$.
  - e.g., $A = 3$
- If $\theta$ forces $A$ to take on one of a specified set of values: $V(A, \sigma_\theta (r)) = \text{number of specified values}$.
  - (e.g., $(A = 1 \ V A = 3 \ V A = 4)$),
- If the selection condition $\theta$ is of the form $A \text{ op } r$ estimated $V(A, \sigma_\theta (r)) = V(A,r) \ast s$
  - where $s$ is the selectivity of the selection.
- In all the other cases: use approximate estimate of $\min(V(A,r), n_{\sigma_\theta (r)})$
  - More accurate estimates can be made using probability theory, but this one works fine generally

Estimation of Distinct Values (Cont.)

Joins: $r \bowtie s$
- If all attributes in $A$ are from $r$ estimated $V(A, r \bowtie s) = \min(V(A,r), n_{r \bowtie s})$
- If $A$ contains attributes $A1$ from $r$ and $A2$ from $s$, then estimated $V(A,r \bowtie s) = \min(V(A1,r) \ast V(A2 - A1,s), V(A1 - A2,r) \ast V(A2,s), n_{r \bowtie s})$
  - More accurate estimates can be made using probability theory, but this one works fine generally
Statistical Information for Cost Estimation

- $n_r$: number of tuples in a relation $r$.
- $b_r$: number of blocks containing tuples of $r$.
  - $l_r$: size of a tuple of $r$.
  - $f_r$: blocking factor of $r$ — i.e., the number of tuples of $r$ that fit into one block.
- $V(A, r)$: number of distinct values that appear in $r$ for attribute $A$; same as the size of $\prod_A(r)$.
- If tuples of $r$ are stored together physically in a file, then:
  $$b_r = \left\lfloor \frac{n_r}{f_r} \right\rfloor$$

More detailed $V(A, r)$ statistics: Histograms

- Histogram on attribute age of relation person

  - Equi-width histograms
  - Equi-depth histograms break up range such that each range has (approximately) the same number of tuples
    - E.g. $(4, 8, 14, 19)$
  - Many databases also store $n$ most-frequent values and their counts
    - Histogram is built on remaining values only