Department of Computer Science

CS 44800: Introduction To Relational Database Systems

Indexing

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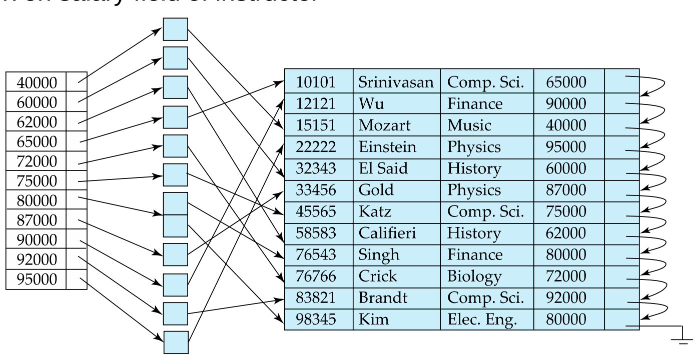
Secondary Indices

- Secondary index on salary field of instructor



- Index record points to a bucket that contains pointers to all the actual records with that particular search-key value.
- Secondary indices have to be dense

Indices on Multiple Keys

- **Composite search key**
  - E.g., index on *instructor* relation on attributes (*name, ID*)
  - Values are sorted lexicographically
    - E.g. (John, 12121) < (John, 13514) and
      (John, 13514) < (Peter, 11223)
  - Can query on just *name*, or on (*name, ID*)

Index Update: Deletion

10101
32343
76766

10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	60000
33456	Gold	Physics	87000
45565	Katz	Comp. Sci.	75000
58583	Califieri	History	62000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

- If deleted record was the only record in the file with its particular search-key value, the search-key is deleted from the index also.
- **Single-level index entry deletion:**
  - **Dense indices** – deletion of search-key is similar to file record deletion.
  - **Sparse indices** –
    - if an entry for the search key exists in the index, it is deleted by replacing the entry in the index with the next search-key value in the file (in search-key order).
    - If the next search-key value already has an index entry, the entry is deleted instead of being replaced.

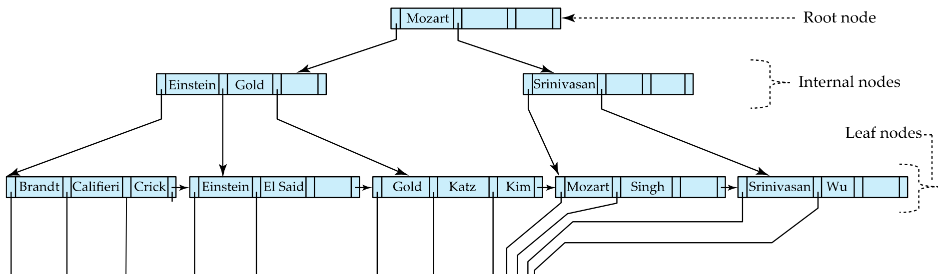
Index Update: Insertion

- **Single-level index insertion:**
  - Perform a lookup using the search-key value of the record to be inserted.
  - **Dense indices** – if the search-key value does not appear in the index, insert it
    - Indices are maintained as sequential files
    - Need to create space for new entry, overflow blocks may be required
  - **Sparse indices** – if index stores an entry for each block of the file, no change needs to be made to the index unless a new block is created.
    - If a new block is created, the first search-key value appearing in the new block is inserted into the index.
- **Multilevel insertion and deletion:** algorithms are simple extensions of the single-level algorithms

B+-Tree Index Files

- Disadvantage of indexed-sequential files
  - Performance degrades as file grows, since many overflow blocks get created.
  - Periodic reorganization of entire file is required.
- Advantage of B+-tree index files:
  - Automatically reorganizes itself with small, local, changes, in the face of insertions and deletions.
  - Reorganization of entire file is not required to maintain performance.
- (Minor) disadvantage of B+-trees:
  - Extra insertion and deletion overhead, space overhead.
- Advantages of B+-trees outweigh disadvantages
  - B+-trees are used extensively

Example of B+-Tree



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B+-Tree Index Files (Cont.)

A B+-tree is a rooted tree satisfying the following properties:

- All paths from root to leaf are of the same length
- Each node that is not a root or a leaf has between $\lceil n/2 \rceil$ and n children.
- A leaf node has between $\lceil (n-1)/2 \rceil$ and $n-1$ values
- Special cases:
  - If the root is not a leaf, it has at least 2 children.
  - If the root is a leaf (that is, there are no other nodes in the tree), it can have between 0 and (n–1) values.

B⁺-Tree Node Structure

- Typical node

P_1	K_1	P_2	…	P_{n-1}	K_{n-1}	P_n

  - K_i are the search-key values
  - P_i are pointers to children (for non-leaf nodes) or pointers to records or buckets of records (for leaf nodes).
- The search-keys in a node are ordered

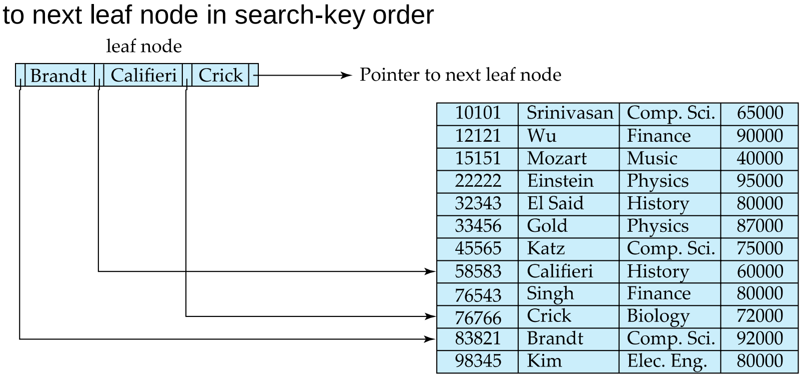
$$K_1 < K_2 < K_3 < . . . < K_{n-1}$$

(Initially assume no duplicate keys, address duplicates later)

Leaf Nodes in B⁺-Trees

Properties of a leaf node:

- For i = 1, 2, . . ., n–1, pointer P_i points to a file record with search-key value K_i,
- If L_i, L_j are leaf nodes and $i < j$, L_i's search-key values are less than or equal to L_j's search-key values
- P_n points to next leaf node in search-key order



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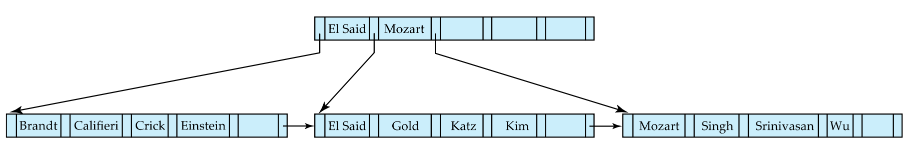
Non-Leaf Nodes in B^+-Trees

- Non leaf nodes form a multi-level sparse index on the leaf nodes. For a non-leaf node with *m* pointers:
  - All the search-keys in the subtree to which P_1 points are less than K_1
  - For $2 \leq i \leq n - 1$, all the search-keys in the subtree to which P_i points have values greater than or equal to K_{i-1} and less than K_i
  - All the search-keys in the subtree to which P_n points have values greater than or equal to K_{n-1}
  - General structure

P_1	K_1	P_2	…	P_{n-1}	K_{n-1}	P_n

Example of B^+-tree

- B^+-tree for *instructor* file ($n = 6$)

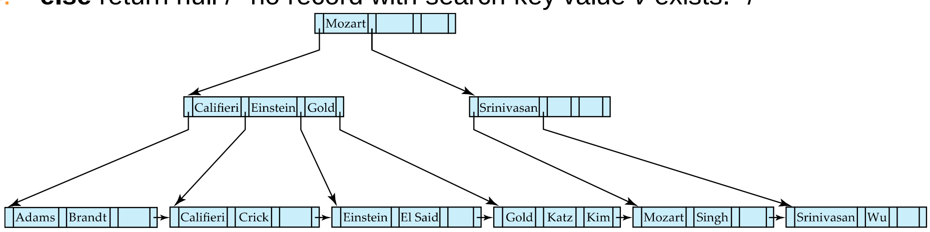


- Leaf nodes must have between 3 and 5 values ($\lceil (n-1)/2 \rceil$ and $n - 1$, with $n = 6$).
- Non-leaf nodes other than root must have between 3 and 6 children ($\lceil n/2 \rceil$ and n with $n = 6$).
- Root must have at least 2 children.

Queries on B+-Trees

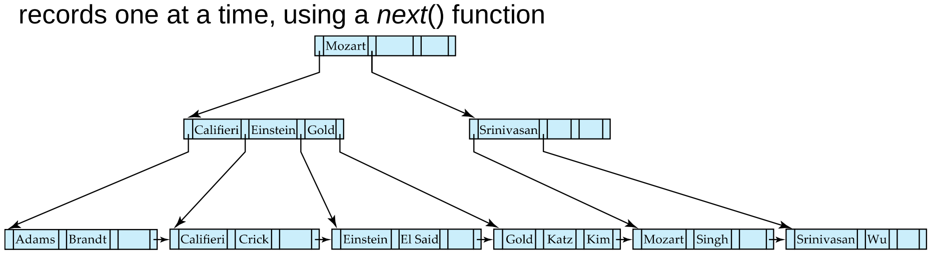
function *find*(*v*)

1. *C=root*
2. **while** (C is not a leaf node)
    1. Let *i* be least number s.t. $V \leq K_i$.
    2. **if** there is no such number *i then*
    3. Set *C = last non-null pointer in C*
    4. **else if** (v = C.K_i) Set C = P_{i+1}
    5. **else set** *C* = C.P_i
3. **if** for some *i*, $K_i = V$ **then** return C.P_i
4. **else** return null /* no record with search-key value *v* exists. */



Queries on B+-Trees (Cont.)

- **Range queries** find all records with search key values in a given range
    - See book for details of **function** *findRange*(*lb, ub*) which returns set of all such records
    - Real implementations usually provide an iterator interface to fetch matching records one at a time, using a *next*() function



Queries on B^+-Trees (Cont.)

- If there are K search-key values in the file, the height of the tree is no more than $\lceil \log_{\lceil n/2 \rceil}(K) \rceil$.
- A node is generally the same size as a disk block, typically 4 kilobytes
  - and n is typically around 100 (40 bytes per index entry).
- With 1 million search key values and n = 100
  - at most $log_{50}(1{,}000{,}000) = 4$ nodes are accessed in a lookup traversal from root to leaf.
- Contrast this with a balanced binary tree with 1 million search key values — around 20 nodes are accessed in a lookup
  - above difference is significant since every node access may need a disk I/O, costing around 20 milliseconds

Updates on B^+-Trees: Insertion

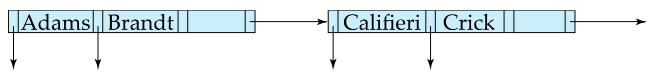
Assume record already added to the file. Let

- *pr* be pointer to the record, and let
- v be the search key value of the record

1. Find the leaf node in which the search-key value would appear
   1. If there is room in the leaf node, insert (v, *pr*) pair in the leaf node
   2. Otherwise, split the node (along with the new (*v, pr*) entry) as discussed in the next slide, and propagate updates to parent nodes.

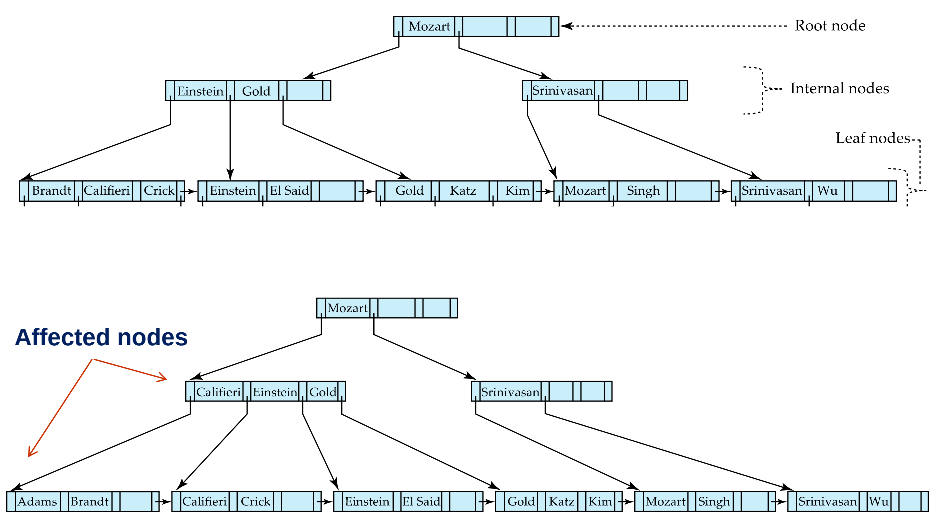
Updates on B+-Trees: Insertion (Cont.)

- Splitting a leaf node:
  - take the *n* (search-key value, pointer) pairs (including the one being inserted) in sorted order. Place the first $\lceil n/2 \rceil$ in the original node, and the rest in a new node.
  - let the new node be *p,* and let *k* be the least key value in *p.* Insert (*k,p*) in the parent of the node being split.
  - If the parent is full, split it and **propagate** the split further up.
- Splitting of nodes proceeds upwards till a node that is not full is found.
  - In the worst case the root node may be split increasing the height of the tree by 1.



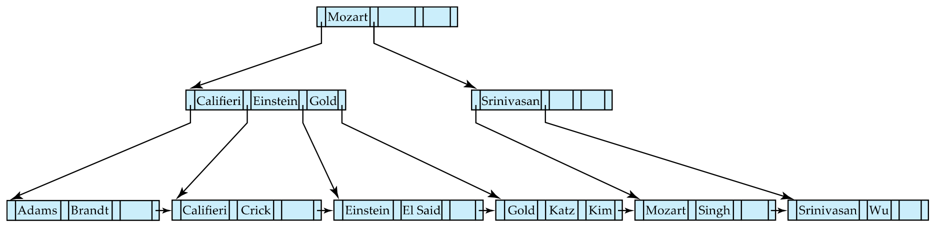
Result of splitting node containing Brandt, Califieri and Crick on inserting Adams
Next step: insert entry with (Califieri, pointer-to-new-node) into parent

B+-Tree Insertion

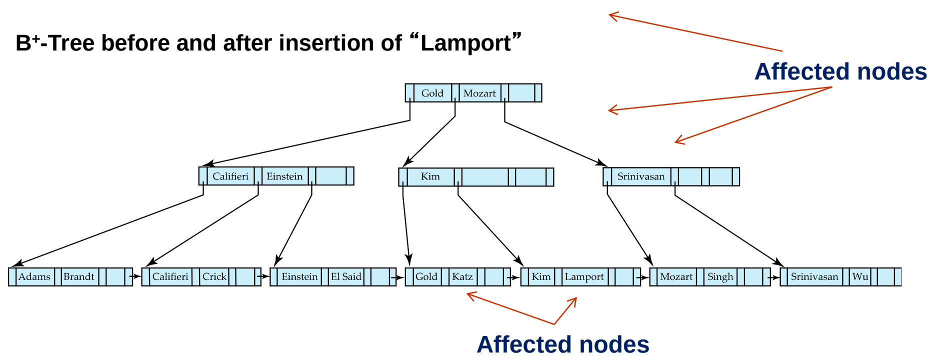


B+-Tree before and after insertion of "Adams"

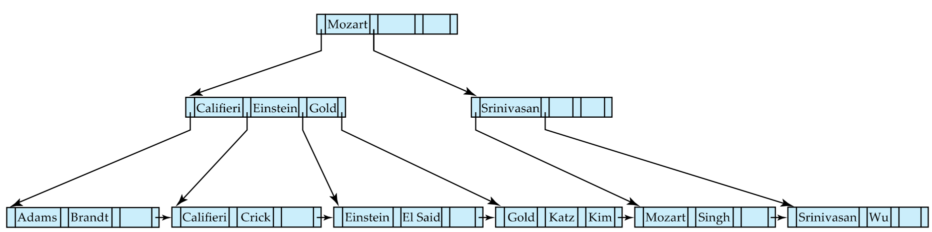
B+-Tree Insertion



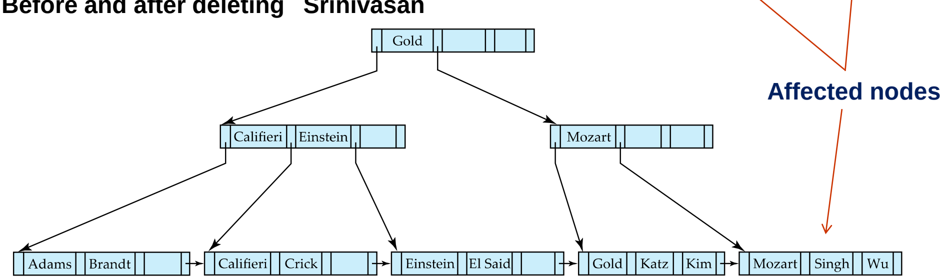
B+-Tree before and after insertion of "Lamport"



Examples of B+-Tree Deletion

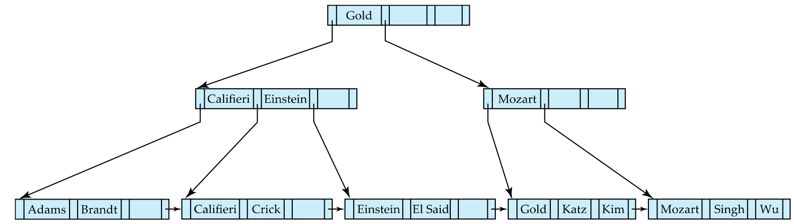


Before and after deleting "Srinivasan"

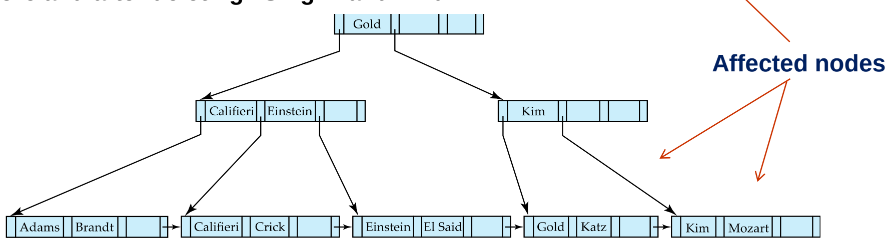


- Deleting "Srinivasan" causes **merging** of under-full leaves

Examples of B⁺-Tree Deletion (Cont.)

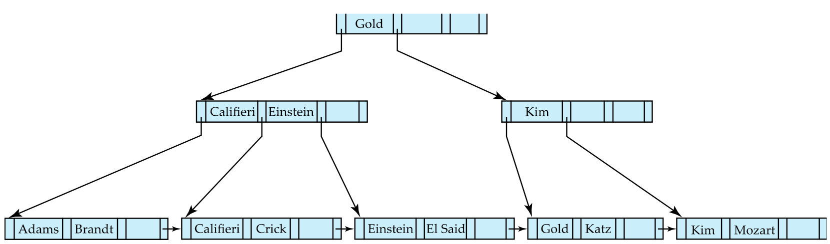


Before and after deleting "Singh" and "Wu"

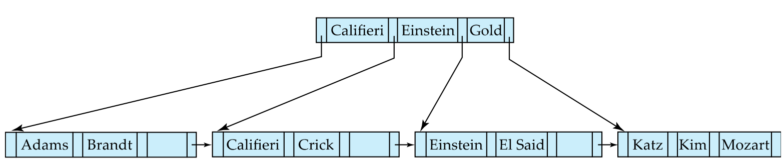


- Leaf containing Singh and Wu became underfull, and **borrowed a value** Kim from its left sibling
- Search-key value in the parent changes as a result

Example of B⁺-tree Deletion (Cont.)



Before and after deletion of "Gold"



- Node with Gold and Katz became underfull, and was merged with its sibling
- Parent node becomes underfull, and is merged with its sibling
  - Value separating two nodes (at the parent) is pulled down when merging
- Root node then has only one child, and is deleted

Updates on B^+-Trees: Deletion

Assume record already deleted from file. Let *V* be the search key value of the record, and *Pr* be the pointer to the record.

- Remove (*Pr, V*) from the leaf node
- If the node has too few entries due to the removal, and the entries in the node and a sibling fit into a single node, then ***merge siblings***:
  - Insert all the search-key values in the two nodes into a single node (the one on the left), and delete the other node.
  - Delete the pair (K_{i-1}, P_i), where P_i is the pointer to the deleted node, from its parent, recursively using the above procedure.

Updates on B^+-Trees: Deletion

- Otherwise, if the node has too few entries due to the removal, but the entries in the node and a sibling do not fit into a single node, then **redistribute pointers**:
  - Redistribute the pointers between the node and a sibling such that both have more than the minimum number of entries.
  - Update the corresponding search-key value in the parent of the node.
- The node deletions may cascade upwards till a node which has $\lceil n/2 \rceil$ or more pointers is found.
- If the root node has only one pointer after deletion, it is deleted and the sole child becomes the root.

Complexity of Updates

- Cost (in terms of number of I/O operations) of insertion and deletion of a single entry proportional to height of the tree
  - With K entries and maximum fanout of n, worst case complexity of insert/delete of an entry is $O(\log_{\lceil n/2 \rceil}(K))$
- In practice, number of I/O operations is less:
  - Internal nodes tend to be in buffer
  - Splits/merges are rare, most insert/delete operations only affect a leaf node
- Average node occupancy depends on insertion order
  - 2/3rds with random, ½ with insertion in sorted order

B^+-Tree File Organization

- B^+-Tree File Organization:
  - Leaf nodes in a B^+-tree file organization store records, instead of pointers
  - Helps keep data records clustered even when there are insertions/deletions/updates
- Leaf nodes are still required to be half full
  - Since records are larger than pointers, the maximum number of records that can be stored in a leaf node is less than the number of pointers in a nonleaf node.
- Insertion and deletion are handled in the same way as insertion and deletion of entries in a B^+-tree index.

