PURDUE UNIVERSITY

CS34800 Information Systems

Functional Dependencies

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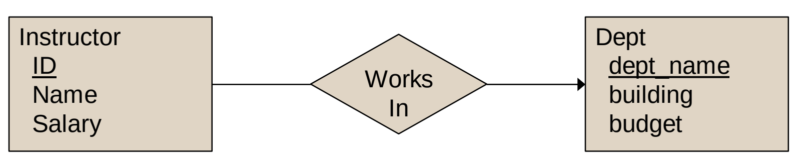
Indiana Center for Database Systems ™

Relational Design



- Instructor(ID number(10) primary key,
  Name varchar(40),
  Salary number(6))
- Dept(dept_name varchar(20) primary key,
  building varchar(30),
  budget number(8))
- Works_in(ID references Instructor(ID),
  dept_name references Dept(dept_name))

Key for Works_in?
A. ID
B. dept_name
C. both
D. neither

Keys of Relations

K is a *key* for relation *R* if:

1. *K* → all attributes of *R*. **(Uniqueness)**
2. For no proper subset of *K* is (1) true. **(Minimality)**

- If *K* at least satisfies (1), then *K* is a *superkey*.

Conventions

- Pick one key; underline key attributes in the relation schema.
- *X*, etc., represent sets of attributes; *A* etc., represent single attributes.

Combine Schemas?

- Suppose we combine *instructor* and *department* into *inst_dept*

ID	*name*	*salary*	*dept_name*	*building*	*budget*
22222	Einstein	95000	Physics	Watson	70000
12121	Wu	90000	Finance	Painter	120000
32343	El Said	60000	History	Painter	50000
45565	Katz	75000	Comp. Sci.	Taylor	100000
98345	Kim	80000	Elec. Eng.	Taylor	85000
76766	Crick	72000	Biology	Watson	90000
10101	Srinivasan	65000	Comp. Sci.	Taylor	100000
58583	Califieri	62000	History	Painter	50000
83821	Brandt	92000	Comp. Sci.	Taylor	100000
15151	Mozart	40000	Music	Packard	80000
33456	Gold	87000	Physics	Watson	70000
76543	Singh	80000	Finance	Painter	120000

- This will
  - A. Duplicate columns
  - B. Duplicate data
  - C. Save space

A Combined Schema Without Repetition

- Consider combining relations
  - *sec_class(sec_id, building, room_number)* and
  - *section(course_id, sec_id, semester, year)*

  into one relation
  - *section(course_id, sec_id, semester, year,*
    building, room_number)
- No repetition in this case

What About Smaller Schemas?

- Suppose we had started with *inst_dept.* How would we know to split up (**decompose**) it into *instructor* and *department*?
- Write a rule "if there were a schema (*dept_name, building, budget*), then *dept_name* would be a candidate key"
- Denote as a **functional dependency**:

  dept_name $\rightarrow$ *building, budget*
- In *inst_dept*, because *dept_name* is not a candidate key, the building and budget of a department may have to be repeated.
  - This indicates the need to decompose *inst_dept*
- Not all decompositions are good. Suppose we decompose *employee(ID, name, street, city, salary)* into

  employee1 (*ID*, *name*)

  employee2 (*name*, *street, city, salary*)
- The next slide shows how we lose information -- we cannot reconstruct the original *employee* relation -- and so, this is a **lossy decomposition**.

A Lossy Decomposition

ID	name	street	city	salary
⋮				
57766	Kim	Main	Perryridge	75000
98776	Kim	North	Hampton	67000
⋮				

employee

ID	name
⋮	
57766	Kim
98776	Kim
⋮	

name	street	city	salary
⋮			
Kim	Main	Perryridge	75000
Kim	North	Hampton	67000
⋮			

natural join

ID	name	street	city	salary
⋮				
57766	Kim	Main	Perryridge	75000
57766	Kim	North	Hampton	67000
98776	Kim	Main	Perryridge	75000
98776	Kim	North	Hampton	67000
⋮				

Lossless Join

- Goal: All legal values can be stored in relations
  - Recover originals through join
- Formally: X, Y is a lossless join decomposition of R w.r.t. F if $\forall r \in R$ satisfying dependencies in F,
  $\pi_X(r) \bowtie \pi_Y(r) = r$

Example of Lossless-Join Decomposition

- **Lossless join decomposition**
- Decomposition of $R = (A, B, C)$

  $R_1 = (A, B)$ $R_2 = (B, C)$

r

A	B	C
α	1	A
β	2	B

$\Pi_{A,B}(r)$

A	B
α	1
β	2

$\Pi_{B,C}(r)$

B	C
1	A
2	B

$\Pi_A(r) \bowtie \Pi_B(r)$

A	B	C
α	1	A
β	2	B

Goal — Devise a Theory for the Following

- Decide whether a particular relation R is in "good" form.
- In the case that a relation R is not in "good" form, decompose it into a set of relations $\{R_1, R_2, ..., R_n\}$ such that
  - each relation is in good form
  - the decomposition is a lossless-join decomposition
- Our theory is based on:
  - functional dependencies
  - multivalued dependencies

Functional Dependencies

- Constraints on the set of legal relations.
- Require that the value for a certain set of attributes determines uniquely the value for another set of attributes.
- A functional dependency is a generalization of the notion of a *key.*

Functional Dependencies

$X \rightarrow A$ = assertion about a relation R that whenever two tuples agree on all the attributes of X, then they must also agree on attribute A

Why do we care?

- Knowing functional dependencies provides a formal mechanism to divide up relations *(normalization)*
  - Saves space
  - Prevents storing data that violates dependencies

Functional Dependencies (Cont.)

- Let R be a relation schema

$$\alpha \subseteq R \text{ and } \beta \subseteq R$$

- The **functional dependency**

$$\alpha \rightarrow \beta$$

holds on R if and only if for any legal relations r(R), whenever any two tuples t_1 and t_2 of r agree on the attributes α, they also agree on the attributes β. That is,

$$t_1[\alpha] = t_2[\alpha] \Rightarrow t_1[\beta] = t_2[\beta]$$

- Example: Consider r(A,B) with the following instance of r.

1	4
1	5
3	7

- On this instance, $A \rightarrow B$ does **NOT** hold, but $B \rightarrow A$ does hold.

Functional Dependencies (Cont.)

- K is a superkey for relation schema R if and only if $K \rightarrow R$
- K is a candidate key for R if and only if
  - $K \rightarrow R$, and
  - for no $\alpha \subset K$, $\alpha \rightarrow R$
- Functional dependencies allow us to express constraints that cannot be expressed using superkeys. Consider the schema:

inst_dept (<u>*ID,*</u> *name, salary,* <u>*dept_name,*</u> *building, budget*).

We expect these functional dependencies to hold:

dept_name → *building*

and *ID* → *building*

but would not expect the following to hold:

dept_name → *salary*

Use of Functional Dependencies

- We use functional dependencies to:
  - test relations to see if they are legal under a given set of functional dependencies.
    - If a relation *r* is legal under a set *F* of functional dependencies, we say that *r* **satisfies** *F.*
  - specify constraints on the set of legal relations
    - We say that *F* **holds on** *R* if all legal relations on *R* satisfy the set of functional dependencies *F.*
- Note: A specific instance of a relation schema may satisfy a functional dependency even if the functional dependency does not hold on all legal instances.
  - For example, a specific instance of *instructor* may, by chance, satisfy $name \rightarrow ID$.

Normalization

Goal = BCNF = Boyce-Codd Normal Form =
all FD's follow from the fact "key $\rightarrow$ everything."

- Formally, *R* is in BCNF if for every nontrivial FD for *R*, say $X \rightarrow A$, then *X* is a superkey.
  - "Nontrivial" = right-side attribute not in left side.

Why?

1. Guarantees no redundancy due to FD's.
2. Guarantees no *update anomalies* = one occurrence of a fact is updated, not all.
3. Guarantees no *deletion anomalies* = valid fact is lost when tuple is deleted.

Boyce-Codd Normal Form

A relation schema *R* is in BCNF with respect to a set *F* of functional dependencies if for all functional dependencies in F^+ of the form

$$\alpha \rightarrow \beta$$

where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following holds:

- $\alpha \rightarrow \beta$ is trivial (i.e., $\beta \subseteq \alpha$)
- α is a superkey for *R*

Example schema *not* in BCNF:

instr_dept (*ID, name, salary, dept_name, building, budget*)

because *dept_name*→ *building, budget*
holds on *instr_dept,* but *dept_name* is not a superkey

- Shorthand: combine FD's with common left side by concatenating their right sides.
- Sometimes, several attributes jointly determine another attribute, although neither does by itself. Example:

```
Department course_number → course_title
```

Functional Dependencies (Cont.)

- *A* functional dependency is **trivial** if it is satisfied by all instances of a relation
  - Example:
    - *ID, name* $\rightarrow$ *ID*
    - *name* $\rightarrow$ *name*
  - In general, $\alpha \rightarrow \beta$ is trivial if $\beta \subseteq \alpha$

Closure of a Set of Functional Dependencies

- Given a set *F* of functional dependencies, there are certain other functional dependencies that are logically implied by *F*.
  - For example: If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow C$
- The set of **all** functional dependencies logically implied by *F* is the **closure** of *F*.
- We denote the *closure* of *F* by F^+.
- F^+ is a superset of *F*.

Example 2

$\underbrace{\text{Lastname} \quad \text{Firstname}}_{\text{Key (2 attributes)}} \qquad \underbrace{\underbrace{\text{Student ID}}_{\text{Key}} \quad \text{Major}}_{\text{Superkey}}$

Note: There are <u>alternate</u> keys

- Keys are {`Lastname, Firstname`} and {`StudentID`}

Who Determines Keys/FD's?

- We could assert a key *K*.
  - Then the only FD's asserted are that $K \rightarrow A$ for every attribute *A*.
  - No surprise: *K* is then the only key for those FD's, according to the formal definition of "key."
- Or, we could assert some FD's and *deduce* one or more keys by the formal definition.
  - E/R diagram implies FD's by key declarations and many-one relationship declarations.
- Rule of thumb: FD's either come from keyness, many-1 relationship, or from physics.
  - *E.g.*, "no two courses can meet in the same room at the same time" yields `room time` → `course`.

Functional-Dependency Theory

- We now consider the formal theory that tells us which functional dependencies are implied logically by a given set of functional dependencies.
- We then develop algorithms to generate lossless decompositions into BCNF and 3NF
- We then develop algorithms to test if a decomposition is dependency-preserving

Functional Dependencies (FD's) and Many-One Relationships

- Consider $R(A1, \ldots, An)$ and X is a key then $X \rightarrow Y$ for any attributes Y in $A1, \ldots, An$ even if they overlap with X. Why?
- Suppose R is used to represent a many $\rightarrow$ one relationship:
  $E1$ entity set $\rightarrow$ $E2$ entity set
  where X key for $E1$, Y key for $E2$,
  Then, $X \rightarrow Y$ holds,
  And $Y \rightarrow X$ does not hold unless the relationship is one-one.
- What about many-many relationships?

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CS34800 Information Systems

Functional Dependencies: Closure

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Inferring FD's

And this is important because …

- When we talk about improving relational designs, we often need to ask "does this FD hold in this relation?"

Given FD's $X1 \rightarrow A1$, $X2 \rightarrow A2$,…, $Xn \rightarrow An$, does FD $Y \rightarrow B$ necessarily hold in the same relation?

- Start by assuming two tuples agree in *Y*. Use given FD's to infer other attributes on which they must agree. If *B* is among them, then yes, else no.

Closure of a Set of Functional Dependencies

- Given a set F set of functional dependencies, there are certain other functional dependencies that are logically implied by F.
  - For e.g.: If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow C$
- The set of **all** functional dependencies logically implied by F is the **closure** of F.
- We denote the *closure* of F by F^+.

Closure of a Set of Functional Dependencies

- We can find F^+, the closure of F, by repeatedly applying **Armstrong's Axioms:**
  - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$ **(reflexivity)**
  - if $\alpha \rightarrow \beta$, then $\gamma \alpha \rightarrow \gamma \beta$ **(augmentation)**
  - if $\alpha \rightarrow \beta$, and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$ **(transitivity)**
- These rules are
  - **sound** (generate only functional dependencies that actually hold), and
  - **complete** (generate all functional dependencies that hold).

FDs: Armstrong's Axioms

- Reflexivity:
  - If $\{B_1, B_2, \ldots, B_m\} \subseteq \{A_1, A_2, \ldots, A_n\} \Rightarrow A_1A_2\cdots A_n \to B_1B_2\cdots B_m$
  - Also called "trivial FDs"
- Augmentation:
  - $A_1A_2\cdots A_n \to B_1B_2\cdots B_m \Rightarrow A_1A_2\cdots A_nC_1C_2\cdots C_k \to B_1B_2\cdots B_mC_1C_2\cdots C_k$
- Transitivity:
  - $A_1A_2\cdots A_n \to B_1B_2\cdots B_m$ and $B_1B_2\cdots B_m \to C_1C_2\cdots C_k \Rightarrow A_1A_2\cdots A_n \to C_1C_2\cdots C_k$

Armstrong's Axioms

- Armstrong's Axioms:
  - if $\beta \subseteq \alpha$, then $\alpha \to \beta$ (reflexivity)
  - if $\alpha \to \beta$, then $\gamma\alpha \to \gamma\beta$ (augmentation)
  - if $\alpha \to \beta$, and $\beta \to \gamma$, then $\alpha \to \gamma$ (transitivity)
- *Owner pet_name age → species*
  species → vaccination
- What rule allows us to combine these two FDs?
  A. Reflexivity
  B. Augmentation
  C. Transitivity
  D. Multiple
  E. None

Armstrong's Axioms

- Armstrong's Axioms:
  - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$ (reflexivity)
  - if $\alpha \rightarrow \beta$, then $\gamma\,\alpha \rightarrow \gamma\,\beta$ (augmentation)
  - if $\alpha \rightarrow \beta$, and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$ (transitivity)
- *Owner pet_name age → species*
  species → vaccination
- Applying transitivity gives:
  A. *pet_name age → species*
  B. *Owner → vaccination*
  C. *Vaccination → species*
  D. *Owner pet_name age → vaccination*
  E. Transitivity can't be applied to these rules

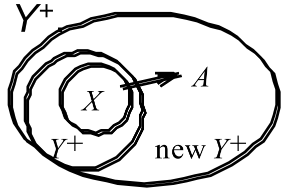
Example

- $R = (A, B, C, G, H, I)$
  $F = \{ A \rightarrow B$
  $A \rightarrow C$
  $CG \rightarrow H$
  $CG \rightarrow I$
  $B \rightarrow H\}$
- some members of F^+
  - $A \rightarrow H$
    - by transitivity from $A \rightarrow B$ and $B \rightarrow H$
  - $AG \rightarrow I$
    - by augmenting $A \rightarrow C$ with G, to get $AG \rightarrow CG$ and then transitivity with $CG \rightarrow I$
  - $CG \rightarrow HI$
    - by augmenting $CG \rightarrow I$ to infer $CG \rightarrow CGI$, and augmenting of $CG \rightarrow H$ to infer $CGI \rightarrow HI$, and then transitivity

Algorithm

Define Y^+ = *closure* of Y = set of attributes functionally determined by Y:

- Basis: $Y^+ := Y$.
- Induction: If $X \subseteq Y^+$, and $X \rightarrow A$ is a given FD, then add A to Y^+.

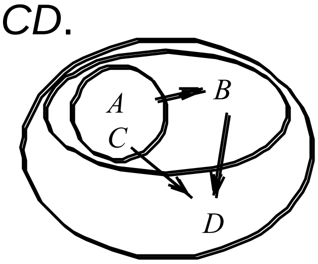


- End when Y^+ cannot be changed.

Example

$A \rightarrow B$, $BC \rightarrow D$.

- $A^+ = AB$.
- $C^+ = C$.
- $(AC)^+ = ABCD$.



Algorithm

- For each set of attributes X compute X^+.
  - But skip $X = \emptyset$, X = all attributes.
  - Add $X \to A$ for each A in $X^+ - X$.
- Drop $XY \to A$ if $X \to A$ holds.
  - Consequence: If X^+ is all attributes, then there is no point in computing closure of supersets of X.
- Finally, project the FD's by selecting only those FD's that involve only the attributes of the projection.
  - Notice that after we project the discovered FD's onto some relation, the eliminated FD's can be inferred *in the projected relation*.

Example

$F = AB \to C$, $C \to D$, $D \to A$. What FD's follow?

- $A^+ = A$; $B^+=B$ (nothing).
- $C^+=ACD$ (add $C \to A$).
- $D^+=AD$ (nothing new).
- $(AB)^+=ABCD$ (add $AB \to D$; skip all supersets of AB).
- $(BC)^+=ABCD$ (nothing new; skip all supersets of BC).
- $(BD)^+=ABCD$ (add $BD \to C$; skip all supersets of BD).
- $(AC)^+=ACD$; $(AD)^+=AD$; $(CD)^+=ACD$ (nothing new).
- $(ACD)^+=ACD$ (nothing new).
- All other sets contain AB, BC, or BD, so skip.
- Thus, the only interesting FD's that follow from F are: $C \to A$, $AB \to D$, $BD \to C$.

Example 2

- Set of FD's in $ABCGHI$:

  $A \rightarrow B$
  $A \rightarrow C$
  $CG \rightarrow H$
  $CG \rightarrow I$
  $B \rightarrow H$

- Compute $(CG)^+$, $(BG)^+$, $(AG)^+$

Example 3

In ABC with FD's $A \rightarrow B$, $B \rightarrow C$, project onto AC.

1. $A^+ = ABC$; yields $A \rightarrow B$, $A \rightarrow C$.
2. $B^+ = BC$; yields $B \rightarrow C$.
3. $AB^+ = ABC$; yields $AB \rightarrow C$;
   - drop in favor of $A \rightarrow C$
4. $AC^+ = ABC$ yields $AC \rightarrow B$;
   - drop in favor of $A \rightarrow B$
5. $C^+ = C$ and $BC^+ = BC$; adds nothing.

- Resulting FD's: $A \rightarrow B$, $A \rightarrow C$, $B \rightarrow C$.
- Projection onto AC: $A \rightarrow C$.

Uses of Attribute Closure

There are several uses of the attribute closure algorithm:

- Testing for superkey:
  - To test if α is a superkey, we compute $\alpha^{+,}$ and check if α^+ contains all attributes of *R*.
- Testing functional dependencies
  - To check if a functional dependency $\alpha \rightarrow \beta$ holds (or, in other words, is in F^+), just check if $\beta \subseteq \alpha^+$.
  - That is, we compute α^+ by using attribute closure, and then check if it contains β.
  - Is a simple and cheap test, and very useful
- Computing closure of F
  - For each $\gamma \subseteq R$, we find the closure γ^+, and for each $S \subseteq \gamma^+$, we output a functional dependency $\gamma \rightarrow S$.

Canonical Cover

- Sets of functional dependencies may have redundant dependencies that can be inferred from the others
  - For example: $A \rightarrow C$ is redundant in: $\{A \rightarrow B,\ \ B \rightarrow C, A \rightarrow C\}$
  - Parts of a functional dependency may be redundant
    - E.g.: on RHS: $\{A \rightarrow B,\ \ B \rightarrow C,\ \ A \rightarrow CD\}$ can be simplified to
      $\{A \rightarrow B,\ \ B \rightarrow C,\ \ A \rightarrow D\}$
    - E.g.: on LHS: $\{A \rightarrow B,\ \ B \rightarrow C,\ \ AC \rightarrow D\}$ can be simplified to
      $\{A \rightarrow B,\ \ B \rightarrow C,\ \ A \rightarrow D\}$
- Intuitively, a canonical cover of F is a "minimal" set of functional dependencies equivalent to F, having no redundant dependencies or redundant parts of dependencies

Extraneous Attributes

- Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F.
  - Attribute A is **extraneous** in α if $A \in \alpha$ and F logically implies $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$.
  - Attribute A is **extraneous** in β if $A \in \beta$ and the set of functional dependencies $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ logically implies F.
- *Note:* implication in the opposite direction is trivial in each of the cases above, since a "stronger" functional dependency always implies a weaker one
- Example: Given $F = \{A \rightarrow C, AB \rightarrow C\}$
  - B is extraneous in $AB \rightarrow C$ because $\{A \rightarrow C, AB \rightarrow C\}$ logically implies $A \rightarrow C$ (I.e. the result of dropping B from $AB \rightarrow C$).
- Example: Given $F = \{A \rightarrow C, AB \rightarrow CD\}$
  - C is extraneous in $AB \rightarrow CD$ since $AB \rightarrow C$ can be inferred even after deleting C

Testing if an Attribute is Extraneous

- Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F.
- To test if attribute $A \in \alpha$ is extraneous in α
  1. compute $(\{\alpha\} - A)^+$ using the dependencies in F
  2. check that $(\{\alpha\} - A)^+$ contains β; if it does, A is extraneous in α
- To test if attribute $A \in \beta$ is extraneous in β
  1. compute α^+ using only the dependencies in $F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$,
  2. check that α^+ contains A; if it does, A is extraneous in β

Canonical Cover

- A **canonical cover** for F is a set of dependencies F_c such that
  - F logically implies all dependencies in $F_{c,}$ and
  - F_c logically implies all dependencies in F, and
  - No functional dependency in F_c contains an extraneous attribute, and
  - Each left side of functional dependency in F_c is unique.
- To compute a canonical cover for F:
  repeat
  Use the union rule to replace any dependencies in F
  $\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_2$ with $\alpha_1 \rightarrow \beta_1 \beta_2$
  Find a functional dependency $\alpha \rightarrow \beta$ with an
  extraneous attribute either in α or in β
  /* Note: test for extraneous attributes done using $F_{c,}$ not F*/
  If an extraneous attribute is found, delete it from $\alpha \rightarrow \beta$
  until F does not change
- Note: Union rule may become applicable after some extraneous attributes have been deleted, so it has to be re-applied

Computing a Canonical Cover

- $R = (A, B, C)$
  $F = \{A \rightarrow BC$
  $B \rightarrow C$
  $A \rightarrow B$
  $AB \rightarrow C\}$
- Combine $A \rightarrow BC$ and $A \rightarrow B$ into $A \rightarrow BC$
  - Set is now $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$
- A is extraneous in $AB \rightarrow C$
  - Check if the result of deleting A from $AB \rightarrow C$ is implied by the other dependencies
    - Yes: in fact, $B \rightarrow C$ is already present!
  - Set is now $\{A \rightarrow BC, B \rightarrow C\}$
- C is extraneous in $A \rightarrow BC$
  - Check if $A \rightarrow C$ is logically implied by $A \rightarrow B$ and the other dependencies
    - Yes: using transitivity on $A \rightarrow B$ *and* $B \rightarrow$ C.
      - Can use attribute closure of A in more complex cases
- The canonical cover is: $A \rightarrow B$
  $B \rightarrow C$

