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- University of Illinois at Urbana Champaign
- Advisor: Jiawei Han
- Thesis topic:
  - Spectral Regression for Dimensionality Reduction

Graph Embedding & Extension for Dimensionality Reduction

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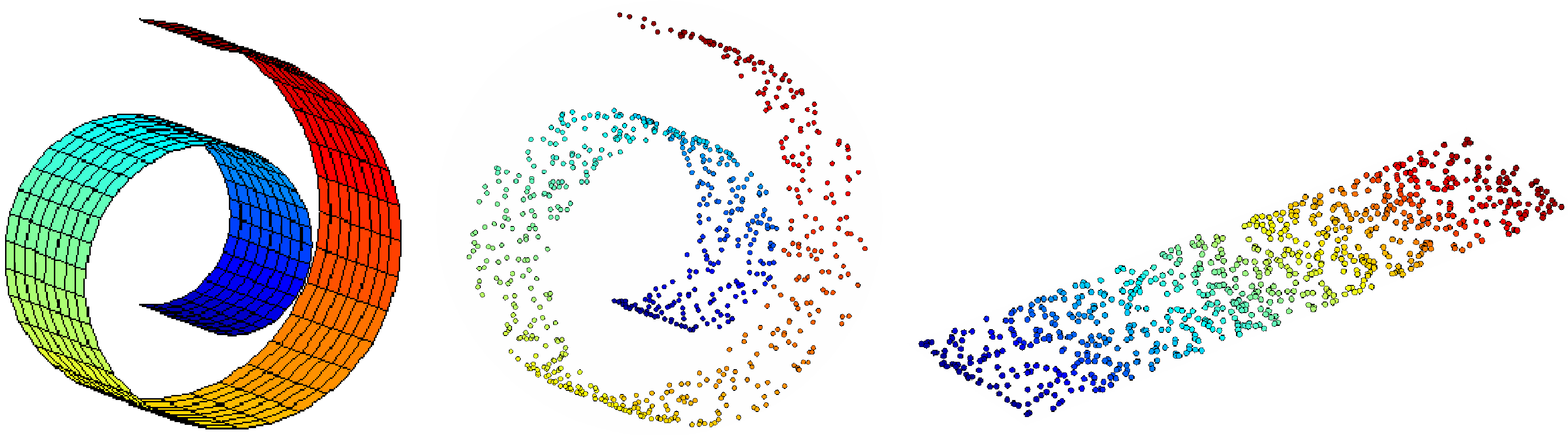
Dimensionality Reduction

- **Question**
  - How can we detect low dimensional structure in high dimensional data?

- **Applications**
  - Digital image and speech processing
  - Gene expression microarray data
  - Visualization of large networks
  - Analysis of neuronal populations

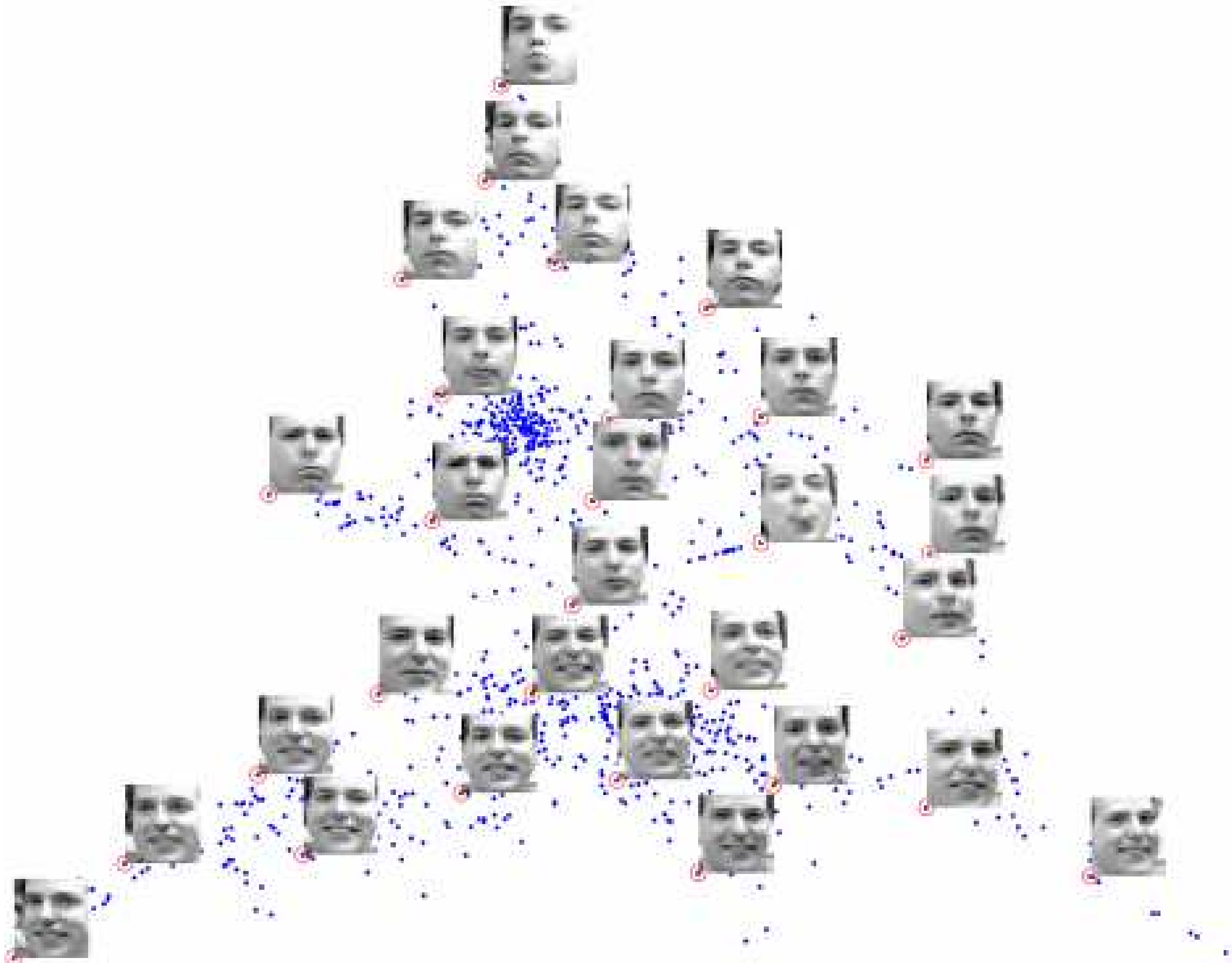
The Big Picture

Given high dimensional data sampled from a low dimensional manifold, how to compute a faithful embedding?

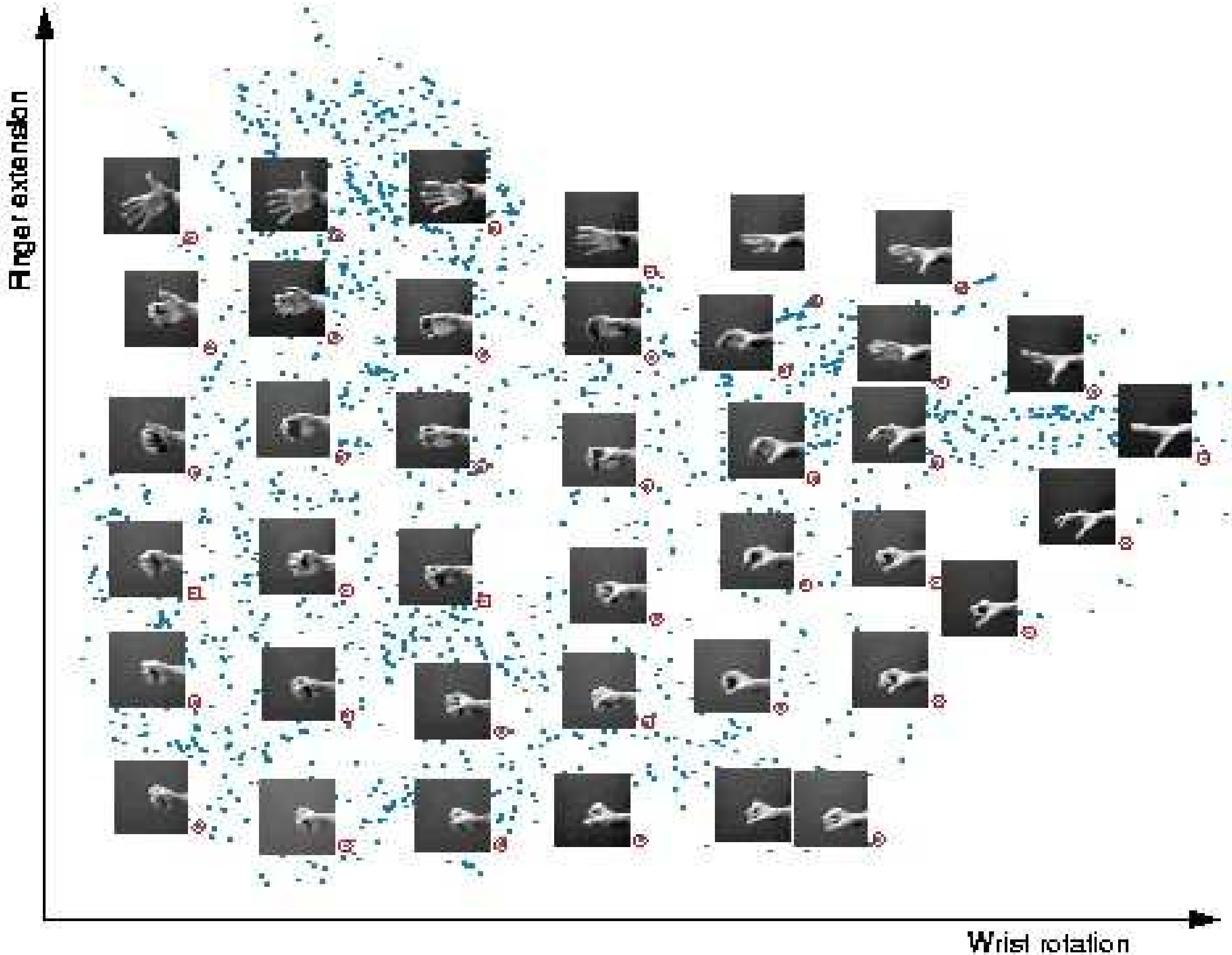


Handwritten Digit

Pose and Expression



Hand Images



Problem

Input:

$$\mathbf{x}_i \in R^n \quad \text{with} \quad i = 1, \cdots, m$$

Output:

$$\mathbf{y}_i \in R^d \quad \text{with} \quad d \ll m$$

Embedding:

Nearby points remain nearby,
distant points remain distant.
Estimate ***d*****.**

Spectral Methods

- Isomap (2000)
- Locally Linear Embedding (2000)
- Laplacian Eigenmap (2001)

- Hessian LLE (2003)
- Maximum Variance Unfolding (2004)
- Conformal Eigenmaps (2005)

Spectral Methods

- **Common framework**
  - Derive sparse graph from *k*-NN
  - Derive matrix from graph weights: W
    - Varied algorithms differ in this step
  - Derive embedding from eigenvectors

$$\mathbf{y}^{opt} = \arg\max \frac{\mathbf{y}^T W \mathbf{y}}{\mathbf{y}^T D \mathbf{y}} \quad \Rightarrow \quad W\mathbf{y} = \lambda D\mathbf{y}$$

- **Weakness**
  - No out-of-sample extension

Linear Extension

- **Require a linear projective function**

$$y = f(\mathbf{x}) = \mathbf{a}^T \mathbf{x}, \qquad \mathbf{y} = X^T \mathbf{a}$$

$$\mathbf{a}^{opt} = \arg\max \frac{\mathbf{a}^T XWX^T \mathbf{a}}{\mathbf{a}^T XDX^T \mathbf{a}} \quad \Rightarrow \quad XWX^T \mathbf{a} = \lambda XDX^T \mathbf{a}$$

- **Examples**
  - Linear Discriminant Analysis
  - Locality Preserving Projection
  - Neighborhood Preserving Embedding
  - More …

Kernel Extension

- **Require a nonlinear projective function**

$$y = f(\mathbf{x}) = \sum \alpha_i K(\mathbf{x}_i, \mathbf{x}) \qquad \mathbf{y} = K\boldsymbol{\alpha}$$

$$\boldsymbol{\alpha}^{opt} = \arg\max \frac{\boldsymbol{\alpha}^T KWK\boldsymbol{\alpha}}{\boldsymbol{\alpha}^T KDK\boldsymbol{\alpha}} \quad \Rightarrow \quad KWK\boldsymbol{\alpha} = \lambda KDK\boldsymbol{\alpha}$$

- **Examples**
  - Kernel Linear Discriminant Analysis
  - Kernel Locality Preserving Projection
  - More…

Platform for Dimensionality Reduction

- **Develop new algorithms**
  - New objective function
  - New graph structure
    - Locality Sensitive Discriminant Analysis
    - Maximum Margin Projection
    - Semi-supervised Discriminant Analysis
    - ….
- **Efficient solution**
  - Cubic-time complexity   Linear-time complexity

Conclusions

- Graph embedding is a way to transform continuous manifold learning problems into discrete ones
- Graph embedding provides a general framework for dimensionality reduction
- Need efficient optimization solutions

Thank You!

http://www.ews.uiuc.edu/~dengcai2