Collaboration and Fairness in Opportunistic Spectrum Access

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Abstract— **The Open Spectrum approach to spectrum access can achieve near-optimal spectrum utilization by letting users sense and utilize available spectrum opportunistically. However, naive spectrum assignment can lead to significant interference. We propose a network controlled spectrum access scheme where users behave collaboratively to optimize spectrum allocation for the entire network. We develop a graph-theoretical model to characterize the spectrum access problem under a number of different optimization functions, and devise rules for users to utilize available spectrum while avoiding interference with its neighbors. Experimental results confirm that user collaboration yields significant benefits (as much as 50% improvement) in opportunistic spectrum access.**

I. INTRODUCTION

Avoiding interference and maximizing utilization of the radio spectrum are the two driving goals of wireless communications. Unfortunately, there is an inherent tradeoff between the two. Previous approaches to the spectrum assignment problem were focused on avoiding interference. They divided the spectrum into frequency range slices and assigned a slice to each wireless technology. While these fixed assignment schemes avoid interference, recently studies and experiments have shown that they lead to significant fragmentation (spectrum holes), resulting in under-utilization of the radio spectrum [1]. These results provide further motivation for the *Open Spectrum* [2], [3], [4] approach to spectrum access. Enabled by software defined radio (SDR) [5] technology, *Open Spectrum* allows users to sense locally available (unallocated) spectrum ranges and utilize them opportunistically. The availability of spectrum ranges fluctuates with both location and time, and is shared among users in close physical proximity. However, a naive implementation of this approach can result in non-ideal performance. A node seizing spectrum ranges without coordination with others can lead to harmful interference with its surrounding neighbors, thus degrading the spectrum usage. This social dilemma is known in the literature about resource allocations as ”The Tragedy of the Commons”.

Our goal is to devise a flexible framework to choose ideal points in this interference / utilization tradeoff customized for each topology and deployment scenario. We study the tradeoff in a network consisting of distributed users or ad hoc users. For these users to coexist, the network needs to devise a set of rules where each user can opportunistically utilize its available spectrum, while controlling its utilization to avoid harmful interference with its neighbors. Spectrum utilization should also be fair, such that each user is able to get a certain amount of spectrum regardless of his location or neighbor environment. The algorithm that determines how much and which spectrum ranges a node can access depends on many factors and is difficult to formulate. In this work, we gain insights into the issue of network controlled spectrum access by studying it under a number of different optimization functions and collaboration rules.

To achieve this, we construct a graph-theoretic model on spectrum allocation in the context of an open spectrum system. We define the set of constraints and optimization functions that characterize the opportunistic spectrum access problem. Based on these, we define a color-sensitive graph coloring model. We also study the spectrum access problem under a number of different optimization functions to address the fairness considerations, and present collaborative and non-collaborative rules in each instance. Finally, we also examine the impact of different optimization functions and rules on user performance.

There has been extensive research on channel allocation, particularly on base station frequency/channel assignment in cellular networks[6]. In order to reduce the probability of call blocking, channel assignment is driven by call requests. In [7], a graph coloring algorithm applying to the channel/slot assignment problem can produce an allocation that avoids all possible collisions for a given network topology. The objective is to minimize the color usage where each vertex is assigned with one color. Our work is different in the following ways. Previous work allocates channels/colors to match the demand, while we perform spectrum allocations to optimize spectrum utilization for the entire network. In addition, previous work treats channels/colors with equal weighting where in practice, spectrum bands can be non-uniformly partitioned depending on the associated technology. In contrast, we consider heterogeneity in both the available spectrum that users perceive and also in the rewards (bandwidth, throughput) that users get from occupying different ranges of available spectrum.

II. SPECTRUM ALLOCATION PROBLEM

A. Assumptions

We assume that the available spectrum is divided into a set of spectrum bands, and that bands differ from each other in

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bandwidth and transmission range. We assume that spectrum bands are completely orthogonal, and that users can utilize any number of spectrum bands at one time. We also assume that nodes can use one of the many available schemes to detect the locally available spectrum. We use a simplified interference metric where two transmissions are within certain distance of each other, then they conflict if using the same spectrum band, and both fail. Environmental conditions such as user location, available spectrum are static during a spectrum assignment. This corresponds to a slow varying spectrum environment where users quickly adapt to environmental changes by re-performing network-wide spectrum allocation.

The essence of spectrum allocation is to find an appropriate distribution of spectrum bands among users so that they can coexist. In this paper, we assume that the distribution depends on network condition as well as user preference. We focus on network controlled spectrum access where users behave in a collaborative fashion to optimize spectrum allocation for the entire network.

B. Definitions

- In a network waiting for spectrum assignment, there are N users or entities indexed from 0 to $N-1$ competing for M spectrum bands indexed 0 to $M-1$.
- Let $L = \{l_{n,m} | l_{n,m} \in \{0,1\}\}_{N \times M}$ characterize the per user available spectrum, *i.e.* spectrum band m is available for user n if $l_{n,m} = 1$. Due to differences in user locations, technology employed in different bands and user requirements, different users will perceive different available spectrum.
- We also consider that the bandwidth/throughput achieved by different bands is different, depending on the user's location/environment. Let $B = \{b_{n,m}\}_{N \times M}$ describe the reward that a user n gets by successfully acquiring available spectrum band m, *i.e.* $b_{n,m}$ represents the maximum bandwidth/throughput that can be acquired (assuming no interference from other neighbors). Let $L_B = \{l_{n,m} \cdot b_{n,m}\}_{N \times M}$ denote the bandwidth weighted available spectrum.
- We characterize interference between two competing users by a constraint set. Let $C = \{c_{n,k,m} | c_{n,k,m} \in \{0,1\}\}_{N \times N \times M}$ represent the interference constraint, where if $c_{n,k,m} = 1$, users n and k would cause interference if they used the spectrum band m simultaneously. Let $c_{n,n,m} = 1 - l_{n,m}$ denote the constraint imposed by spectrum availability. Here the constraints are spectrum band specific. Note that two users who are constrained by one spectrum band (they cannot use this band simultaneously) does not imply that they are constrained by other commonly available spectrum bands. This is due to the dependence of interference on transmission power of the spectrum band and distance between transmitter and receiver[1].
- We define a valid spectrum assignment $A = \{a_{n,m} | a_{n,m} \in \{0,1\}\}_{N \times M}$ where $a_{n,m} = 1$ denotes that spectrum band m is assigned to user n. A satisfies all the constraints defined by C, that is,

  $$a_{n,m} \cdot a_{k,m} = 0, \;\; if \;\; c_{n,k,m} = 1, \forall\, n, k < N, m < M. \tag{1}$$

  Finally, we use $\Lambda_{N,M}$ to denote the set of valid spectrum assignments for a given set of N users and M spectrum bands.

C. Optimization Problem

The objective of a general resource allocation problem can be defined in terms of a utility function. For a specific type of application, the utility function may be obtained by sophisticated subjective surveys. Another method is to design utility functions based on the habits of the traffic and appropriate fairness in the network. In spectrum related resource allocation problems, we usually need to solve the optimization problem expressed as follows:

1) *Max-Sum-Bandwidth (MSB)*: This aims to maximize the total spectrum utilization in the system. The optimization problem is expressed as

$$\max_{A \in \Lambda_{N,M}} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} a_{n,m} \cdot b_{n,m}. \tag{2}$$

2) *Max-Min-Bandwidth (MMB)*: This aims to maximize the bottleneck user's spectrum utilization. The optimization problem can be expressed by

$$\max_{A \in \Lambda_{N,M}} \min_{n<N} \sum_{m=0}^{M-1} a_{n,m} \cdot b_{n,m}. \tag{3}$$

3) *Max-Proportional-Fair(MPF)*: Applying PF criterion[8], the corresponding fairness driven spectrum allocation problem can be presented as

$$\max_{A \in \Lambda_{N,M}} \sum_{n=0}^{N-1} log_{10}(\sum_{m=0}^{M-1} a_{n,m} \cdot b_{n,m}). \tag{4}$$

D. Color-Sensitive Graph Coloring Problem

By mapping each spectrum band into a color, we abstract the above spectrum allocation problem into a graph coloring problem. We define a bidirectional graph $G = (U, E_C, L_B)$ where U is a set of vertices denoting the users that share the spectrum, L_B represents the bandwidth weighted available spectrum as defined in II-B, or the color list at each vertex, and E_C is a set of undirected edges between vertices representing interference constraints between two vertices defined by C. For any two distinct vertices u,$v \in U$, a m-color edge between u and v, is in E_C if and only if $c_{u,v,m} = 1$. Hence, any two distinct vertices can have multiple colored edges between them. We define the color m specific degree of a vertex u, *i.e.*, $D_{u,m}$ to represent the number of neighbors that are color m mutually constrained with u (those who can not use m if u uses color m). It is also a relatively good measure of the impact (to neighbors) when assigning a color to a vertex. The

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[1] For example, since the transmission range of UWB is much smaller than that of WiFi, transmissions of two neighbor nodes could interfere on the WiFi band but not on the UWB band.

equivalent graph coloring problem is to color each vertex using a number of colors from its color list, such that if a color m edge exists between any two distinct vertices, they can't be colored with m simultaneously. We name this the color-sensitive graph coloring (CSGC) problem. Fig. 1 illustrates an example CSGC graph. The edges between any two distinct vertices are color-specific.

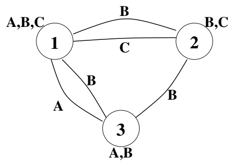

Fig. 1. An example of CSGS graph

III. Spectrum Allocation Algorithms

The optimal coloring problem is known to be NP-hard. In this section, we discuss a set of heuristic based approaches that produce good coloring solutions. In particularly, we extend some of the well-known graph coloring solutions toward our problem settings and optimization goals. In [7], progressive minimum neighbor first (PMNF) was proposed to solve the traditional graph coloring problem. The algorithm chooses the vertex with the largest number of neighbors, and colors it with the lowest indexed color without violating the constraints. The colored vertex and the associated edges are deleted from the graph and the process repeats until all the vertices are colored.

In this work, we consider the heterogeneity in both the color list and also the color rewards (bandwidth, throughput). The colors are assigned in a greedy fashion. In each stage, the algorithm labels all the vertices with a non-empty color list according to a labeling rule. Each label is associated with a color. The algorithm picks the vertex with the highest label, and assigns the color associated with the label, e.g. color m. The algorithm then deletes the color from the vertex's color list, and also from the color lists of the m color-constrained neighbors. It should be noted that the neighborhood of a vertex keeps on changing as other vertices are processed. The labels of the colored vertex and his neighbor vertices are modified according to the new graph. The algorithm enters the next stage until every vertex's color list becomes empty.

Note that our graph coloring problem wants to maximize utility while the conventional graph coloring problem [7] wants to minimize the number of colors used. While the labeling rule in our approach is different from PMNF, the intuition is similar. We choose to color the "most valuable" vertices first, *i.e.* the vertices that contribute to the system utility the most.

In the following, we propose a set of heuristics based labeling rules. We claim that a rule is collaborative if it considers the impact of interference to the neighbors when performing labeling and coloring.

- *Collaborative-Max-Sum-Bandwidth (CMSB) rule*
  This rule aims to maximize the sum of bandwidth weighted color usage, corresponding to MSB optimization defined in (2). When a vertex n is assigned with a color m, his contribution to the sum bandwidth in a local neighborhood can be computed as $b_{n,m}/D_{n,m}$ since his neighbors can not use the color. Here $D_{n,m}$ represents the number of m color constrained neighbor of a vertex n in the current graph. We propose to label the vertex according to

$$label_n = \max_{m \in l_n} b_{n,m}/(D_{n,m}+1), \tag{5}$$

$$color_n = arg \max_{m \in l_n} b_{n,m}/(D_{n,m}+1) \tag{6}$$

  where l_n represents the color list available at vertex n at this assignment stage. This rule considers the tradeoff between spectrum utilization (in terms of selecting the color with the largest bandwidth) and interference to neighbors. This rule enables collaboration by taking into account the impact to neighbors. If two vertices have the same label, then the vertex with lower assigned bandwidth weighted colors will get a higher label.

- *Non-collaborative-Max-Sum-Bandwidth (NMSB) rule*
  This rule aims to maximize the sum of bandwidth weighted color usage without considering the impact of interference to neighbors. The vertex with the maximum bandwidth-weighted color will be colored, *i.e.* a vertex n is labeled with

$$label_n = \max_{m \in l_n} b_{n,m}, \tag{7}$$

$$color_n = arg \max_{m \in l_n} b_{n,m}. \tag{8}$$

  When colors have the same property, this corresponds to a random labeling. Comparing to CMSB rule, this rule is relatively selfish or non-collaborative.

- *Collaborative-Max-Min-Bandwidth (CMMB) rule*
  This rule aims to assign equal number of colors to vertices in order to improve the minimum bandwidth weighted colors that a vertex can get, while considering interference to neighbors. It is targeted to solve MMB optimization defined in (3). In each stage, the vertices are labeled according to

$$label_n = -\sum_{m=0}^{N-1} a_{n,m} \cdot b_{n,m}, \tag{9}$$

$$color_n = arg \max_{m \in l_n} b_{n,m}/(D_{n,m}+1). \tag{10}$$

  If two vertices have the same label, then the vertex with larger $\max_{m \in l_n} b_{n,m}/(D_{n,m}+1)$ value gets a higher label.

- *Non-collaborative-Max-Min-Bandwidth (NMMB) rule*
  This rule is a non-collaborative version of CMMB rule where the impact of interference is not considered in the vertex labeling, and coloring. In each stage, the vertices are still labeled according to (9), but the

associated color is determined as $arg\max_{m\in l_n} b_{n,m}$. If two vertices have the same label, then the vertex with larger $\max_{m\in l_n} b_{n,m}$ is assigned with a higher label.

- *Collaborative-Max-Proportional-Fair (CMPF) rule*
  This rule aims to achieve a specific fairness among vertices, corresponding to MPF optimization defined in (4). It is well known that proportional fair scheduling [8] assigns resource (time slot) to the user with the highest $r_n/\hat{R}_n$, where r_n represents the reward generated by using the resource and $\hat{R}_n$ is the average reward that the user n gets in the past. The concept of proportional fair scheduling is applied to this problem by viewing color as time slot. In each stage, the vertices are labeled according to
  $$label_n = \frac{\max_{m\in l_n} b_{n,m}/(D_{n,m}+1)}{\sum_{m=0}^{M-1} a_{n,m}\cdot b_{n,m}}, \tag{11}$$
  $$color_n = arg\max_{m\in l_n} b_{n,m}/(D_{n,m}+1). \tag{12}$$
  where $label_n$ represents the ratio of the interference-weighted bandwidth using one color and the accumulated bandwidth in the past. This rule is in general different from the traditional proportional fair rule as it captures the difference in the impact of interference generated by a color (resource) assignment.

- *Non-collaborative-Max-Proportional-Fair (NMPF) rule*
  This is a non-collaborative version of the CMF rule. Each vertex n is labeled according to
  $$label_n = \frac{\max_{m\in l_n} b_{n,m}}{\sum_{m=0}^{M-1} a_{n,m}\cdot b_{n,m}}, \tag{13}$$
  $$color_n = arg\max_{m\in l_n} b_{n,m}. \tag{14}$$
- *Random (RAND) rule*
  Each vertex is assigned with a random label, and the chosen vertex is colored with a randomly picked color from his color list.

The implementation of the above coloring algorithm can be divided into two categories.

- *Centralized*: If there is a central controller who makes decisions on color assignment, the corresponding implementation is quite straightforward. The controller collects spectrum and interference information from all the vertices, and executes the rule to distribute colors among vertices and broadcast the assignment.
- *Distributed*: In this case, each vertex executes the rule to select the appropriate color(s). The colors are assigned in a greedy fashion. In each stage, each vertex labels itself according to one of the above labeling rules, and broadcasts the label to his neighbors. A vertex with the maximum label within his neighborhood gets to grab the color associated with his label and broadcasts the color assignment to his neighbors. After collecting assignment information from surrounding neighbors, each vertex updates his color list and recalculates the label. This process is repeated until the color list at each vertex is exhausted or all the vertices are satisfied.

We observe that the above two implementations are complex when the number of colors are large. In the worst case, the number of iterations equal to the number of colors. We are currently investigating a low complexity implementation.

IV. Simulation Results and Discussions

Our simulations have been conducted under the assumption of a noiseless, immobile radio network, where the nodes are distributed in a given area and may each have a different transmission range. We are only interested in spectrum allocation among those secondary users or links among secondary users. We convert this network into a graph $G=(U,E_C,L_B)$ that captures the interference among transmissions and the spectrum availability, according to the procedures described in II-D. We conduct extensive studies on various topologies. Due to space limit, we focus on two fixed topologies and one random topology to illustrate the results. The fixed topologies are shown in Fig. 2, corresponding to two extreme cases. In topology I, degree of vertex 1 is significantly larger than that of the other vertices while in topology II, the vertices have the same degree of 2. In all the simulations related to topology I and II, the total number of colors is fixed to 5. The random topology (topology III) consists of 20 random vertices with mean degree of 4. The total number of colors is fixed to 10.

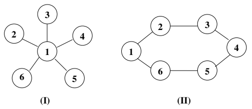

Fig. 2. Fixed Topolgy

Due to space limit, we only show the performance of centralized implementation and the related results using distributed implementations will be included in a more detailed study [9]. We examine the performance of different rules defined in III in terms of performance metrics abstracted based on II-C. The metrics corresponding to MSB,MMB and MPF problems are indexed "Sum Bandwidth", "Min Bandwidth" and "Fair Bandwidth", respectively. We set "Fair Bandwidth" to -4 when there is at least one node who gets zero spectrum. The CMSB, NMSB, CMMB, NMMB, CMF, NMF, and RAND rules are indexed from 0 to 6, respectively. Note rule 0 and 1 aim to solve MSB problem, 2 and 3 to solve MMB, 4 and 5 to solve MPF. In addition, relatively, rule 0, 2 and 4 enable collaborations. For fixed topologies, we also include the results of the optimal solution (for each metric) obtained through exhaustive search, indexed by 7. Our goal is to examine the role of collaboration in terms of integrating degree information in the labeling rule, and the impact of different optimization functions on user performance.

First, we study the performance with homogeneous color availability. Each vertex has a full color list of 0 to $M-1$. Fig. 3 compares the performance of different labeling rules using different performance metrics. We observe that among rules 0 to 6, CMSB (rule 0) maximizes “Sum Bandwidth”, CMMB (rule 2) maximizes “Min Bandwidth” and CMF (rule 4) maximizes “Fair Bandwidth”. This shows that the proposed rules match to their design goal. And for fixed topologies, the performance is very close to that of the optimal (rule 7). Random rule (rule 6) is relatively good for MSB problem but not for MMB and MPF.

We observe that collaborative rules 0,2,4 outperform non-collaborative rules 1,3,5 in corresponding metrics, and the advantage of collaboration applies to all three topologies. In particular, comparing to non-collaborative rules, collaborative rules achieve 25% improvement in MSB, 50% in MMB and 0.18 difference(in log_{10} scale) in MPF for topology I, 11%,13% and 0.33 for topology II, and 15%,12% and 0.28 for topology III, respectively.

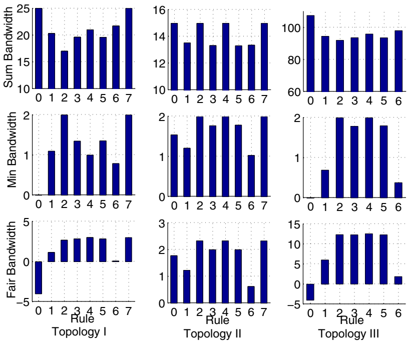

Fig. 3. Performance with homogeneous color availability and color bandwidth

Intuitively one might think that when topology is highly symmetric (*i.e.* topology II), degree information is not useful. However, it should be noted that in the proposed algorithms, since the neighborhood of a vertex keeps changing as the other vertices are processed, including degree information helps to choose the best color for each node. Using topology II as an example, if vertex 1 is colored with color m, then vertex 2 and 6 are not allowed use m. Hence, the degree of vertex 3 and 5 respect to color m reduces to 1 and should be colored with m. Unaware of this, a non-collaborative rule could assign color m to vertex 4 only. Therefore, by considering the vertex degree and thus the impact to neighbors, a collaborative rule leads to better spectrum utilization for the whole network.

Next, we study the performance under heterogeneous color availability to reflect the impact of spectrum occupation by prioritized users. The color list at each vertex varies randomly according to a uniform distribution, and the average performance is shown in Fig.4. The conclusions are similar to those in homogeneous case: collaborative rules gain superior performance in all three topologies. Collaborative rules achieve 10% improvement in MSB, 20% in MMB and 0.80 in MPF for topology I, 6%,24% and 0.75 for topology II, and 8%,34% and 1.80 for topology III. Comparing the results in Fig. 3 and 4, we see the negative impact of reduced color availability.

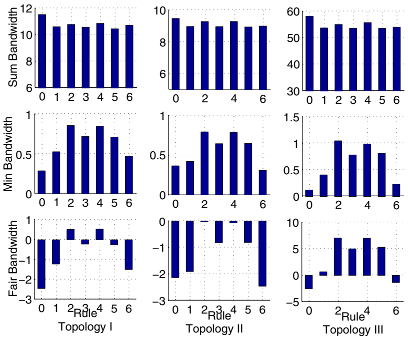

Fig. 4. Performance with heterogeneous color availability and homogeneous color bandwidth

V. CONCLUSION AND FUTURE WORK

In this paper, we explore the tradeoff in spectrum utilization and interference mitigation in open spectrum system. We focus on network controlled spectrum access where users behave in a collaborative fashion to optimize spectrum allocation for the entire network based on a network-wise optimization function. We develop a new graph-theoretical model to characterize the spectrum access problem under a number of different optimization functions, taking into account heterogeneity in both spectrum availability, reward and interference constraint. We then devise a set of collaborative rules where each user can opportunistically utilize its available spectrum, while controlling its utilization to avoid harmful interference with its neighbors. Experimental results confirm that user collaboration yields significant benefits in opportunistic spectrum access.

In this work, we assume that the environment is static and focus on per snapshot optimization. If environment changes, network-wide spectrum allocation has to be performed. This leads to significant overhead and delay. It is thus important to devise low complexity schemes that allow user to perform fast local adaptations and to explore opportunities in temporal variations. It is also important to investigate the statistics of spectrum availability and spectrum reward distributions.

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