CS 50010 Module 1 Practice Exam

The exam will be closed notes, but you may bring a 3x5 two-sided handwritten notecard if you wish.

1) Provide the contraposition for the following statement
   "If I throw a ball up, it will come down"

2) Negate the following statement
   For all x > 0 there exists some y such that y > x

3) Using set builder notation, write the set consisting of all (x,y) points that lie above the line y = x

4) What is the cardinality of the set formed by the intersection of the set of prime numbers and the set of even natural numbers?

5) Give the big-O notation for the following function

```
int foo(int n) {
      int sum = 1;
      for(int i = 0; i < n; i*=10){
            sum += 10 * i;
            sum /= 2;
      }
      return sum;
}
```

6) Give the big-Ω notation for the following function

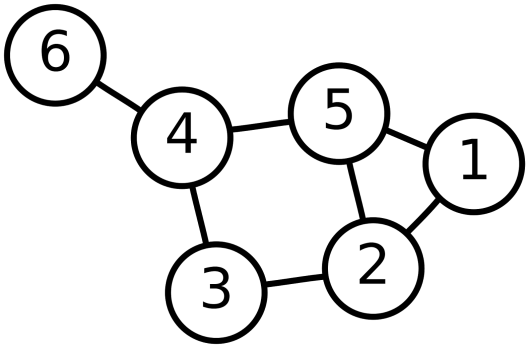
```
int bar(int n, int m) {
      int sum = 0;
      if(n < m) return;
      else for(int i = 0; i < n*m; i+=2)
            Sum++;
      return sum;
}
```

7) Draw both the array-based (begin at 2 cells) and linked-list versions of the following commands run on a Queue
   a) insert(1), insert(6), insert(1), insert(8), remove()

8) Consider both the recursive and iterative versions of binary search. What are the runtimes and memory usages of each version? Provide these in big-O notation.

9) Provide the edge list and adjacency list representation for a complete bipartite graph with 2 vertices in each disjoint set

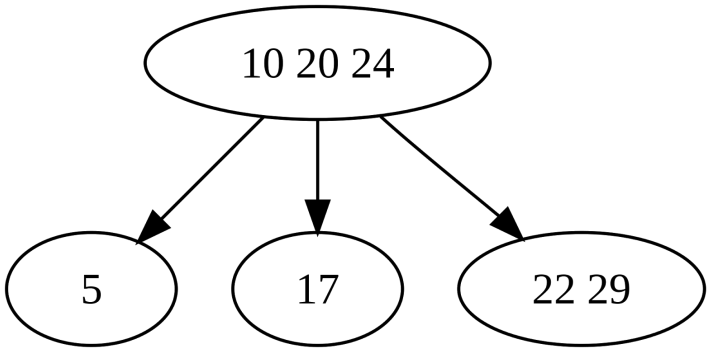
10) Consider the following graph:



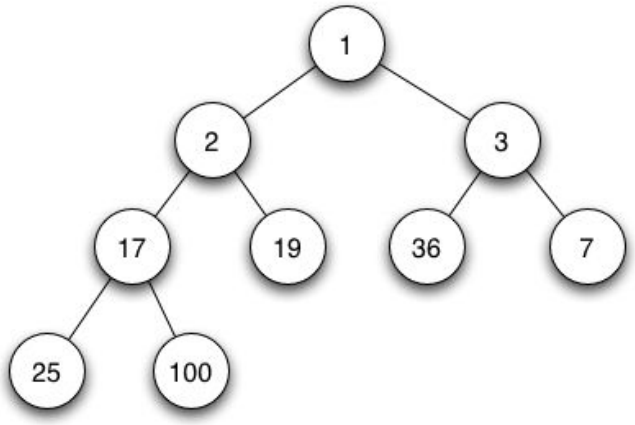
Draw the tree generated from a depth-first search starting at vertex 4. When deciding which vertex to visit, choose the one with the smallest Number.

11) Consider a hash table for integers with an array of size 10 using linked lists for collision handling. Your hash function is integers mod 10. Draw the hash map as it would appear after inserting 1, 2, 3, 5, 8, 13, and 21.

12) Consider the following 2-3 tree in the middle of an insertion. Finish the last step of this insertion into the tree.



13) Show the following heap after a removeMin() operation is run.



14) Of the following algorithms, which is a greedy algorithm, and what part of the algorithm makes it greedy? (BFS, DFS, Dijkstra)

15) Consider the following algorithm that tries to find hash collisions:

```
string collide() {
      HashTable t;
      Hash a random value
      Check if the hash is already in t
      If not, place the input and hash into t
      Continue until a collision is found
}
```

Is this an example of a Las Vegas or a Monte-Carlo algorithm?

16) Classify these three problems as being in NP, coNP, or both

a) Determine if a given path from a to b is the shortest possible path from a to b
b) Determine if a number is the product of two prime numbers
c) Determine if a graph has a Hamiltonian Path

17) Alice claims that adding two numbers a and b can be solved in linear time by repeatedly adding one to a b times. Why is this wrong? Provide an algorithm that actually accomplishes this task.

18) Determine if each of these languages is decidable, recognizable, or co-recognizable

a) The language of binary representations of prime numbers

b) The language of strings with fewer than 10 0's in a row

c) The language of strings containing "ababab" as a substring

Long answer:

Prove that a full binary tree (a complete binary tree where the last row is also full) has exactly 2^n-1 nodes. (Hint: Use induction)

In your homework you saw that 4-way mergesort works in n log n time. Show that this works for any constant *k* using the Master Theorem