1) (pt. 20) Let R(D,E,F) be a relational schema. Determine for each of the following equalities whether the equality is true or false.

(a) \( \Pi_{D,E}((\sigma_{(D > 10 \text{ AND } F=7)}(\Pi_{D,E,F}((\sigma_{(D > 15)}(R)))))) = \Pi_{D,E}((\sigma_{(D > 15 \text{ AND } F=7)}(R))) \)

(b) \( \Pi_{D,E}((\sigma_{(D > 10 \text{ OR } F=7)}(\Pi_{D,E,F}((\sigma_{(D > 15)}(R)))))) = \Pi_{D,E}((\sigma_{(D > 15 \text{ OR } F=7)}(R))) \)

2) (pt. 25) Consider the following relational schema; the schema describes a database containing data about theses carried out by students of a master degree. Theses have supervisors and co-supervisors. The underlined attributes denote the primary keys of the relations. The column Prof# in the relation Theses indicates the professors that are supervisors.

Students (S#, FName, LName, Street, City, Phone#)
Theses (T#, Title, Topic, Prof#, S#, Evaluation)
Professors (Prof#, PName, Rank, Phone#, Dept#)
Theses-Co-Supervisors (T#, Prof#)

(a) Determine for each relation, the foreign keys.
(b) Based on the primary and foreign keys of the relations, answer the following questions:
   - Can a professor supervise more than one thesis?
   - Can a thesis have multiple supervisors?
   - Can a thesis have multiple co-supervisors?

3) (pt. 15) Consider the relational schema in question (2) above. Discuss if and when \( \Pi_{\text{TitleTheses}} \neq \Pi_{\text{Title}(\text{Theses} |X| \text{Theses-Co-Supervisors})} \)

4) (pt. 30)

(a) Given the query “Retrieve the topic of the thesis by student Bob Rossi” determine which of the following RA expressions is a correct expression for this query:
   (i) \( (\sigma_{(\text{FName} = 'Bob' \text{ AND } \text{LName} = 'Rossi')}(\text{Students})) |X| (\Pi_{\text{Topic}}(\text{Theses})) \)
   (ii) \( \Pi_{\text{Topic}}((\sigma_{(\text{FName} = 'Bob' \text{ AND } \text{LName} = 'Rossi')}(\text{Students})) |X| \text{Theses}) \)
   (iii) \( \Pi_{\text{Topic}, \text{Fname}, \text{Lname}}((\sigma_{(\text{FName} = 'Bob' \text{ AND } \text{LName} = 'Rossi')}(\text{Students})) |X| \text{Theses}) \)
   (iv) none of the above

(b) Given the query “Retrieve the last name of the co-supervisor(s) of the thesis by student Bob Rossi” determine which of the following RA expressions is a correct expression for this query:
(i) \(\sigma_{(\text{FName} = \text{Bob} \text{ AND LName} = \text{Rossi})} \text{Students } | \text{Theses} | \text{Theses-Co-Supervisors} | \text{Professors}\)

(ii) \(\Pi_{\text{Pname}}(((\sigma_{(\text{FName} = \text{Bob} \text{ AND LName} = \text{Rossi})} \text{Students } | \text{Theses}) | \text{Theses-Co-Supervisors} | \text{Professors})\)

(iii) \(\Pi_{\text{Pname}}(\Pi_{\text{T#}} ((\sigma_{(\text{FName} = \text{Bob} \text{ AND LName} = \text{Rossi})} \text{Students } | \text{Theses}) | \text{Theses-Co-Supervisors} | \text{Professors})\)

(iv) none of the above

(c) Given the query “Select the professor number of all professors that supervise no thesis” determine which of the following RA expressions is a correct expression for this query:

(i) \(\Pi_{\text{Pname}}(\text{Professors}) - \Pi_{\text{Pname}}(\text{Theses})\)

(ii) \(\Pi_{\text{Prof#}}(\text{Professors}) - \Pi_{\text{Prof#}}(\text{Theses})\)

(iii) \(\Pi_{\text{Prof#}}(\text{Professors} | \text{Prof#} \neq \text{Prof#} \text{Theses})\)

(iv) None of the above

5) (pt. 10) Let \(R\) be a relation of degree \(K\); which is the maximum number of candidate keys that \(R\) can have?