# Chapter 2: Relational Model 

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## Chapter 2: Relational Model

- Structure of Relational Databases
- Fundamental Relational-Algebra-Operations
- Additional Relational-Algebra-Operations
- Extended Relational-Algebra-Operations
- Null Values
- Modification of the Database


## Example of a Relation

| account_number | branch_name | balance |
| :---: | :--- | :---: |
| A-101 | Downtown | 500 |
| A-102 | Perryridge | 400 |
| A-201 | Brighton | 900 |
| A-215 | Mianus | 700 |
| A-217 | Brighton | 750 |
| A-222 | Redwood | 700 |
| A-305 | Round Hill | 350 |

## Basic Structure

- Formally, given sets $D_{1}, D_{2}, \ldots D_{n}$ a relation $r$ is a subset of

$$
D_{1} \times D_{2} \times \ldots \times D_{n}
$$

Thus, a relation is a set of $n$-tuples $\left(a_{1}, a_{2}, \ldots, a_{n}\right)$ where each $a_{i} \in D_{i}$

- Example: If
customer_name $=$ \{Jones, Smith, Curry, Lindsay $\}$
customer_street $=\{$ Main, North, Park $\}$
customer_city = \{Harrison, Rye, Pittsfield\}
Then $r=\{$ (Jones, Main, Harrison),
(Smith, North, Rye),
(Curry, North, Rye),
(Lindsay, Park, Pittsfield) \}
is a relation over
customer_name x customer_street x customer_city


## Attribute Types

- Each attribute of a relation has a name
- The set of allowed values for each attribute is called the domain of the attribute
- Attribute values are (normally) required to be atomic; that is, indivisible
- Note: multivalued attribute values are not atomic
- Note: composite attribute values are not atomic
- The special value null is a member of every domain
- The null value causes complications in the definition of many operations
- We shall ignore the effect of null values in our main presentation and consider their effect later


## Relation Schema

- $A_{1}, A_{2}, \ldots, A_{n}$ are attributes
- $R=\left(A_{1}, A_{2}, \ldots, A_{n}\right)$ is a relation schema

Example:
Customer_schema = (customer_name, customer_street, customer_city)

- $r(R)$ is a relation on the relation schema $R$ Example:
customer (Customer_schema)


## Relation Instance

- The current values (relation instance) of a relation are specified by a table
- An element $t$ of $r$ is a tuple, represented by a row in a table



## Relations are Unordered

■ Order of tuples is irrelevant (tuples may be stored in an arbitrary order)

- Example: account relation with unordered tuples


## account_number branch_name balance

| A-101 | Downtown | 500 |
| :--- | :--- | :--- |
| A-215 | Mianus | 700 |
| A-102 | Perryridge | 400 |
| A-305 | Round Hill | 350 |
| A-201 | Brighton | 900 |
| A-222 | Redwood | 700 |
| A-217 | Brighton | 750 |

## Database

- A database consists of multiple relations
- Information about an enterprise is broken up into parts, with each relation storing one part of the information


## account: stores information about accounts

depositor : stores information about which customer owns which account
customer : stores information about customers

- Storing all information as a single relation such as bank(account_number, balance, customer_name, ..) results in
- repetition of information (e.g., two customers own an account)
- the need for null values (e.g., represent a customer without an account)
- Normalization theory (Chapter 7) deals with how to design relational schemas


## The customer Relation

| customer_name | customer_street | customer_city |
| :---: | :---: | :---: |
| Adams | Spring | Pittsfield |
| Brooks | Senator | Brooklyn |
| Curry | North | Rye |
| Glenn | Sand Hill | Woodside |
| Green | Walnut | Stamford |
| Hayes | Main | Harrison |
| Johnson | Alma | Palo Alto |
| Jones | Main | Harrison |
| Lindsay | Park | Pittsfield |
| Smith | North | Rye |
| Turner | Putnam | Stamford |
| Williams | Nassau | Princeton |

## The depositor Relation

$$
\begin{array}{c|c}
\hline \text { customer_name } & \text { account_number } \\
\hline \hline \text { Hayes } & \text { A-102 } \\
\text { Johnson } & \text { A-101 } \\
\text { Johnson } & \text { A-201 } \\
\text { Jones } & \text { A-217 } \\
\text { Lindsay } & \text { A-222 } \\
\text { Smith } & \text { A-215 } \\
\text { Turner } & \text { A-305 }
\end{array}
$$

## Keys

- Let $\mathrm{K} \subseteq \mathrm{R}$
- $K$ is a superkey of $R$ if values for $K$ are sufficient to identify a unique tuple of each possible relation $r(R)$
- by "possible $r$ " we mean a relation $r$ that could exist in the enterprise we are modeling.
- Example: \{customer_name, customer_street\} and \{customer_name\}
are both superkeys of Customer, if no two customers can possibly have the same name.
- $K$ is a candidate key if $K$ is minimal

Example: \{customer_name\} is a candidate key for Customer, since it is a superkey (assuming no two customers can possibly have the same name), and no subset of it is a superkey.

- Primary Key


## Query Languages

- Language in which user requests information from the database.
- Categories of languages
- Procedural
- Non-procedural, or declarative
- "Pure" languages:
- Relational algebra
- Tuple relational calculus
- Domain relational calculus
- Pure languages form underlying basis of query languages that people use.


## Relational Algebra

- Procedural language
- Six basic operators
- select: $\sigma$
- project: П
- union: $\cup$
- set difference: -
- Cartesian product: x
- rename: $\rho$
- The operators take one or two relations as inputs and produce a new relation as a result.


## Select Operation - Example

- Relation r

| $A$ | $B$ | $C$ | $D$ |
| :---: | :---: | :---: | :---: |
| $\alpha$ | $\alpha$ | 1 | 7 |
| $\alpha$ | $\beta$ | 5 | 7 |
| $\beta$ | $\beta$ | 12 | 3 |
| $\beta$ | $\beta$ | 23 | 10 |

- $\sigma_{A=B \wedge D>5}(r)$

| $A$ | $B$ | $C$ | $D$ |
| :---: | :---: | :---: | :---: |
| $\alpha$ | $\alpha$ | 1 | 7 |
| $\beta$ | $\beta$ | 23 | 10 |

## Select Operation

- Notation: $\sigma_{p}(r)$
- $p$ is called the selection predicate
- Defined as:

$$
\sigma_{p}(\boldsymbol{r})=\{t \mid t \in r \text { and } p(t)\}
$$

Where $p$ is a formula in propositional calculus consisting of terms connected by : ^ (and), $\vee$ (or), $\neg$ (not)
Each term is one of:
<attribute> op <attribute> or <constant>
where $o p$ is one of: $=, \neq,>, \geq .<. \leq$

- Example of selection:

$$
\sigma_{\text {branch_name="Perryridge"(account) }}
$$

## Project Operation - Example

| - Relation $r$ : |  | A | B | C |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \alpha \\ & \alpha \\ & \beta \\ & \beta \end{aligned}$ | 10 20 30 40 | 1 1 1 2 |  |
| $\prod_{\mathrm{A}, \mathrm{C}}(r)$ | A | c |  | A | C |
|  | $\alpha$ $\alpha$ $\beta$ $\beta$ | 1 1 1 2 | $=$ | $\alpha$ $\beta$ $\beta$ | 1 1 2 |

## Project Operation

- Notation:

$$
\prod_{A_{1}, A_{2}, \ldots, A_{k}}(r)
$$

where $A_{1}, A_{2}$ are attribute names and $r$ is a relation name.

- The result is defined as the relation of $k$ columns obtained by erasing the columns that are not listed
- Duplicate rows removed from result, since relations are sets
- Example: To eliminate the branch_name attribute of account

$$
\Pi_{a c c o u n t \_n u m b e r, ~ b a l a n c e ~} \text { (account) }
$$

## Union Operation - Example

- Relations $r$, $s$ :

- $\quad$ U $:$

| $A$ | $B$ |
| :---: | :---: |
| $\alpha$ | 1 |
| $\alpha$ | 2 |
| $\beta$ | 1 |
| $\beta$ | 3 |

## Union Operation

- Notation: $r \cup s$
- Defined as:

$$
r \cup s=\{t \mid t \in r \text { or } t \in s\}
$$

- For $r \cup s$ to be valid.

1. $r$, s must have the same arity (same number of attributes)
2. The attribute domains must be compatible (example: $2^{\text {nd }}$ column of $r$ deals with the same type of values as does the $2^{\text {nd }}$ column of $s$ )

- Example: to find all customers with either an account or a loan

$$
\Pi_{\text {customer_name }}(\text { depositor }) \cup \prod_{\text {customer_name }} \text { (borrower) }
$$

## Set Difference Operation - Example

- Relations $r$, $s$ :

- $r$ - $s$ :



## Set Difference Operation

- Notation $r-s$
- Defined as:

$$
r-s=\{t \mid t \in r \text { and } t \notin s\}
$$

- Set differences must be taken between compatible relations.
- $r$ and $s$ must have the same arity
- attribute domains of $r$ and $s$ must be compatible


## Cartesian-Product Operation - Example

- Relations $r$, $s$ :

| $A$ | $B$ |
| :---: | :---: |
| $\alpha$ | 1 |
| $\beta$ | 2 |
| $r$ |  |


| $C$ | $D$ | $E$ |
| :---: | :---: | :---: |
| $\alpha$ | 10 | $a$ |
| $\beta$ | 10 | $a$ |
| $\beta$ | 20 | $b$ |
| $\gamma$ | 10 | $b$ |

S

- $r \times s$ :

| $A$ | $B$ | $C$ | $D$ | $E$ |
| :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | 1 | $\alpha$ | 10 | $a$ |
| $\alpha$ | 1 | $\beta$ | 10 | $a$ |
| $\alpha$ | 1 | $\beta$ | 20 | $b$ |
| $\alpha$ | 1 | $\gamma$ | 10 | $b$ |
| $\beta$ | 2 | $\alpha$ | 10 | $a$ |
| $\beta$ | 2 | $\beta$ | 10 | $a$ |
| $\beta$ | 2 | $\beta$ | 20 | $b$ |
| $\beta$ | 2 | $\gamma$ | 10 | $b$ |

## Cartesian-Product Operation

- Notation $r \times s$
- Defined as:

$$
r \times s=\{t q \mid t \in r \text { and } q \in s\}
$$

- Assume that attributes of $r(R)$ and $s(S)$ are disjoint. (That is, $R \cap S=\varnothing$ ).
- If attributes of $r(R)$ and $s(S)$ are not disjoint, then renaming must be used.


## Composition of Operations

- Can build expressions using multiple operations
- Example: $\sigma_{\mathrm{A}=\mathrm{C}}(r \times s)$
- $r x s$

| $A$ | $B$ | $C$ | $D$ | $E$ |
| :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | 1 | $\alpha$ | 10 | $a$ |
| $\alpha$ | 1 | $\beta$ | 10 | $a$ |
| $\alpha$ | 1 | $\beta$ | 20 | $b$ |
| $\alpha$ | 1 | $\gamma$ | 10 | $b$ |
| $\beta$ | 2 | $\alpha$ | 10 | $a$ |
| $\beta$ | 2 | $\beta$ | 10 | $a$ |
| $\beta$ | 2 | $\beta$ | 20 | $b$ |
| $\beta$ | 2 | $\gamma$ | 10 | $b$ |

- $\sigma_{\mathrm{A}=\mathrm{C}}(r \times s)$

| $A$ | $B$ | $C$ | $D$ | $E$ |
| :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | 1 | $\alpha$ | 10 | $a$ |
| $\beta$ | 2 | $\beta$ | 10 | $a$ |
| $\beta$ | 2 | $\beta$ | 20 | $b$ |

## Rename Operation

- Allows us to name, and therefore to refer to, the results of relationalalgebra expressions.
- Allows us to refer to a relation by more than one name.
- Example:

$$
\rho_{X}(E)
$$

returns the expression $E$ under the name $X$

- If a relational-algebra expression $E$ has arity $n$, then

$$
\rho_{x\left(A_{1}, A_{2}, \ldots, A_{n}\right)}(E)
$$

returns the result of expression $E$ under the name $X$, and with the attributes renamed to $A_{1}, A_{2}, \ldots, A_{n}$.

## Banking Example

```
branch (branch_name, branch_city, assets)
customer (customer_name, customer_street, customer_city)
account (account_number, branch_name, balance)
loan (loan_number, branch_name, amount)
depositor (customer_name, account_number)
borrower (customer_name, loan_number)
```


## Example Queries

- Find all loans of over $\$ 1200$

$$
\sigma_{\text {amount }>1200}(l o a n)
$$

- Find the loan number for each loan of an amount greater than \$1200

$$
\Pi_{\text {loan_number }}\left(\sigma_{\text {amount }>1200}(\text { loan })\right)
$$

## Example Queries

- Find the names of all customers who have a loan, an account, or both, from the bank

$$
\Pi_{\text {customer_name }}(\text { borrower }) \cup \Pi_{\text {customer_name }} \text { (depositor) }
$$

- Find the names of all customers who have a loan and an account at bank.

$$
\Pi_{\text {customer_name }} \text { (borrower) } \cap \Pi_{\text {customer_name }} \text { (depositor) }
$$

## Example Queries

- Find the names of all customers who have a loan at the Perryridge branch.
$\prod_{\text {customer_name }}\left(\sigma_{\text {branch_name="Perryridge" }}\right.$
$\left(\sigma_{\text {borrower.loan_number }=\text { loan.loan_number }}(\right.$ borrower $x$
loan $)))$
- Find the names of all customers who have a loan at the Perryridge branch but do not have an account at any branch of the bank.
$\Pi_{\text {Customer_name }}\left(\sigma_{\text {branch_name }}=\right.$ "Perryridge"
$\left(\sigma_{\text {borrower.loan_number }=\text { loan.loan_number }}(\right.$ borrower x loan) $\left.)\right)$ $\Pi_{\text {customer_name }}$ (depositor)


## Example Queries

- Find the names of all customers who have a loan at the Perryridge branch.
- Query 1
$\Pi_{\text {customer_name }}\left(\sigma_{\text {branch_name }}=\right.$ "Perryridge" $($
$\sigma_{\text {borrower.loan_number }}=$ loan.loan_number $($ borrower x loan)))
- Query 2
$\Pi_{\text {customer_name }}\left(\sigma_{\text {loan.loan_number }}=\right.$ borrower.loan_number $($

$$
\left.\left.\left(\sigma_{\text {branch_name }}=\text { "Perryridge" }(\text { loan })\right) \times \text { borrower }\right)\right)
$$

## Example Queries

- Find the largest account balance
- Strategy:
- Find those balances that are not the largest
- Rename account relation as $d$ so that we can compare each account balance with all others
- Use set difference to find those account balances that were not found in the earlier step.
- The query is:
$\Pi_{\text {balance }}($ account $)-\prod_{\text {account.balance }}$
$\quad\left(\sigma_{\text {account.balance }}<\right.$ d.balance $\left(\right.$ account $\times \rho_{d}($ account $\left.\left.)\right)\right)$


## Formal Definition

- A basic expression in the relational algebra consists of either one of the following:
- A relation in the database
- A constant relation
- Let $E_{1}$ and $E_{2}$ be relational-algebra expressions; the following are all relational-algebra expressions:
- $E_{1} \cup E_{2}$
- $E_{1}-E_{2}$
- $E_{1} \times E_{2}$
- $\sigma_{p}\left(E_{1}\right), P$ is a predicate on attributes in $E_{1}$
- $\Pi_{s}\left(E_{1}\right), S$ is a list consisting of some of the attributes in $E_{1}$
- $\rho_{x}\left(E_{1}\right), \mathrm{x}$ is the new name for the result of $E_{1}$


## Additional Operations

We define additional operations that do not add any power to the relational algebra, but that simplify common queries.

- Set intersection
- Natural join
- Division
- Assignment


## Set-Intersection Operation

- Notation: $r \cap s$
- Defined as:
- $r \cap s=\{t \mid t \in r$ and $t \in s\}$
- Assume:
- $r, s$ have the same arity
- attributes of $r$ and $s$ are compatible
- Note: $r \cap s=r-(r-s)$


## Set-Intersection Operation - Example

- Relation $r$, $s$ :

| $A$ | $B$ |
| :--- | :--- |
| $\alpha$ | 1 |
| $\alpha$ | 2 |
| $\beta$ | 1 |

$r$


S

- $r \cap s$



## Natural-Join Operation

- Notation: $\mathrm{r} \bowtie \mathrm{s}$
- Let $r$ and $s$ be relations on schemas $R$ and $S$ respectively. Then, $r \bowtie s$ is a relation on schema $R \cup S$ obtained as follows:
- Consider each pair of tuples $t_{r}$ from $r$ and $t_{S}$ from $s$.
- If $t_{r}$ and $t_{s}$ have the same value on each of the attributes in $R \cap S$, add a tuple $t$ to the result, where
- $t$ has the same value as $t_{r}$ on $r$
, $t$ has the same value as $t_{s}$ on $s$
- Example:

$$
\begin{aligned}
& R=(A, B, C, D) \\
& S=(E, B, D)
\end{aligned}
$$

- Result schema $=(A, B, C, D, E)$
- $r \bowtie s$ is defined as:

$$
\Pi_{r . A, r . B, r . C, r . D, s . E}\left(\sigma_{r . B=s . B} \wedge_{r . D=s . D}(r \times s)\right)
$$

## Natural Join Operation - Example

- Relations $\mathrm{r}, \mathrm{s}$ :

| $A$ | $B$ | $C$ | $D$ |
| :---: | :---: | :---: | :---: |
| $\alpha$ | 1 | $\alpha$ | a |
| $\beta$ | 2 | $\gamma$ | a |
| $\gamma$ | 4 | $\beta$ | b |
| $\alpha$ | 1 | $\gamma$ | a |
| $\delta$ | 2 | $\beta$ | b |
| $r$ |  |  |  |


| $B$ | $D$ | $E$ |
| :---: | :---: | :---: |
| 1 | a | $\alpha$ |
| 3 | a | $\beta$ |
| 1 | a | $\gamma$ |
| 2 | b | $\delta$ |
| 3 | b | $\epsilon$ |
| s |  |  |

- $r \bowtie s$

| $A$ | $B$ | $C$ | $D$ | $E$ |
| :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | 1 | $\alpha$ | a | $\alpha$ |
| $\alpha$ | 1 | $\alpha$ | a | $\gamma$ |
| $\alpha$ | 1 | $\gamma$ | a | $\alpha$ |
| $\alpha$ | 1 | $\gamma$ | a | $\gamma$ |
| $\delta$ | 2 | $\beta$ | b | $\delta$ |

## Division Operation

- Notation: $r \div s$
- Suited to queries that include the phrase "for all".
- Let $r$ and $s$ be relations on schemas $R$ and $S$ respectively where
- $R=\left(A_{1}, \ldots, A_{m}, B_{1}, \ldots, B_{n}\right)$
- $S=\left(B_{1}, \ldots, B_{n}\right)$

The result of $r \div s$ is a relation on schema

$$
\begin{aligned}
& R-S=\left(A_{1}, \ldots, A_{m}\right) \\
& r \div s=\left\{t \mid t \in \Pi_{R-S}(r) \wedge \forall u \in s(t u \in r)\right\}
\end{aligned}
$$

Where tu means the concatenation of tuples $t$ and $u$ to produce a single tuple

## Division Operation - Example

- Relations $r$, $s$ :
- $r \div s$ :

| $A$ | $B$ |
| :--- | :--- |
| $\alpha$ | $\alpha$ 1 <br> $\alpha$ 2 <br> $\alpha$ 3 <br> $\beta$ 1 <br> $\gamma$ 1 <br> $\delta$ 1 <br> $\delta$ 3 <br> $\delta$ 4 <br> $\in$ 6 <br> $\in$ 1 <br> $\beta$ 2 |


| $B$ |
| :---: |
| 1 |
| 2 |
| $s$ |

## Another Division Example

- Relations $r, s$ :

| $A$ | $B$ | $C$ | $D$ | $E$ |
| :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | a | $\alpha$ | a | 1 |
| $\alpha$ | a | $\gamma$ | a | 1 |
| $\alpha$ | a | $\gamma$ | b | 1 |
| $\beta$ | a | $\gamma$ | a | 1 |
| $\beta$ | a | $\gamma$ | b | 3 |
| $\gamma$ | a | $\gamma$ | a | 1 |
| $\gamma$ | a | $\gamma$ | b | 1 |
| $\gamma$ | a | $\beta$ | b | 1 |



- $r \div s$ :

| $A$ | $B$ | $C$ |
| :---: | :---: | :---: |
| $\alpha$ | a | $\gamma$ |
| $\gamma$ | a | $\gamma$ |

## Division Operation (Cont.)

- Property
- Let $q=r \div s$
- Then $q$ is the largest relation satisfying $q \times s \subseteq r$
- Definition in terms of the basic algebra operation

Let $r(R)$ and $s(S)$ be relations, and let $S \subseteq R$

$$
r \div s=\Pi_{R-S}(r)-\Pi_{R-S}\left(\left(\Pi_{R-S}(r) \times s\right)-\Pi_{R-S, S}(r)\right)
$$

To see why

- $\Pi_{R-S, S}(r)$ simply reorders attributes of $r$
- $\left.\quad \Pi_{R-S}\left(\Pi_{R-S}(r) \times s\right)-\Pi_{R-S, S}(r)\right)$ gives those tuples t in $\Pi_{R-S}(r)$ such that for some tuple $u \in s, t u \notin r$.


## Assignment Operation

- The assignment operation $(\leftarrow)$ provides a convenient way to express complex queries.
- Write query as a sequential program consisting of
- a series of assignments
- followed by an expression whose value is displayed as a result of the query.
- Assignment must always be made to a temporary relation variable.
- Example: Write $r \div s$ as

$$
\begin{aligned}
& \text { temp1 } \leftarrow \Pi_{R-S}(r) \\
& \text { temp2 } \leftarrow \Pi_{R-S}\left((\text { temp1 xs })-\Pi_{R-S, S}(r)\right) \\
& \text { result }=\text { temp1 }- \text { temp2 }
\end{aligned}
$$

- The result to the right of the $\leftarrow$ is assigned to the relation variable on the left of the $\leftarrow$.
- May use variable in subsequent expressions.


## Bank Example Queries

- Find the names of all customers who have a loan and an account at bank.

$$
\Pi_{\text {customer_name }} \text { (borrower) } \cap \Pi_{\text {customer_name }} \text { (depositor) }
$$

- Find the name of all customers who have a loan at the bank and the loan amount
$\prod_{\text {Customer-name, loan-number, amount }}$ (borrower $\bowtie$ loan)


## Bank Example Queries

- Find all customers who have an account from at least the "Downtown" and the Uptown" branches.
- Query 1

$$
\begin{gathered}
\Pi_{\text {customer_name }}\left(\sigma_{\text {branch_name }}=\text { "Downtown" }\left(\text { depositor } \bowtie_{\text {account }}\right)\right) \cap \\
\Pi_{\text {customer_name }}\left(\sigma_{\text {branch_name }}=\text { "Uptown" }(\text { depositor } \bowtie \text { account })\right)
\end{gathered}
$$

- Query 2

$$
\begin{aligned}
& \Pi_{\text {customer_name, branch_name }}(\text { depositor } \bowtie \text { account }) \\
& \quad \div \rho_{\text {temp(branch_name) }}(\{\text { ("Downtown" ), ("Uptown" ) })
\end{aligned}
$$

Note that Query 2 uses a constant relation.

## Example Queries

- Find all customers who have an account at all branches located in Brooklyn city.

$$
\begin{aligned}
& \prod_{\text {customer_name, branch_name }}(\text { depositor凶 account }) \\
& \div \prod_{\text {branch_name }}\left(\sigma_{\text {branch_city }}=\right.\text { "Brooklyn" } \\
& (\text { branch }))
\end{aligned}
$$

## Extended Relational-Algebra-Operations

- Generalized Projection
- Aggregate Functions
- Outer Join


## Generalized Projection

- Extends the projection operation by allowing arithmetic functions to be used in the projection list.

$$
\prod_{F_{1}, F_{2}, \ldots, F_{n}}(E)
$$

- $E$ is any relational-algebra expression
- Each of $F_{1}, F_{2}, \ldots, F_{n}$ are are arithmetic expressions involving constants and attributes in the schema of $E$.
- Given relation credit_info(customer_name, limit, credit_balance), find how much more each person can spend:

$$
\Pi_{\text {customer_name, limit - credit_balance }} \text { (credit_info) }
$$

## Aggregate Functions and Operations

- Aggregation function takes a collection of values and returns a single value as a result.
avg: average value
min: minimum value
max: maximum value
sum: sum of values
count: number of values
- Aggregate operation in relational algebra

$$
G_{1}, G_{2}, \ldots, G_{n} \vartheta_{F_{1}\left(A_{1}\right), F_{2}\left(A_{2}, \ldots, F_{n}\left(A_{n}\right)\right.}(E)
$$

$E$ is any relational-algebra expression

- $G_{1}, G_{2} \ldots, G_{n}$ is a list of attributes on which to group (can be empty)
- Each $F_{i}$ is an aggregate function
- Each $A_{i}$ is an attribute name


## Aggregate Operation - Example

- Relation $r$ :

| $A$ | $B$ | $C$ |
| :---: | :---: | :---: |
| $\alpha$ | $\alpha$ | 7 |
| $\alpha$ | $\beta$ | 7 |
| $\beta$ | $\beta$ | 3 |
| $\beta$ | $\beta$ | 10 |

- $g_{\text {sum(c) }}(\mathrm{r})$
sum(c)


## Aggregate Operation - Example

- Relation account grouped by branch-name:

| branch_name | account_number | balance |
| :--- | :---: | :---: |
| Perryridge | A-102 | 400 |
| Perryridge | A-201 | 900 |
| Brighton | A-217 | 750 |
| Brighton | A-215 | 750 |
| Redwood | A-222 | 700 |

branch_name $g_{\text {sum(balance) }}$ (account)

| branch_name | sum(balance) |
| :--- | :---: |
| Perryridge | 1300 |
| Brighton | 1500 |
| Redwood | 700 |

## Aggregate Functions (Cont.)

- Result of aggregation does not have a name
- Can use rename operation to give it a name
- For convenience, we permit renaming as part of aggregate operation
branch_name 9 sum(balance) as sum_balance (account)


## Outer Join

- An extension of the join operation that avoids loss of information.
- Computes the join and then adds tuples form one relation that does not match tuples in the other relation to the result of the join.
- Uses null values:
- null signifies that the value is unknown or does not exist
- All comparisons involving null are (roughly speaking) false by definition.
- We shall study precise meaning of comparisons with nulls later


## Outer Join - Example

- Relation loan

| loan_number | branch_name | amount |
| :--- | :--- | :---: |
| L-170 | Downtown | 3000 |
| L-230 | Redwood | 4000 |
| L-260 | Perryridge | 1700 |

- Relation borrower

| Customer_name | loan_number |
| :--- | :--- |
| Jones | $\mathrm{L}-170$ |
| Smith | $\mathrm{L}-230$ |
| Hayes | $\mathrm{L}-155$ |

## Outer Join - Example

- Inner Join

Ioan $\bowtie$ Borrower

| loan_number | branch_name | amount | customer_name |
| :--- | :--- | :---: | :--- |
| L-170 | Downtown | 3000 | Jones |
| L-230 | Redwood | 4000 | Smith |

■ Left Outer Join
Ioan $\triangle \mathbb{X}$ Borrower

| loan_number | branch_name | amount | customer_name |
| :--- | :--- | :---: | :--- |
| L-170 | Downtown | 3000 | Jones |
| L-230 | Redwood | 4000 | Smith |
| L-260 | Perryridge | 1700 | null |

## Outer Join - Example

- Right Outer Join
loan $\bowtie$ borrower

| loan_number | branch_name | amount | customer_name |
| :--- | :--- | :---: | :--- |
| L-170 | Downtown | 3000 | Jones |
| L-230 | Redwood | 4000 | Smith |
| L-155 | null | null | Hayes |

■ Full Outer Join
loan $\triangle \bigvee_{-}$borrower

| loan_number | branch_name | amount | customer_name |
| :--- | :--- | :---: | :--- |
| L-170 | Downtown | 3000 | Jones |
| L-230 | Redwood | 4000 | Smith |
| L-260 | Perryridge | 1700 | null |
| L-155 | null | null | Hayes |

## Null Values

- It is possible for tuples to have a null value, denoted by null, for some of their attributes
- null signifies an unknown value or that a value does not exist.
- The result of any arithmetic expression involving null is null.
- Aggregate functions simply ignore null values (as in SQL)
- For duplicate elimination and grouping, null is treated like any other value, and two nulls are assumed to be the same (as in SQL)


## Null Values

- Comparisons with null values return the special truth value: unknown
- If false was used instead of unknown, then not $(A<5)$ would not be equivalent to $\quad A>=5$
- Three-valued logic using the truth value unknown:
- OR: (unknown or true) = true, (unknown or false) = unknown (unknown or unknown) = unknown
- AND: (true and unknown) = unknown, (false and unknown) = false, (unknown and unknown) = unknown
- NOT: (not unknown) = unknown
- In SQL " $P$ is unknown" evaluates to true if predicate $P$ evaluates to unknown
- Result of select predicate is treated as false if it evaluates to unknown


## Modification of the Database

- The content of the database may be modified using the following operations:
- Deletion
- Insertion
- Updating
- All these operations are expressed using the assignment operator.


## Deletion

- A delete request is expressed similarly to a query, except instead of displaying tuples to the user, the selected tuples are removed from the database.
- Can delete only whole tuples; cannot delete values on only particular attributes
- A deletion is expressed in relational algebra by:

$$
r \leftarrow r-E
$$

where $r$ is a relation and $E$ is a relational algebra query.

## Deletion Examples

- Delete all account records in the Perryridge branch. account $\leftarrow$ account $-\sigma_{\text {branch_name }=\text { "Perryridge" (account }) ~}^{\text {( }}$ )
- Delete all loan records with amount in the range of 0 to 50
- Delete all accounts at branches located in Needham.

$$
\begin{aligned}
& r_{1} \leftarrow \sigma_{\text {branch_city }}=\text { "Needham" }(\text { account } \bowtie \text { branch }) \\
& r_{2} \leftarrow \Pi_{\text {branch_name, account_number, balance }}\left(r_{1}\right) \\
& r_{3} \leftarrow \Pi_{\text {customer_name, account_number }}\left(r_{2} \bowtie \text { depositor }\right) \\
& \text { account } \leftarrow \text { account }-r_{2} \\
& \text { depositor } \leftarrow \text { depositor }-r_{3}
\end{aligned}
$$

## Insertion

- To insert data into a relation, we either:
- specify a tuple to be inserted
- write a query whose result is a set of tuples to be inserted
- in relational algebra, an insertion is expressed by:

$$
r \leftarrow r \cup E
$$

where $r$ is a relation and $E$ is a relational algebra expression.

- The insertion of a single tuple is expressed by letting $E$ be a constant relation containing one tuple.


## Insertion Examples

- Insert information in the database specifying that Smith has $\$ 1200$ in account A-973 at the Perryridge branch.

```
account }\leftarrow\mathrm{ account }\cup{("Perryridge", A-973, 1200)
depositor }\leftarrow depositor \cup {("Smith", A-973)
```

- Provide as a gift for all loan customers in the Perryridge branch, a $\$ 200$ savings account. Let the loan number serve as the account number for the new savings account.

$$
\begin{aligned}
& r_{1} \leftarrow\left(\sigma_{\text {branch_name }}=\right.\text { "Perryridge" } \\
& \text { account } \leftarrow \text { account } \cup \prod_{\text {branch_name, loan_number,_200 }}\left(r_{1}\right) \\
& \text { depositor } \leftarrow \text { depositor } \cup \prod_{\text {customer_name, loan_number }}\left(r_{1}\right)
\end{aligned}
$$

## Updating

- A mechanism to change a value in a tuple without charging all values in the tuple
- Use the generalized projection operator to do this task

$$
r \leftarrow \prod_{F_{1}, F_{2}, \ldots, F_{1},}(r)
$$

- Each $F_{i}$ is either
- the $I^{\text {th }}$ attribute of $r$, if the $l^{\text {th }}$ attribute is not updated, or,
- if the attribute is to be updated $F_{i}$ is an expression, involving only constants and the attributes of $r$, which gives the new value for the attribute


## Update Examples

- Make interest payments by increasing all balances by 5 percent.

```
account }\leftarrow\mp@subsup{\Pi}{\mathrm{ account_number, branch_name, balance * 1.05 (account)}}{
```

- Pay all accounts with balances over $\$ 10,0006$ percent interest and pay all others 5 percent

$\cup \Pi_{\text {account_number, branch_name, balance } * 1.05}\left(\sigma_{\text {BAL }} \leq 10000(\right.$ account $\left.)\right)$


## End of Chapter 2

## Database System Concepts, $5^{\text {th }}$ Ed.

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## Figure 2.3. The branch relation

| branch_name | branch_city | assets |
| :--- | :--- | ---: |
| Brighton | Brooklyn | 7100000 |
| Downtown | Brooklyn | 9000000 |
| Mianus | Horseneck | 400000 |
| North Town | Rye | 3700000 |
| Perryridge | Horseneck | 1700000 |
| Pownal | Bennington | 300000 |
| Redwood | Palo Alto | 2100000 |
| Round Hill | Horseneck | 8000000 |

## Figure 2.6: The loan relation

| loan_number | branch_name | amount |
| :---: | :--- | ---: |
| L-11 | Round Hill | 900 |
| L-14 | Downtown | 1500 |
| L-15 | Perryridge | 1500 |
| L-16 | Perryridge | 1300 |
| L-17 | Downtown | 1000 |
| L-23 | Redwood | 2000 |
| L-93 | Mianus | 500 |

## Figure 2.7: The borrower relation

## customer_name loan_number

Adams<br>Curry<br>Hayes<br>Jackson<br>Jones<br>Smith<br>Smith<br>Williams<br>\[ \begin{aligned} \& \hline L-16<br>\& L-93<br>\& L-15<br>\& L-14<br>\& L-17<br>\& L-11<br>\& L-23<br>\& L-17 \end{aligned} \]

## Figure 2.8: Schema diagram



## Figure 2.9

Result of $\sigma_{\text {branch_name }}=$ "Perryridge" $($ loan $)$

## loan_number branch_name amount

$$
\begin{aligned}
& \mathrm{L}-15 \\
& \mathrm{~L}-16
\end{aligned}
$$

## Perryridge <br> 1500 Perryridge <br> 1300

## Figure 2.10:

## Loan number and the amount of the Ioan

| loan_number | amount |
| :---: | :---: |
| $\mathrm{L}-11$ | 900 |
| $\mathrm{~L}-14$ | 1500 |
| $\mathrm{~L}-15$ | 1500 |
| $\mathrm{~L}-16$ | 1300 |
| $\mathrm{~L}-17$ | 1000 |
| $\mathrm{~L}-23$ | 2000 |
| $\mathrm{~L}-93$ | 500 |

## Figure 2.11: Names of all customers who have either an account or an loan

Adams
Curry
Hayes
Jackson
Jones
Smith
Williams
Lindsay
Johnson
Turner

## Figure 2.12:

## Customers with an account but no loan

## customer_name

## Johnson Lindsay

 Turner
## Figure 2.13: Result of borrower |X| loan

| customer_name | borrower. <br> loan_number | loan. loan_number | branch_name | amount |
| :---: | :---: | :---: | :---: | :---: |
| Adams | L-16 | L-11 | Round Hill | 900 |
| Adams | L-16 | L-14 | Downtown | 1500 |
| Adams | L-16 | L-15 | Perryridge | 1500 |
| Adams | L-16 | L-16 | Perryridge | 1300 |
| Adams | L-16 | L-17 | Downtown | 1000 |
| Adams | L-16 | L-23 | Redwood | 2000 |
| Adams | L-16 | L-93 | Mianus | 500 |
| Curry | L-93 | L-11 | Round Hill | 900 |
| Curry | L-93 | L-14 | Downtown | 1500 |
| Curry | L-93 | L-15 | Perryridge | 1500 |
| Curry | L-93 | L-16 | Perryridge | 1300 |
| Curry | L-93 | L-17 | Downtown | 1000 |
| Curry | L-93 | L-23 | Redwood | 2000 |
| Curry | L-93 | L-93 | Mianus | 500 |
| Hayes | L-15 | L-11 |  | 900 |
| Hayes | L-15 | L-14 |  | 1500 |
| Hayes | L-15 | L-15 |  | 1500 |
| Hayes | L-15 | L-16 |  | 1300 |
| Hayes | L-15 | L-17 |  | 1000 |
| Hayes | L-15 | L-23 |  | 2000 |
| Hayes | L-15 | L-93 |  | 500 |
| -•• | -•• | -•• | * | $\cdots$ |
| . . | . $\cdot$ | . | $\cdots$ | . $\cdot$ |
| Smith | . ${ }^{\text {c }}$ | $\cdots$ | Round Hill | $\cdots$ |
| Smith | L-23 | L-11 | Round Hill | 900 |
| Smith | L-23 | L-14 | Downtown | 1500 |
| Smith | L-23 | L-15 | Perryridge | 1500 |
| Smith | L-23 | L-16 | Perryridge | 1300 |
| Smith | L-23 | L-17 | Downtown | 1000 |
| Smith | L-23 | L-23 | Redwood | 2000 |
| Smith | L-23 | L-93 | Mianus | 500 |
| Williams | L-17 | L-11 | Round Hill | 900 |
| Williams | L-17 | L-14 | Downtown | 1500 |
| Williams | L-17 | L-15 | Perryridge | 1500 |
| Williams | L-17 | L-16 | Perryridge | 1300 |
| Williams | L-17 | L-17 | Downtown | 1000 |
| Williams | L-17 | L-23 | Redwood | 2000 |
| Williams | L-17 | L-93 | Mianus | 500 |

## Figure 2.14

| customer_name | borrower. <br> loan_number | loan. <br> loan_number | branch_name | amount |
| :--- | :---: | :---: | :---: | :---: |
| Adams | $\mathrm{L}-16$ | $\mathrm{~L}-15$ | Perryridge | 1500 |
| Adams | $\mathrm{L}-16$ | $\mathrm{~L}-16$ | Perryridge | 1300 |
| Curry | $\mathrm{L}-93$ | $\mathrm{~L}-15$ | Perryridge | 1500 |
| Curry | $\mathrm{L}-93$ | $\mathrm{~L}-16$ | Perryridge | 1300 |
| Hayes | $\mathrm{L}-15$ | $\mathrm{~L}-15$ | Perryridge | 1500 |
| Hayes | $\mathrm{L}-15$ | $\mathrm{~L}-16$ | Perryridge | 1300 |
| Jackson | $\mathrm{L}-14$ | $\mathrm{~L}-15$ | Perryridge | 1500 |
| Jackson | $\mathrm{L}-14$ | $\mathrm{~L}-16$ | Perryridge | 1300 |
| Jones | $\mathrm{L}-17$ | $\mathrm{~L}-15$ | Perryridge | 1500 |
| Jones | $\mathrm{L}-17$ | $\mathrm{~L}-16$ | Perryridge | 1300 |
| Smith | $\mathrm{L}-11$ | $\mathrm{~L}-15$ | Perryridge | 1500 |
| Smith | $\mathrm{L}-11$ | $\mathrm{~L}-16$ | Perryridge | 1300 |
| Smith | $\mathrm{L}-23$ | $\mathrm{~L}-15$ | Perryridge | 1500 |
| Smith | $\mathrm{L}-23$ | $\mathrm{~L}-16$ | Perryridge | 1300 |
| Williams | $\mathrm{L}-17$ | $\mathrm{~L}-15$ | Perryridge | 1500 |
| Williams | $\mathrm{L}-17$ | $\mathrm{~L}-16$ | Perryridge | 1300 |

## Figure 2.15

## customer_name

## Adams

Hayes

## Figure 2.16

| balance |
| :---: |
| 500 |
| 400 |
| 700 |
| 750 |
| 350 |

## Figure 2.17 <br> Largest account balance in the bank



## Figure 2.18: Customers who live on the same street and in the same city as Smith

## customer_name

## Curry Smith

Figure 2.19: Customers with both an account and a loan at the bank

## customer_name

# Hayes <br> Jones Smith 

## Figure 2.20

| customer_name | loan_number | amount |
| :--- | :---: | ---: |
| Adams | $\mathrm{L}-16$ | 1300 |
| Curry | $\mathrm{L}-93$ | 500 |
| Hayes | $\mathrm{L}-15$ | 1500 |
| Jackson | $\mathrm{L}-14$ | 1500 |
| Jones | $\mathrm{L}-17$ | 1000 |
| Smith | $\mathrm{L}-23$ | 2000 |
| Smith | $\mathrm{L}-11$ | 900 |
| Williams | $\mathrm{L}-17$ | 1000 |

## Figure 2.21

## branch_name

## Brighton

Perryridge

## Figure 2.22

## branch_name

Brighton
Downtown

## Figure 2.23

## customer_name branch_name

Hayes Johnson Johnson Jones Lindsay Smith Turner<br>Perryridge<br>Downtown<br>Brighton<br>Brighton<br>Redwood<br>Mianus<br>Round Hill

## Figure 2.24: The credit_info relation

| customer_name | limit | credit_balance |
| :---: | :---: | :---: |
| Curry | 2000 | 1750 |
| Hayes | 1500 | 1500 |
| Jones | 6000 | 700 |
| Smith | 2000 | 400 |

## Figure 2.25

| customer_name | credit_available |
| :---: | :---: |
| Curry | 250 |
| Jones | 5300 |
| Smith | 1600 |
| Hayes | 0 |

## Figure 2.26: The pt_works relation

| employee_name | branch_name | salary |
| :--- | :--- | ---: |
| Adams | Perryridge | 1500 |
| Brown | Perryridge | 1300 |
| Gopal | Perryridge | 5300 |
| Johnson | Downtown | 1500 |
| Loreena | Downtown | 1300 |
| Peterson | Downtown | 2500 |
| Rao | Austin | 1500 |
| Sato | Austin | 1600 |

## Figure 2.27

## The pt_works relation after regrouping

| employee_name | branch_name | salary |
| :---: | :--- | :--- |
| Rao | Austin | 1500 |
| Sato | Austin | 1600 |
| Johnson | Downtown | 1500 |
| Loreena | Downtown | 1300 |
| Peterson | Downtown | 2500 |
| Adams | Perryridge | 1500 |
| Brown | Perryridge | 1300 |
| Gopal | Perryridge | 5300 |

## Figure 2.28

## branch_name sum of salary <br> Austin <br> Downtown Perryridge <br> 3100 5300 8100

## Figure 2.29

\section*{| branch_name | sum_salary | max_salary |
| :--- | :--- | :--- | <br> | Austin | 3100 | 1600 |
| :--- | :--- | :--- |
| Downtown | 5300 | 2500 |
| Perryridge | 8100 | 5300 |}

## Figure 2.30

## The employee and fitworks relations

| employee_name | street | city |
| :---: | :--- | :--- |
| Coyote | Toon | Hollywood |
| Rabbit | Tunnel | Carrotville |
| Smith | Revolver | Death Valley |
| Williams | Seaview | Seattle |
|  |  |  |
| employee_name branch_name salary <br> Coyote Mesa 1500 <br> Rabbit Mesa 1300 <br> Gates Redmond 5300 <br> Williams Redmond 1500 |  |  |

## Figure 2.31

| employee_name | street | city | branch_name | salary |
| :---: | :--- | :--- | :--- | :--- |
| Coyote | Toon | Hollywood | Mesa | 1500 |
| Rabbit | Tunnel | Carrotville | Mesa | 1300 |
| Williams | Seaview | Seattle | Redmond | 1500 |

## Figure 2.32

| employee_name | street | city | branch_name | salary |
| :---: | :--- | :--- | :---: | :---: |
| Coyote | Toon | Hollywood | Mesa | 1500 |
| Rabbit | Tunnel | Carrotville | Mesa | 1300 |
| Williams | Seaview | Seattle | Redmond | 1500 |
| Smith | Revolver | Death Valley | null | null |

## Figure 2.33

| employee_name | street | city | branch_name | salary |
| :---: | :--- | :--- | :--- | :--- |
| Coyote | Toon | Hollywood | Mesa | 1500 |
| Rabbit | Tunnel | Carrotville | Mesa | 1300 |
| Williams | Seaview | Seattle | Redmond | 1500 |
| Gates | null | null | Redmond | 5300 |

## Figure 2.34

| employee_name | street | city | branch_name | salary |
| :--- | :--- | :--- | :--- | :--- |
| Coyote | Toon | Hollywood | Mesa | 1500 |
| Rabbit | Tunnel | Carrotville | Mesa | 1300 |
| Williams | Seaview | Seattle | Redmond | 1500 |
| Smith | Revolver | Death Valley | null | null |
| Gates | null | null | Redmond | 5300 |

