| | | |
|---|---|---|
| $a$ | $b$ | $c$ |
| $d$ | $a$ | $f$ |
| $c$ | $b$ | $d$ |
| $b$ | $g$ | $a$ |

(a) $R \cup S$

| | | |
|---|---|---|
| $a$ | $b$ | $c$ |
| $c$ | $b$ | $d$ |

(b) $R - S$

| $A$ | $B$ | $C$ | $D$ | $E$ | $F$ |
|---|---|---|---|---|---|
| $a$ | $b$ | $c$ | $b$ | $g$ | $a$ |
| $a$ | $b$ | $c$ | $d$ | $a$ | $f$ |
| $d$ | $a$ | $f$ | $b$ | $g$ | $a$ |
| $d$ | $a$ | $f$ | $d$ | $a$ | $f$ |
| $c$ | $b$ | $d$ | $b$ | $g$ | $a$ |
| $c$ | $b$ | $d$ | $d$ | $a$ | $f$ |

(c) $R \times S$

| $A$ | $C$ |
|---|---|
| $a$ | $c$ |
| $d$ | $f$ |
| $c$ | $d$ |

(d) $\pi_{A,C}(R)$

| $A$ | $B$ | $C$ |
|---|---|---|
| $a$ | $b$ | $c$ |
| $c$ | $b$ | $d$ |

(e) $\sigma_{B=b}(R)$

**Fig. 5.4.** Results of some relational algebra operations.

| A | B | C |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

(a) Relation $R$

| D | E |
|---|---|
| 3 | 1 |
| 6 | 2 |

(b) Relation $S$

| A | B | C | D | E |
|---|---|---|---|---|
| 1 | 2 | 3 | 3 | 1 |
| 1 | 2 | 3 | 6 | 2 |
| 4 | 5 | 6 | 6 | 2 |

(c) $R \underset{B<D}{\bowtie} S$

**Fig. 5.6.** Example of a $<$-join.

| A | B | C |
|---|---|---|
| a | b | c |
| d | b | c |
| b | b | f |
| c | a | d |

(a) Relation $R$

| B | C | D |
|---|---|---|
| b | c | d |
| b | c | e |
| a | d | b |

(b) Relation $S$

| A | B | C | D |
|---|---|---|---|
| a | b | c | d |
| a | b | c | e |
| d | b | c | d |
| d | b | c | e |
| c | a | d | b |

(c) $R \bowtie S$

**Fig. 5.7.** Example of a natural join.

*Example 5.8*: The union of $R$ and $S$ is expressed by the calculus expression

$$\{ t \mid R(t) \vee S(t) \}$$

In words, the above is "the set of tuples $t$ such that $t$ is in $R$ or $t$ is in $S$." Note that union only makes sense if $R$ and $S$ have the same arity, and similarly, the formula $R(t) \vee S(t)$ only makes sense if $R$ and $S$ have the same arity, since tuple variable $t$ is assumed to have some fixed length.

The difference $R - S$ is expressed by $\{ t \mid R(t) \wedge \neg S(t) \}$. If $R$ and $S$ are relations of arity $r$ and $s$, respectively, then $R \times S$ can be expressed in calculus by:

$$\begin{aligned} \{ t^{(r+s)} \mid & (\exists u^{(r)})(\exists v^{(s)})(R(u) \wedge S(v) \\ & \wedge t[1] = u[1] \wedge \cdots \wedge t[r] = u[r] \\ & \wedge t[r+1] = v[1] \wedge \cdots \wedge t[r+s] = v[s]) \} \end{aligned}$$

Recall that $t^{(i)}$ indicates that $t$ has arity $i$. In words, $R \times S$ is the set of tuples $t$ (which we understand to be of length $r+s$) such that there exist $u$ and $v$, with $u$ in $R$, $v$ in $S$; the first $r$ components of $t$ form $u$, and the next $s$ components of $t$ form $v$.

The projection $\pi_{i_1, i_2, \ldots, i_k}(R)$ is expressed by

$$\{ t^{(k)} \mid (\exists u)(R(u) \wedge t[1] = u[i_1] \wedge \cdots \wedge t[k] = u[i_k]) \}$$

The selection $\sigma_F(R)$ is expressed by $\{ t \mid R(t) \wedge F' \}$, where $F'$ is the formula $F$ with each operand $i$, denoting the $i^{th}$ component, replaced by $t[i]$.

As a last example, if $R$ is a relation of arity two, then

$$\{ t^{(2)} \mid (\exists u)(R(t) \wedge R(u) \wedge (t[1] \neq u[1] \vee t[2] \neq u[2])) \}$$

is a calculus expression that denotes $R$ if $R$ has two or more members and denotes the empty relation if $R$ is empty or has only one member.

*Example 5.10:* If $R$ and $S$ are binary relations, their composition in the ordinary set-theoretic sense is expressed by the relational algebra expression $\pi_{1,4}(\sigma_{2=3}(R \times S))$. Using the algorithm of Theorem 5.1, we construct for $R \times S$ the relational calculus expression

$$\{ t \mid (\exists u)(\exists v)(R(u) \wedge S(v) \\ \wedge t[1] = u[1] \wedge t[2] = u[2] \wedge t[3] = v[1] \wedge t[4] = v[2]) \}$$

For $\sigma_{2=3}(R \times S)$ we add to the above formula the term $\wedge\ t[2] = t[3]$. Then, for $\pi_{1,4}(\sigma_{2=3}(R \times S))$ we get the expression

$$\{ w \mid (\exists t)(\exists u)(\exists v)(R(u) \wedge S(v) \\ \wedge t[1] = u[1] \wedge t[2] = u[2] \wedge t[3] = v[1] \wedge t[4] = v[2] \\ \wedge t[2] = t[3] \wedge w[1] = t[1] \wedge w[2] = t[4]) \}$$

| *employee-name* | *street* | *city* |
|---|---|---|
| Coyote | Toon | Hollywood |
| Rabbit | Tunnel | Carrotville |
| Smith | Revolver | Death Valley |
| Williams | Seaview | Seattle |

| *employee-name* | *branch-name* | *salary* |
|---|---|---|
| Coyote | Mesa | 1500 |
| Rabbit | Mesa | 1300 |
| Gates | Redmond | 5300 |
| Williams | Redmond | 1500 |

**Figure 3.31** The *employee* and *ft-works* relations.

| *employee-name* | *street* | *city* | *branch-name* | *salary* |
|---|---|---|---|---|
| Coyote | Toon | Hollywood | Mesa | 1500 |
| Rabbit | Tunnel | Carrotville | Mesa | 1300 |
| Williams | Seaview | Seattle | Redmond | 1500 |

**Figure 3.32** The result of $employee \bowtie ft\text{-}works$.

| *employee-name* | *street* | *city* | *branch-name* | *salary* |
|---|---|---|---|---|
| Coyote | Toon | Hollywood | Mesa | 1500 |
| Rabbit | Tunnel | Carrotville | Mesa | 1300 |
| Williams | Seaview | Seattle | Redmond | 1500 |
| Smith | Revolver | Death Valley | *null* | *null* |

**Figure 3.33** Result of $employee ⟕ ft\text{-}works$.

| *employee-name* | *street* | *city* | *branch-name* | *salary* |
|---|---|---|---|---|
| Coyote | Toon | Hollywood | Mesa | 1500 |
| Rabbit | Tunnel | Carrotville | Mesa | 1300 |
| Williams | Seaview | Seattle | Redmond | 1500 |
| Gates | *null* | *null* | Redmond | 5300 |

**Figure 3.34** Result of *employee* ⟖ *ft-works*.

| *employee-name* | *street* | *city* | *branch-name* | *salary* |
|---|---|---|---|---|
| Coyote | Toon | Hollywood | Mesa | 1500 |
| Rabbit | Tunnel | Carrotville | Mesa | 1300 |
| Williams | Seaview | Seattle | Redmond | 1500 |
| Smith | Revolver | Death Valley | *null* | *null* |
| Gates | *null* | *null* | Redmond | 5300 |

**Figure 3.35** Result of *employee* ⟗ *ft-works*.