MAXIMA Examples and Internals

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Factorial function:

```
(C1) 30!;
(D1)                     265252859812191058636308480000000
(C2) fac (n) := product (i, i, 1, n);
(D2)                       FAC(N) := PRODUCT(I, I, 1, N)
(C3) fac (30);
(D3)                     265252859812191058636308480000000
(C4) fac (n) := block (
  [res],

  res : 1,
  for i : 1 thru n do res : res * i,
  return (res)
);
(D4) FAC(N) := BLOCK([RES], RES : 1, FOR I THRU N DO RES : RES I, RETURN(RES
(C5) fac (30);
(D5)                     265252859812191058636308480000000
(C6) kill(fac);
(D6)                                   DONE
(C7) fac(n) := if (n <= 1) then 1 else n*fac(n-1);
(D7)               FAC(N) := IF N <= 1 THEN 1 ELSE N FAC(N - 1)
(C8) fac(30);
(D8)                     265252859812191058636308480000000
```

Let's use subscripted functions to re-use previously computed factorials:

```
(C1) fac[1](x) := 1;
(D1)                                FAC (X) := 1
                                       1
(C2) fac[n](x) := n * fac[n-1](x);
(D2)                         FAC (X) := N FAC     (X)
```

```
                               N            N - 1
(C3) fac[30];
(D3)           LAMBDA([X], 265252859812191058636308480000000)
(C4) fac[30](x);
(D4)                  265252859812191058636308480000000
(C5) showtime;
(D5)                                FALSE
(C6) showtime : true;
Evaluation took 0.00 seconds (0.00 elapsed)
(D6)                                TRUE
(C7) fac[40](x);
Evaluation took 00.60 seconds (0.00 elapsed)
(D7)          815915283247897734345611269596115894272000000000
(C8) fac[40](x);
Evaluation took 0.00 seconds (0.00 elapsed)
(D8)          815915283247897734345611269596115894272000000000
(C9) 40!;
Evaluation took 0.00 seconds (0.00 elapsed)
(D9)          815915283247897734345611269596115894272000000000
(C10) 40!;
Evaluation took 0.00 seconds (0.00 elapsed)
(D10)         815915283247897734345611269596115894272000000000
(C11) kill(fac);
Evaluation took 00.10 seconds (0.00 elapsed)
(D11)                               DONE
(C12) fac(n) := product(i, i, 1, n);
Evaluation took 0.00 seconds (0.00 elapsed)
(D12)                    FAC(N) := PRODUCT(I, I, 1, N)
(C13) fac(40);
Evaluation took 0.13 seconds (0.00 elapsed)
(D13)         815915283247897734345611269596115894272000000000
(C14) fac(40);
Evaluation took 0.13 seconds (0.00 elapsed)
(D14)         815915283247897734345611269596115894272000000000
(C15) showtime : false;
(D15)                               FALSE
```

More subscripted functions:

Chebyshev polynomials:

$$T_0(x) \equiv 1$$
$$T_1(x) \equiv x$$
$$T_j(x) \equiv 2\,x\,T_{j-1}(x) - T_{j-2}(x)$$

```
(C1) t[0](x) := 1;
(D1)                                   T (X) := 1
                                        0
(C2) t[1](x) := x;
(D2)                                   T (X) := X
                                        1
(C3) t[j](x) := 2*x*t[j-1](x) - t[j-2](x);
(D3)                        T (X) := 2 X T     (X) - T     (X)
                             J             J - 1       J - 2
(C4) t[2](x);
                                           2
(D4)                                    2 X  - 1
(C5) t[3](x);
                                           2
(D5)                                2 X (2 X  - 1) - X
(C6) expand (%);
                                          3
(D6)                                   4 X  - 3 X
(C7) t[7](x), expand;
                                  7        5       3
(D7)                          64 X  - 112 X  + 56 X  - 7 X
(C8) t[5];
                                            5       3
(D8)                        LAMBDA([X], 16 X  - 20 X  + 5 X)
```

Your very own differentiation function:

Consider the following simple differentiation system:

$$\frac{dx}{dx} = 1$$

$$\frac{dy}{dx} = 0, \text{ when } x \neq y$$

$$\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

```
mydiff (expr,var) :=
  if atom(expr) then (
    if expr = var then 1 else 0
  )
  else if part(expr,0) = "+" then (
    mydiff(part(expr,1),var) + mydiff(part(expr,2),var)
  )
  else if part(expr,0) = "*" then (
    part(expr,1)*mydiff(part(expr,2),var) +
    part(expr,2)*mydiff(part(expr,1),var)
  )
  else if part(expr,0) = "//" then (
    'mydiff(expr,var)
  )
  else (
    'mydiff(expr,var)
);
```

```
(C1) mydiff (expr,var) :=
  if atom(expr) then (
    if expr = var then 1 else 0
  )
  else if part(expr,0) = "+" then (
    mydiff(part(expr,1),var) + mydiff(part(expr,2),var)
  )
  else if part(expr,0) = "*" then (
    part(expr,1)*mydiff(part(expr,2),var) +
    part(expr,2)*mydiff(part(expr,1),var)
  )
  else if part(expr,0) = "//" then (
    'mydiff(expr,var)
  )
  else (
    'mydiff(expr,var)
);
(D1) MYDIFF(EXPR, VAR) := IF ATOM(EXPR) THEN (IF EXPR = VAR THEN 1 ELSE 0)

 ELSE (IF PART(EXPR, 0) = "+" THEN MYDIFF(PART(EXPR, 1), VAR)

 + MYDIFF(PART(EXPR, 2), VAR) ELSE (IF PART(EXPR, 0) = "*"

 THEN PART(EXPR, 1) MYDIFF(PART(EXPR, 2), VAR)

 + PART(EXPR, 2) MYDIFF(PART(EXPR, 1), VAR)

 ELSE (IF PART(EXPR, 0) = "//" THEN 'MYDIFF(EXPR, VAR)

 ELSE 'MYDIFF(EXPR, VAR))))
(C2) mydiff(x,x);
(D2)                                    1
(C3) mydiff(a,a);
(D3)                                    1
(C4) mydiff(foo,bar);
(D4)                                    0
(C5) mydiff((x+1)*(x+2), x);
```

```
(D5)                               2 X + 3
(C6) mydiff^U
diff ((x+1)*(x+2),x);
(D6)                               2 X + 3
(C7) mydiff(u(x)*v(x), x);
(D7)              U(X) MYDIFF(V(X), X) + V(X) MYDIFF(U(X), X)
(C8) mydiff (u(x) + v(x), x);
(D8)                   MYDIFF(V(X), X) + MYDIFF(U(X), X)
(C9) mydiff (u(x) / v(x), x);
                                      U(X)
(D9)                           MYDIFF(----, X)
                                      V(X)
```

HOMEWORK:

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1. Complete the "mydiff" function to handle quotients.