Row-Cholesky Factorization

for $j = 1$ to n
  for k in $S(L_{*,j})$
    rmod(j,k);
  rdiv(j)

Forms of Cholesky Factorizations

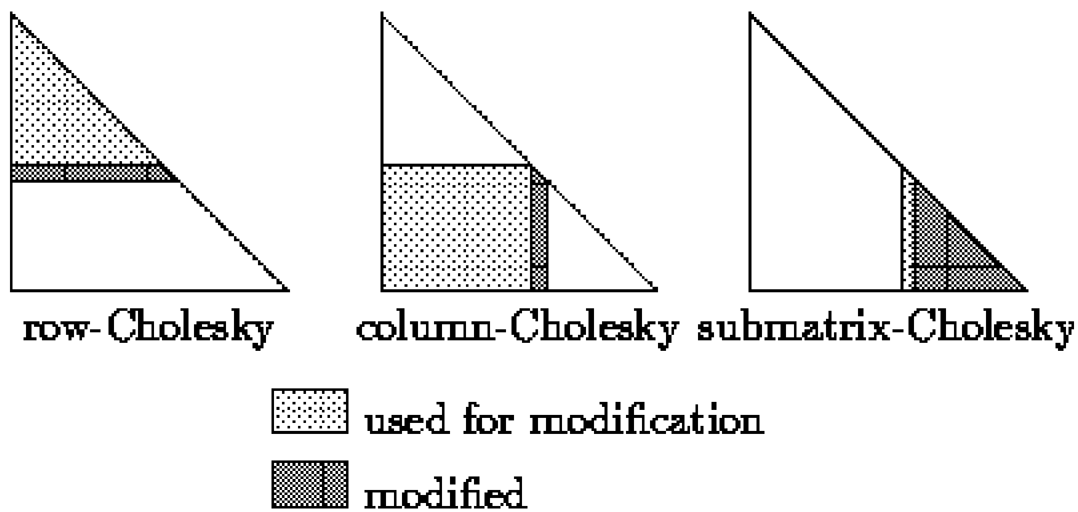


Figure 5: Three forms of Cholesky factorization.

Column Cholesky Factorization

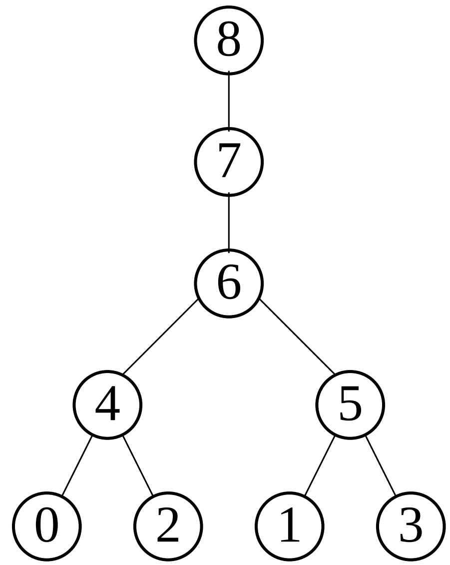
for $j = 1$ to n
    for $k = 1$ to $j\text{-}1$
        for $i = j, n$ {cmod (j, k)}
            $a_{ij} = a_{ij} - a_{ik}.a_{jk}$
    $a_{jj} = \mathrm{sqrt}(a_{jj})$
    for $k = j+1, n$ {cdiv(j)}
        $a_{kj} = a_{kj} / a_{jj}$

for $j = 1$ to n
    for k in $S(L_{j,*})$
        cmod(j,k);
    cdiv(j)

Sparse matrix and its corresponding elimination tree:

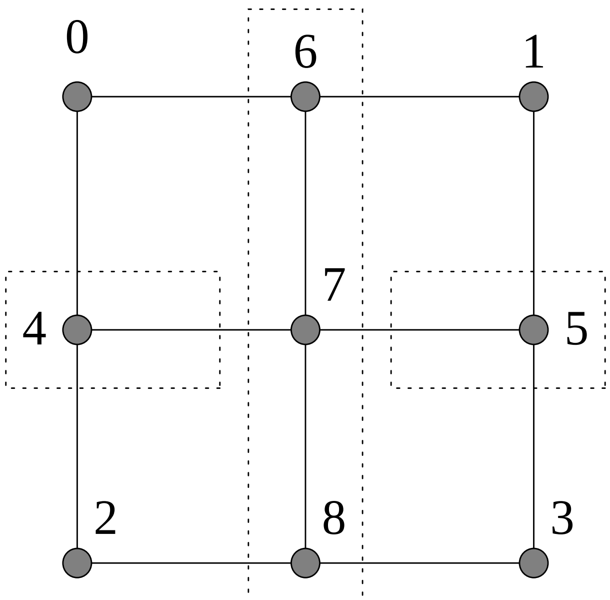
	0	1	2	3	4	5	6	7	8
0	X				X		X		
1		X				X	X		
2			X		X				X
3				X		X			X
4	X		X		X		□	X	□
5		X		X		X	□	X	□
6	X	X			□	□	X	X	□
7					X	X	X	X	X
8			X	X	□	□	□	X	X

(a)



(b)

Nested Dissection Ordering of a 3x3 Grid:



(a) A 3 X 3 finite difference grid with nested dissection numbering

	0	1	2	3	4	5	6	7	8
0	X				X		X		
1		X				X	X		
2			X		X				X
3				X		X			X
4	X		X		X		□	X	□
5		X		X		X	□	X	□
6	X	X			□	□	X	X	□
7					X	X	X	X	X
8			X	X	□	□	□	X	X

(b) The resulting sparse matrix with fill-in on factorization

Sparse Matrix Factorization:

Fill-in and the role of re-ordering to reduce fill.

	0	1	2	3	4	5	6	7
0	X	X			X			X
1	X	X	X					
2		X	X	X				
3			X	X				
4	X				X	X		X
5					X	X	X	
6						X	X	
7	X				X			X

(a) A random sparse matrix

	0	1	2	3	4	5	6	7
0	X	X			X			X
1	X	X	X		□			□
2		X	X	X	□			□
3			X	X	□			□
4	X	□	□	□	X	X		X
5					X	X	X	□
6						X	X	□
7	X	□	□	□	X	□	□	X

(b) Fill-in on factorization

	(3)	(6)	(5)	(4)	(7)	(0)	(1)	(2)
0 (3)	X							X
1 (6)		X	X					
2 (5)		X	X	X				
3 (4)			X	X	X	X		
4 (7)				X	X	X		
5 (0)				X	X	X	X	
6 (1)						X	X	X
7 (2)	X						X	X

(c) A no-fill-in reordering

Minimum Degree Ordering

- After k steps of elimination, let c_i be the number of non-zeros in column i of remaining matrix.
- Let r_i be the number of non-zeros in row i of remaining matrix.
- For each row, compute $(r_i-1)(c_i-1)$.
- Pick the row with the min value as the pivot.

CS590C: Introduction to Computational Science

Lecture 10: Sparse Matrix Methods.

This lecture is intended to familiarize you with:

- ❏ Ordering sparse matrices for factorization.
- ❏ Algorithms for sparse factorization.
- ❏ Parallelism in sparse factorization.