CS 590V: Parallel Computing
Spring 1997: Midterm 1
March 24, 1998, 9:00 - 10:15 AM

1. (20 points) A *mesh of trees* is a network that imposes a tree interconnection on a grid of processors. A $\sqrt{p} \times \sqrt{p}$ mesh of trees is constructed as follows. Starting with a $\sqrt{p} \times \sqrt{p}$ grid of processors a complete binary tree is imposed on each row of the grid. Then a complete binary tree is imposed on each column of the grid. Figure 1 illustrates the construction of a 4×4 mesh of trees. Assume that the nodes at intermediate levels are switching elements. Determine the bisection width, diameter, and total number of switching elements in a $\sqrt{p} \times \sqrt{p}$ mesh of processors.

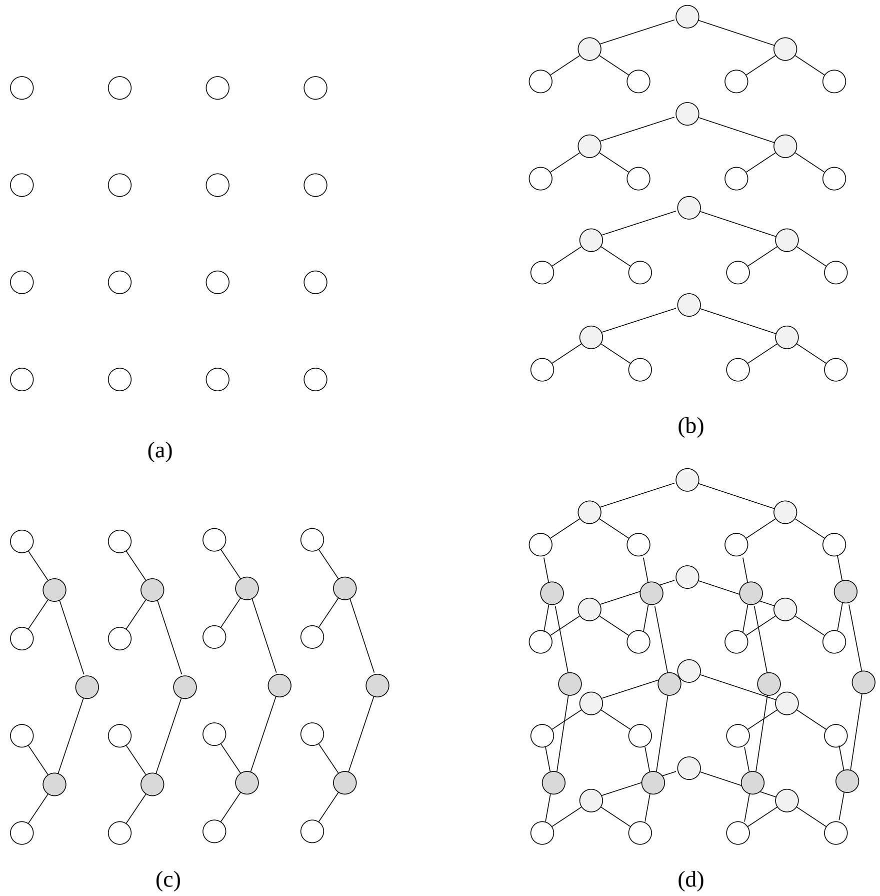

Figure 1: The construction of a 4×4 mesh of trees: (a) a 4×4 processor grid, (b) complete binary trees imposed over individual rows, (c) complete binary trees imposed over each column, and (c) the complete 4×4 mesh of trees.

2. (20 points) An alternate way of performing a one-to-all broadcast of a large message is to break it into p parts. The broadcast is performed in two steps: (i) a one-to-all personalized communication; and (ii) an all to all broadcast. What is the parallel time of this algorithm (do not ignore any constants)? Is it better than the tree-based broadcast algorithm?

3. (10 points) The parallel runtime of FFT on a hypercube connected parallel computer is given by $t_p = t_c(n/p)\log p + t_w(n/p)\log p$ (assuming $t_s = 0$). The serial runtime of an n-point FFT is $t_c n \log n$. What is the isoefficiency of this formulation? (Note: you will have to be careful about constants when computing isoefficiency in order terms.)

4. (10 points) What is the largest number of processors you can use to add a list of n numbers cost-optimally?

5. (20 points) Consider an $n \times n$ matrix that is distributed in a checkerboard fashion (2-D partitioning) on p processors. What is the runtime for computing the transpose of this matrix on (i) a hypercube; (ii) a mesh.

6. (20 points) Given a weighted graph G(V,E), the minimum spanning tree contains all the nodes in the graph and a subset of the edges of minimum weight to keep the graph connected. The minimum spanning tree starts by inserting an arbitrarily chosen node into the current tree T. It then selects one of the other nodes in V - T such that the weight on the edge connecting this node to any node in T is minimum. This process is continued until T = V.

   Write a MPI-like pseudocode for this algorithm. The input to the program is the adjacency matrix W (such that $w_{i,j} \in W$ represents the weight of the edge between nodes i and j). The program uses num_procs processors and outputs the list of edges R in the minimum spanning tree.