CS 525: Parallel Computing

Spring 1999

Problem Set 4: Analysis of Parallel Algorithms

Due Date: Apr 15, 1999 in class

No late submissions will be accepted

General Instructions: Do not copy homework solutions (either verbatim or in ideas). Any evidence of copying will be dealt with in accordance with established Purdue Policies. Write algorithms clearly in steps. Where analysis is required, derive time for each step and write along with the step.

Collective Communication Operations

1. In a k-all broadcast, there are k processors simultaneously performing one-to-all broadcasts of m word messages. Give an algorithm that accomplishes this in time $t_s \log p + t_w m(\log(p/k) + k - 1)$. Assume that the messages cannot be split and that k is a power of 2.

2. Show that if the messages can be split and their parts routed independently, then the same operation can be performed in time $t_s \log(p^2/k) + t_w m(k+1)$.

3. Show that a k-all personalized communication can be accomplished in time $t_s(\log(p/k) + k - 1) + t_w m(p-1)$.

Metrics and Analysis Techniques

4. Scaled speedup is the speedup function when the problem size (input size) is increased linearly with the number of processors. Compute speedup and scaled speedup for adding a list of n numbers with $n = 10^6$ (on 1 processor), $t_s = 50\mu s$, $t_w = 0.05\mu s/w$, $t_c = 2$ns as a function of p. Plot these curves for $p = 1$ to 256.

5. What is the maximum number of processors you can use to cost-optimally add a list of n numbers on a PRAM? Note that a PRAM is a shared memory model in which all memory can be accessed in the same time.

6. The parallel runtime of FFT on a p processor machine is given by:

$$T_p = t_c \frac{n \log n}{p} + t_w \frac{n \log p}{p}$$

The corresponding serial runtime is $t_c n \log n$. Compute the isoefficiency of this algorithm. Pay particular attention to constants.

Dense Matrix Algorithms

7. Cannon's matrix multiplication algorithm has a parallel runtime of

$$T_p = t_c \frac{n^3}{p} + 4\sqrt{p}\, t_s + 4 t_w \frac{n^2}{\sqrt{p}}$$

Compute the isoefficiency of this formulation.

8. Show that the cumulative idle time over all processors in gaussian elimination for a block mapping is $\Theta(n^3)$ whether the matrix is striped or checkerboarded. Show that this time reduces to $\Theta(n^2)$ if cyclic mapping is used.

Sorting

9. What is the worst case parallel runtime of quicksort on a hypercube (for the algorithm discussed in class in which processors are paired with corresponding processor in high half and numbers are exchanged).

10. If at each step of the quicksort algorithm, it can be guaranteed that the longer of the two sublists is no larger than μn where n is the length of the original list, derive the parallel runtime of the above algorithm.

Graph Algorithms

11. In Prim's algorithm for computing MSTs, what is the maximum number of processors that can be used cost-optimally? What is the runtime using $\Theta(n)$ processors?