CS514 Fall '00
Numerical Analysis
Homework 5
Due date: Dec 7, 2000 (before class)

1. Consider the differential equation:

$$\frac{d^2U}{dx^2} + \frac{1}{4}U = 0, 0 < x < \pi$$

With boundary conditions $U(0) = 1$ and $U(\pi) = 0$. Using a cubic polynomial $U\ (x : a) = a_1 + a_2x + a_3x^2 + a_4x^3$, apply the collocation method with $x = \pi/3$ and $x = 2\pi/3$ as collocation points. The exact solution is $U(x) = cosx/2$.

2. Repeat problem 1 with the galerkin method.

3. Repeat problem 1 using the least squares method.

4. Consider the differential equation:

$$\frac{d}{dx}((x+1)\frac{dU(x)}{dx}) = 0, 1 < x < 2$$

with boundary conditions $U(1) = 1$, $\tau(2) = 1$. The flux τ is given by $-(x+1)dU/dx$.

   (a) Using the Galerkin method with a quadratic polynomial for the trial solution $U\ (x : a) = a_1 + a_2x + a_3x^2$, obtain an approximate solution for the function.

   (b) Obtain a second solution using a cubic polynomial for the trial solution.

   (c) Compare the two solutions and estimate their accuracy. Also compare with the exact solution $U(x) = 1 - ln((x+1)/2)$. Do your solutions appear to converge to the exact solution.

5. Repeat above question, except, in this case, use linear approximation $U\ (x : a) = a_1 + a_2x$ but with 3 elements with interior joints at 4/3 and 5/3.