CS514 Fall '00
Numerical Analysis
Homework 1
Due date: Sept 12, 2000 (before class)

1. Answer the following questions from your text in Chapter 1:
   Problems: 10, 11, 24, 25, 31, 41.

2. **(a)** Show that the following three schemes can be used to recursively generate the sequence $\{\frac{1}{2^n}\}_{n=0}^{\infty}$.

   (1) $r_n = (\frac{1}{2})r_{n-1}$, for $n = 1, 2, \cdots$.

   (2) $p_n = (\frac{3}{2})p_{n-1} - (\frac{1}{2})p_{n-2}$, for $n = 2, 3, \cdots$.

   (3) $p_n = (\frac{5}{2})q_{n-1} - q_{n-2}$, for $n = 2, 3, \cdots$.

   (b) Use MATLAB to generate the first ten numerical approximations to the sequence $\{x_n\} = \{\frac{1}{2^n}\}$ using the schemes in (a):

   For (1) $r_0 = 0.994$,
   For (2) $p_0 = 1$ and $p_1 = 0.497$,
   For (3) $q_0 = 1$ and $_1 = 0.497$.

   Produce the numerical results to two tables: one for approximation values and the other for errors. The table formats are as:

   Table 1. For approximation values

n	x	r	p	q
1				
...	...	...	...	...

   Table 2. For errors, $|x_n - r_n|, |x_n - p_n|$, and $|x_n - q_n|$

n	x-r	x-p	x-q
1			
...	...	...	...

   (c) Use MATLAB to plot the errors of the three schemes and indicate which scheme is stable or unstable.

3. **(a)** Consider the evaluation of $I_n = \int_0^1 x^n e^{x-1} dx$, for some $n > 1$. Note that $I_1 = \frac{1}{e} \approx 0.3678794$. Please show that I_n can be evaluated recursively by

$$I_n = 1 - nI_{n-1}.$$

(b) Use MATLAB to evaluate I_{12}, output the results to a table,

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   n  |   In
   1  |
  ... |   ...
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plot the result, and discuss its condition (ill-condition or well-condition).

(c) Above method seems ill-conditioned, how to improve it? Also, write a MATLAB program to output the results in a table (i.e. record each iteration result to the table) and plot it. Discuss why the new method is better.