ROADSTRIX

RADIX SORT

Radix Sort

- Unlike other sorting methods, radix sort considers the structure of the keys

- Assume keys are represented in a base M number system (M = radix), i.e., if M = 2, the keys are represented in binary

$$\begin{array}{ccccccc} & & 8 & 4 & 2 & 1 & \text{weight} \\ 9 & = & \boxed{1} & \boxed{0} & \boxed{0} & \boxed{1} & (b = 4) \\ & & 3 & 2 & 1 & 0 & \text{bit \#} \end{array}$$

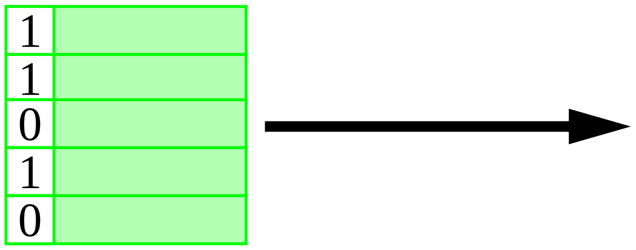
- Sorting is done by comparing bits in the same position

- Extension to keys that are alphanumeric strings

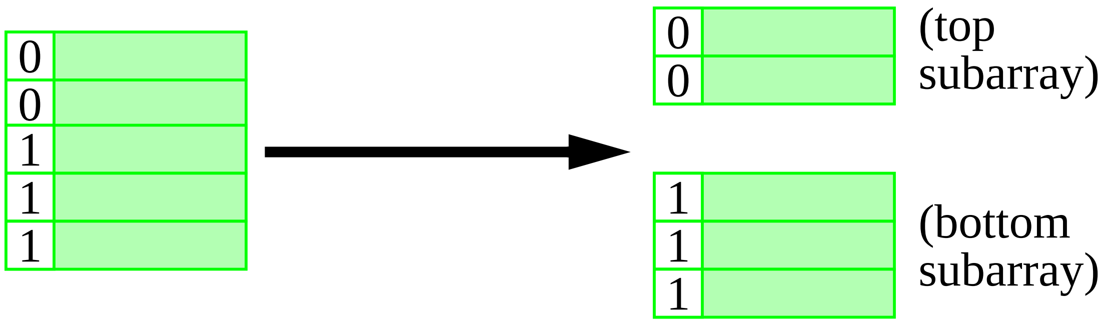
Radix Exchange Sort

Examine bits from ***left*** to ***right***:

1. **Sort array with respect to leftmost bit**



2. **Partition array**



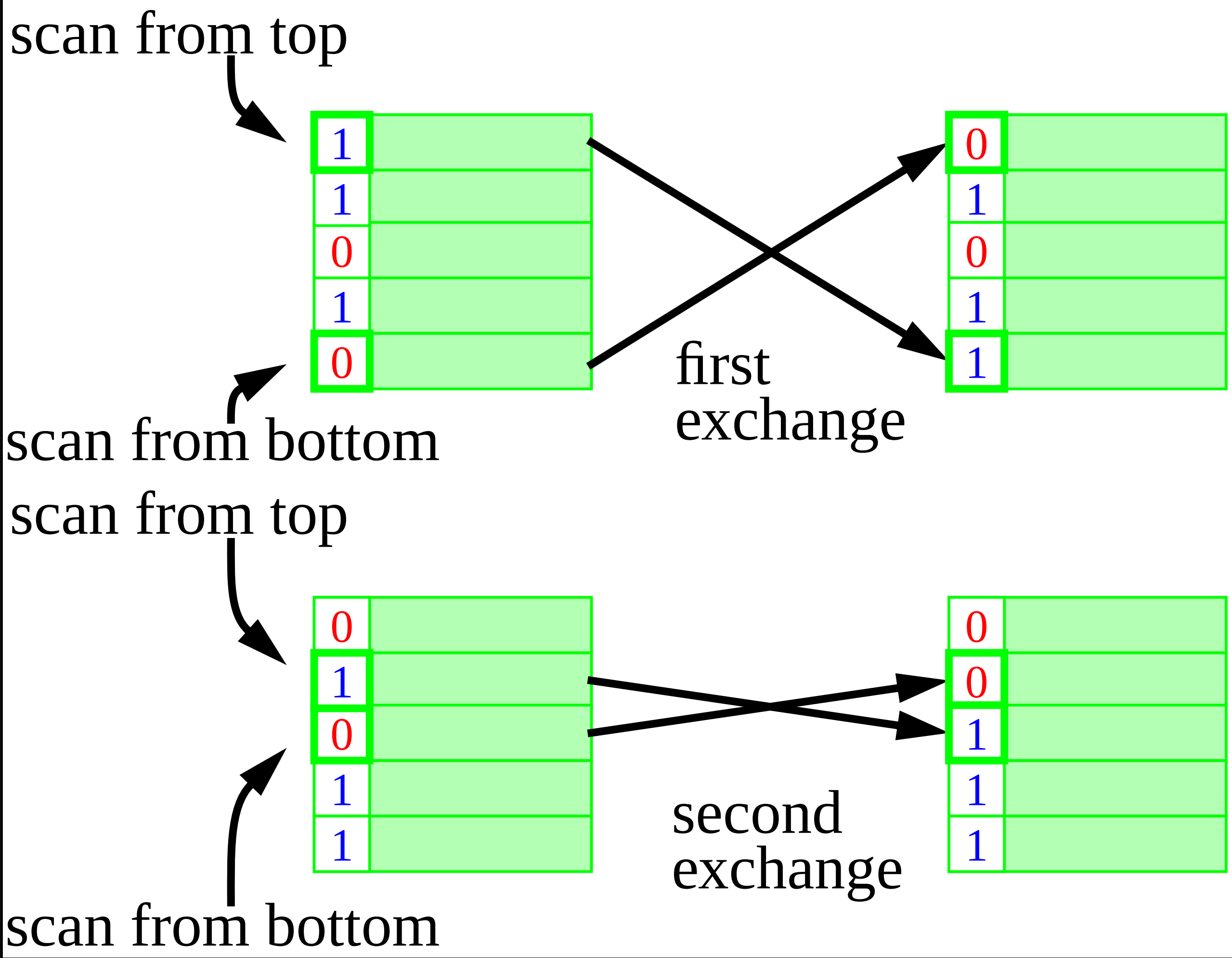
3. **Recursion**
   - recursively sort top subarray, ignoring leftmost bit
   - recursively sort bottom subarray, ignoring leftmost bit

Time: O(b N)

Radix Exchange Sort

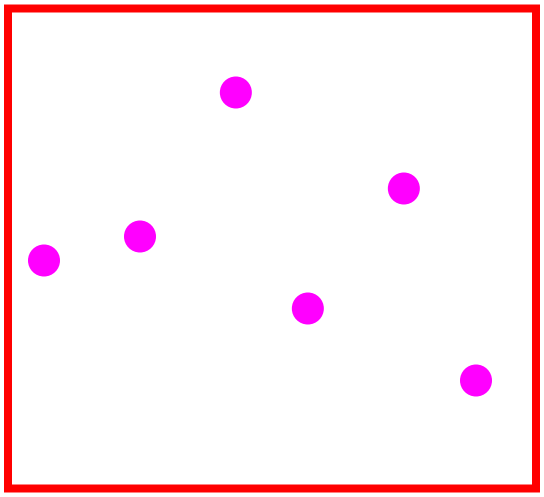
How do we do the sort from the previous page?
Same idea as partition in Quicksort.

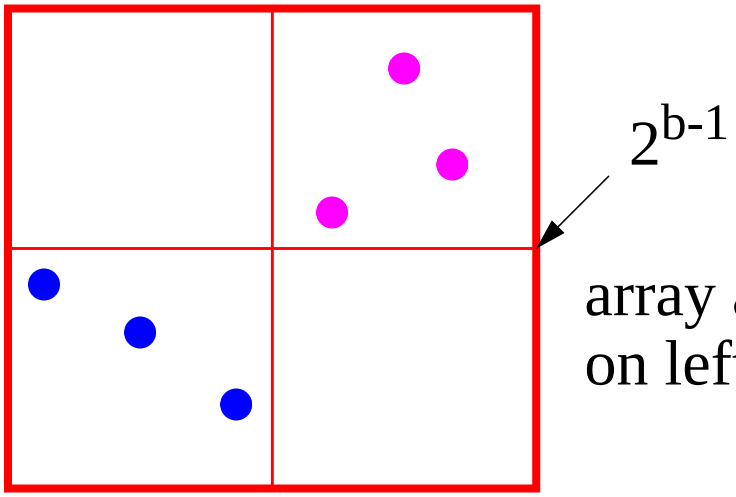
```
repeat
    scan top-down to find key starting with 1;
    scan bottom-up to find key starting with 0;
    exchange keys;
until  scan indices cross;
```



Radix Exchange Sort

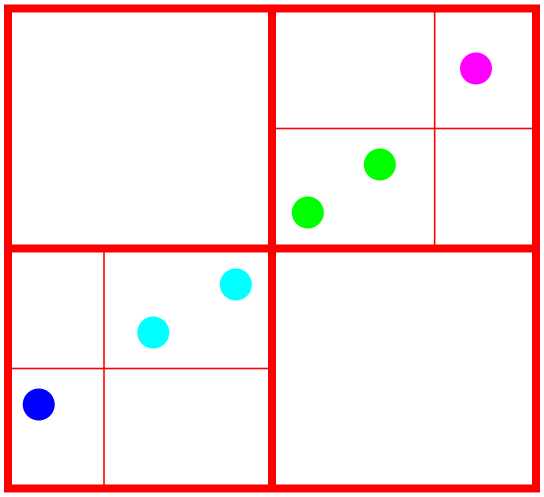
array before sort





array after sort
on leftmost bit

array after recursive
sort on second from
leftmost bit



Radix Exchange Sort vs. Quicksort

Similarities

- both partition array
- both recursively sort sub-arrays

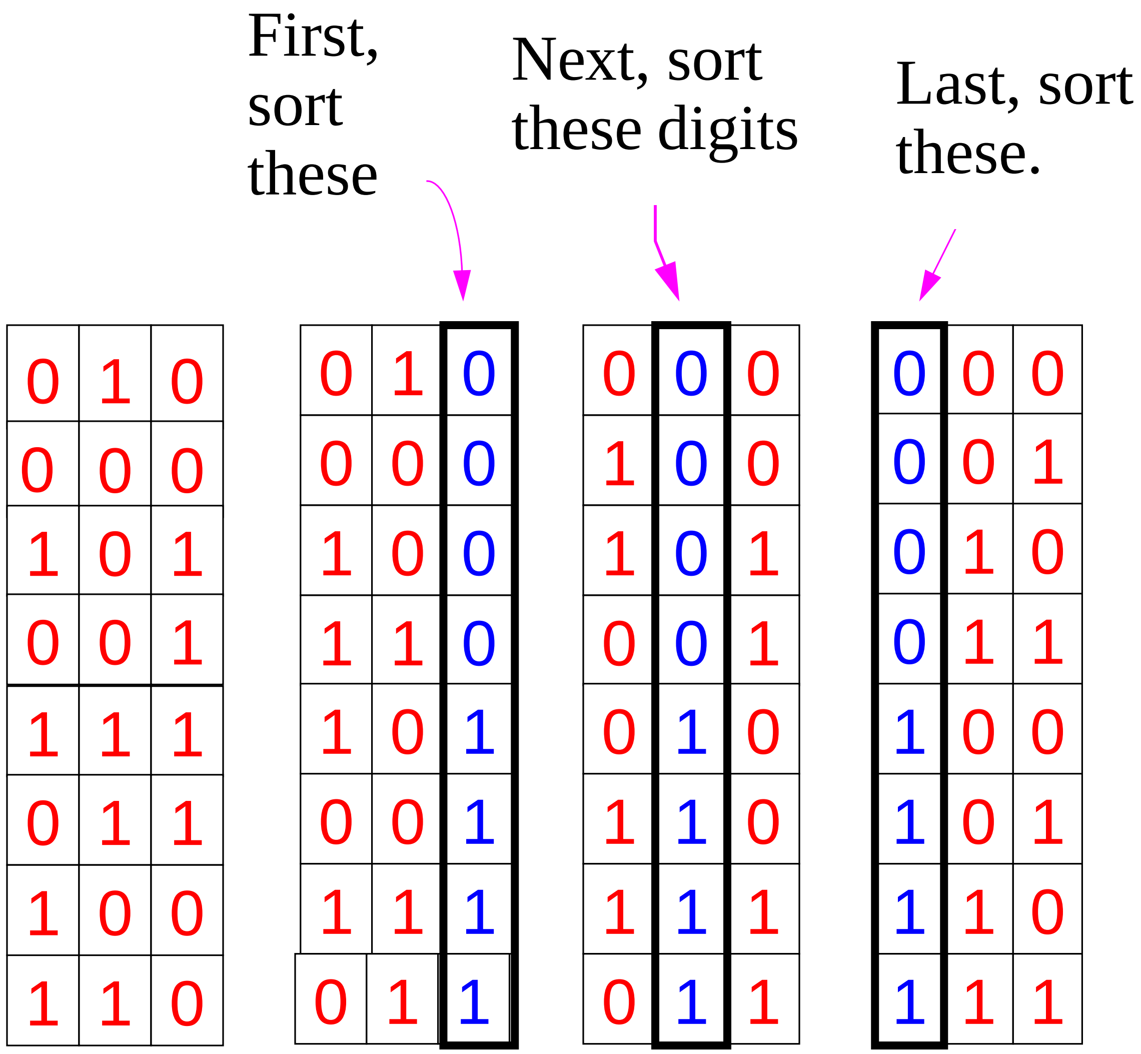
Differences

- ***Method of partitioning***
  - radix exchange divides array based on greater than or less than 2^{b-1}
  - quicksort partitions based on greater than or less than some element of the array
- ***Time complexity***
  - Radix exchange O (bN)
  - Quicksort average case O (N log N)
  - Quicksort worst case O (N^2)

Straight Radix Sort

Examines bits from ***right*** to ***left***

```
for  k := 0  to  b–1
    sort the array in a stable way,
    looking only at bit k
```



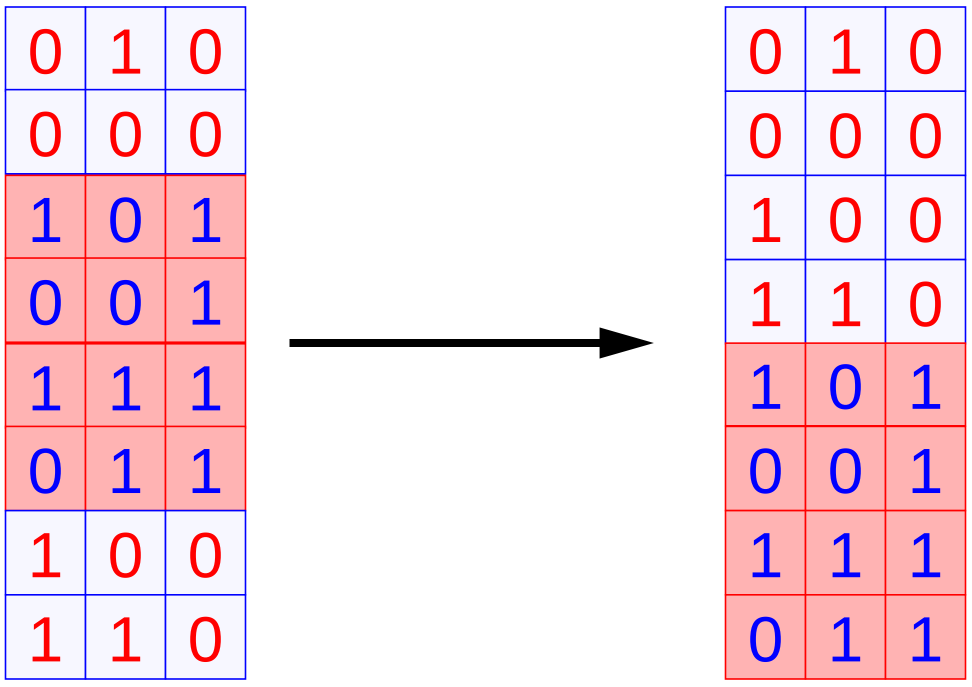
0	1	0		0	1	0		0	0	0		0	0	0
0	0	0		0	0	0		1	0	0		0	0	1
1	0	1		1	0	0		1	0	1		0	1	0
0	0	1		1	1	0		0	0	1		0	1	1
1	1	1		1	0	1		0	1	0		1	0	0
0	1	1		0	0	1		1	1	0		1	0	1
1	0	0		1	1	1		1	1	1		1	1	0
1	1	0		0	1	1		0	1	1		1	1	1

Note order of these bits after sort.

I forgot what it means to “sort in a stable way”!!!

In a stable sort, the initial relative order of equal keys is unchanged.

For example, observe the first step of the sort from the previous page:



Note that the relative order of those keys ending with 0 is unchanged, and the same is true for elements ending in 1

The Algorithm is Correct (right?)

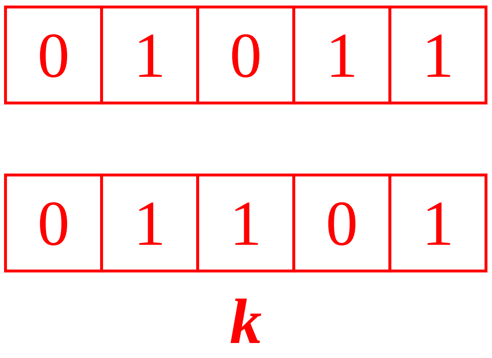
- We show that any two keys are in the correct relative order at the end of the algorithm
- Given two keys, let ***k*** be the leftmost bit-position where they differ

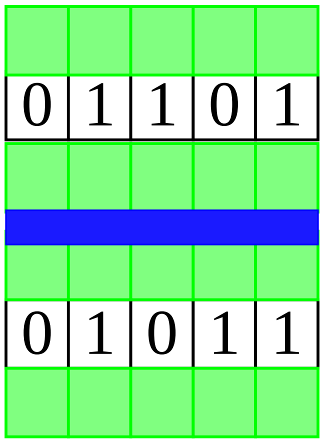
0	1	0	1	1
0	1	1	0	1
		k		

- At step ***k*** the two keys are put in the correct relative order
- Because of stability, the successive steps do not change the relative order of the two keys

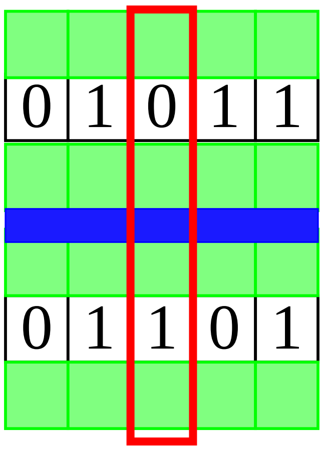
For Instance,

Consider a sort on an array with these two keys:

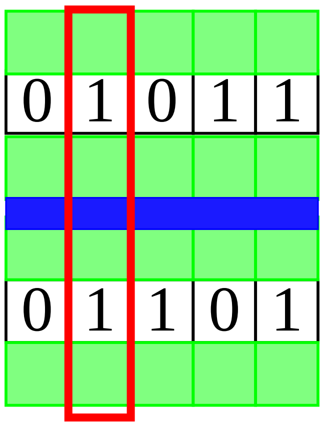




It makes no difference what order they are in when the sort begins.

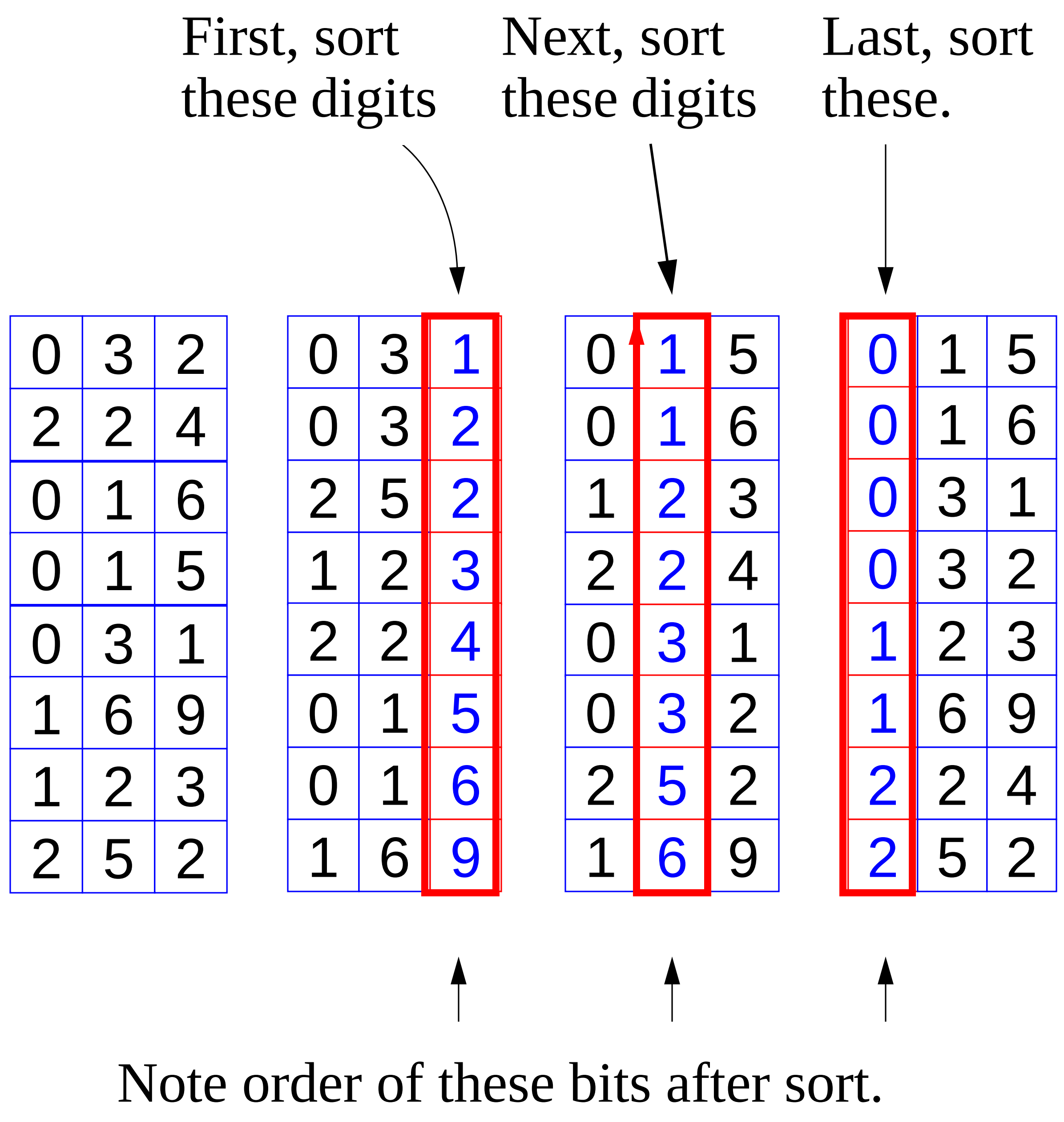


When the sort visits bit ***k*,** the keys are put in the correct relative order.



Because the sort is stable, the order of the two keys will not be changed when bits > ***k*** are compared.

Radix sorting can be applied to decimal numbers



0	3	2
2	2	4
0	1	6
0	1	5
0	3	1
1	6	9
1	2	3
2	5	2

0	3	1
0	3	2
2	5	2
1	2	3
2	2	4
0	1	5
0	1	6
1	6	9

0	1	5
0	1	6
1	2	3
2	2	4
0	3	1
0	3	2
2	5	2
1	6	9

0	1	5
0	1	6
0	3	1
0	3	2
1	2	3
1	6	9
2	2	4
2	5	2

Voila!

Straight Radix Sort Time Complexity

```
for k := 0 to b-1
    sort the array in a *stable* way,
    looking only at bit k
```

Suppose we can perform the stable sort above in O(N) time. The total time complexity would be

O(bN).

As you might have guessed, we can perform a stable sort based on the keys' ***k***th digit in O(N) time.

The method, you ask? Why it's Bucket Sort, of course.

$\longrightarrow$

Bucket Sort

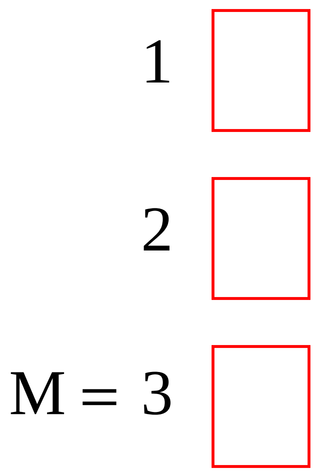
- N numbers
- Each number $\in$ {1, 2, 3, ... M}
- Stable
- Time: O (N + M)

For example, M = 3 and our array is:

2	1	3	1	2

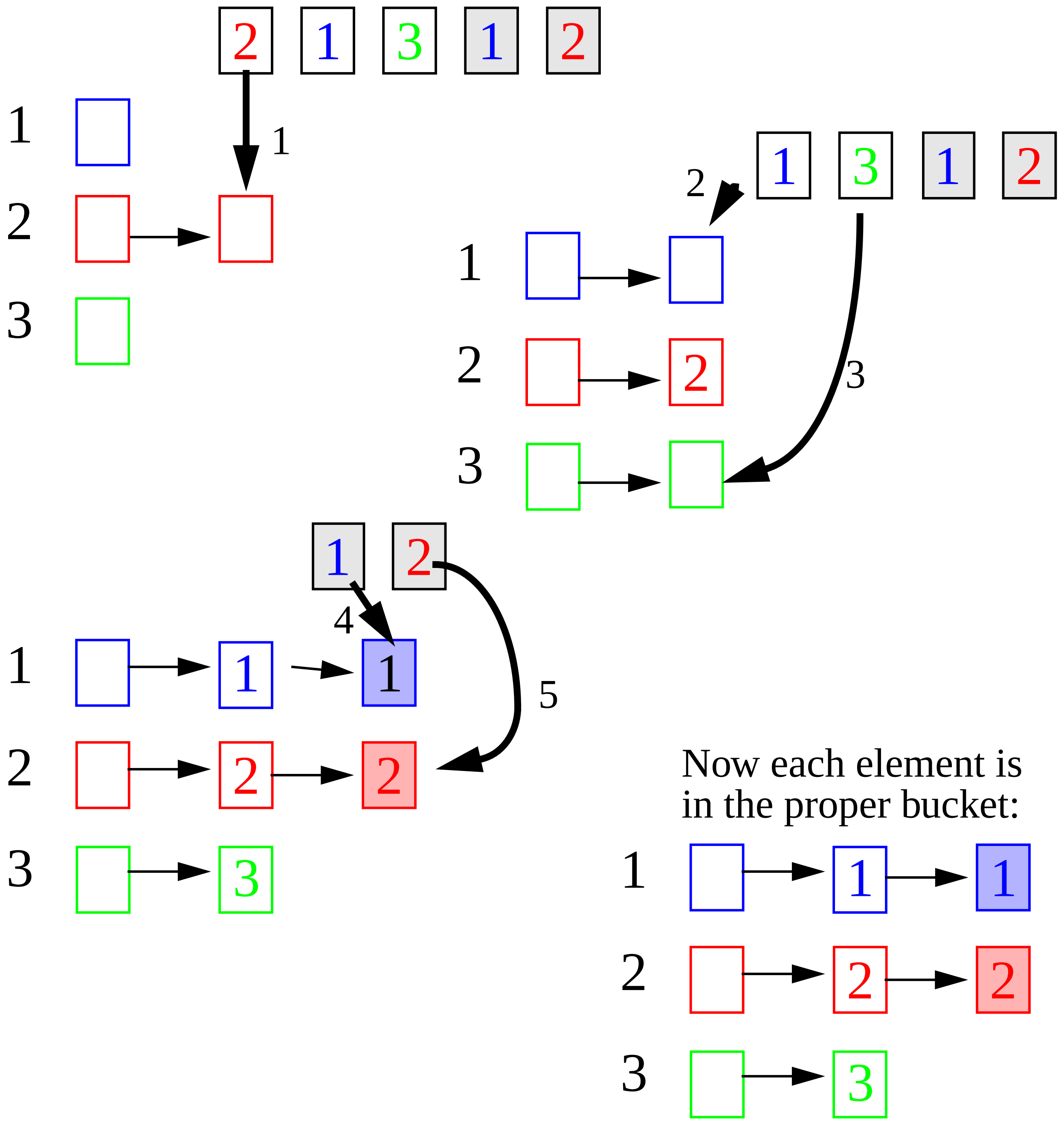
(note that there are two “2”s and two “1”s)

First, we create M “buckets”



Bucket Sort

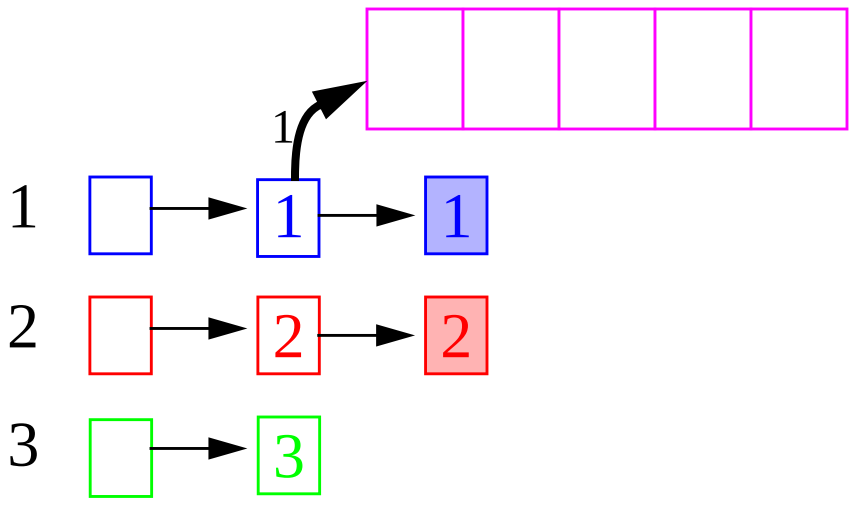
Each element of the array is put in one of the M "buckets"

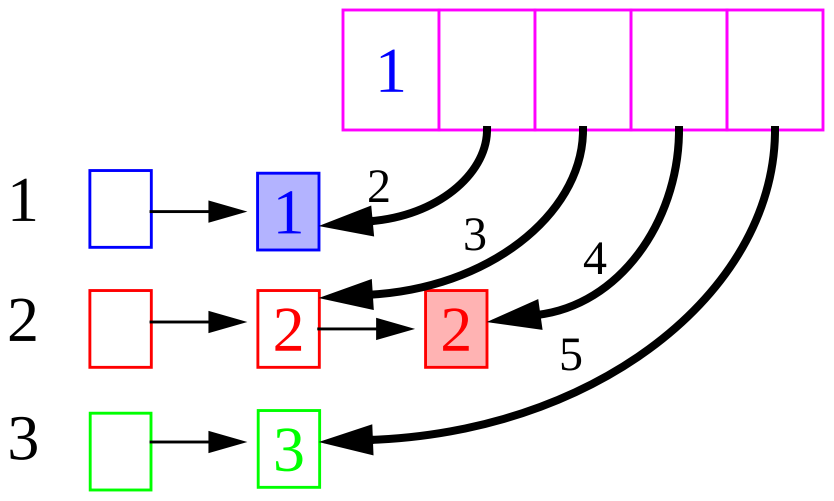


Now each element is in the proper bucket:

Bucket Sort

Now, pull the elements from the buckets into the array





At last, the sorted array (sorted in a stable way):