(Geometric) Camera Calibration

CS635

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Camera Calibration

- Digital Cameras
- Perspective Projection
- Aberrations
- Calibration
Cameras

• First photograph due to Nicéphore Niépce (1826):
  – Heliography: pewter covered with “Bitumen of Judea” (derived from crude oil) and then areas exposed to sunlight resisted dissolution in oil of lavender and petroleum
Digital Camera vs. Film Camera

• CCD: Charge-Coupled Device
• CMOS: Complementary Metal-oxide Semiconductor
  – (better for low light)

• CCD/CMOS:
  – Image plane is a CCD/CMOS array instead of film
  – Device is typically ¼ or ½ inch in size
  – What is sensitivity of film vs CCD/CMOS?
    • Approximately up to 40MP on consumer 35mm cameras
    • But depends on focus, light and numerous other conditions
Digital Camera

- Resolution
  - Lenses and light play a large role.
  - But, CCD/CMOS have various resolutions (e.g., 640x480, 100MP, 250GP)
• Resolution
  – Lenses and light play a large role.
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100 MP
(astrophotography)
Digital Camera

- **Resolution**
  - Lenses and light play a large role.
  - But, CCD/CMOS have various resolutions (e.g., 640x480, 100MP, 250GP)
Digital Camera

• Resolution
  – Lenses and light play a large role.
  – But, CCD/CMOS have various resolutions (e.g., 640x480, 100MP, 250GP)

Used 12,000 images….Calibration is crucial!!!!
Digital Camera

- Number of CCD/CMOS
  - 1: captures RGB simultaneously, reducing the resolution by 1/3 (kinda) using Bayer color filter (e.g., RGGB)
  - 3: typical RGB
  - 200+ bands: hyperspectral cameras using multiple CCDs and filters

- Video
  - Digital cameras have a maximum “frame rate”, usually determined by the hardware and bandwidth
    - Interlaced: only “half” of the horizontal lines of pixels are present in each frame
    - Progressive scan: each frame has a full-set of pixels
  - Shutter:
    - Rolling: output line-by-line (e.g., CMOS, low-light)
    - Global: output all pixels at once
Digital Camera (D-SLR)

Exclusively developed 14 element, 11 group 4 X zoom for the E-10, designed as a very high precision and highly accurate optical system.

[Diagram of camera lens components]
Digital Camera (Satellite)
Digital Camera

• Thus not such a simple device...
• So what do we do?
The simplest 1-CCD camera in town

CCD array

image plane

object
Exposures

image plane

CCD array

aperture

object

“effective” shutter (e.g., often just turn CCD on/off)
Exposures

• An “exposure” is when the CCD is exposed to the scene, typically for a brief amount of time and with a particular set of camera parameters.

• The characteristics of an “exposure” are determined by multiple factors, in particular:
  – Camera aperture
    • Determines amount of light that shines onto CCD
  – Camera shutter speed
    • Determines time during which aperture is “open” and light shines on CCD
Camera Calibration

• Digital Cameras

• Perspective Projection

• Aberrations

• Calibration
Perspective Projection

\[ x = \frac{f X}{\bar{Z}} \quad \quad y = \frac{f Y}{\bar{Z}} \]
Perspective Projection

\[
y = \frac{Y}{f} \frac{Z}{Z}
\]  

\[
x = f \frac{X}{Z}
\]  

\[
y = f \frac{Y}{Z}
\]  &  

\[
x = f \frac{X}{Z}
\]
Camera Calibration

- Digital Cameras
- Perspective Projection
- Aberrations
- Calibration
Thins Lens Assumption

• A “real” lens system does not produce a perfect image

• Aberrations are caused by i) imperfect manufacturing and ii) our approximate but practical models
  – Lenses typically have a spherical surface (easy manufacture)
    • However, aspherical lenses would better compensate for refraction but are more difficult to manufacture
  – Typically 1\textsuperscript{st} order approximations are used
    • Remember \( \sin \Omega = \Omega - \Omega^3/3! + \Omega^5/5! - \ldots \)
    • Thus, thin-lens equations only valid iff \( \sin \Omega \approx \Omega \)
    • Thin-lens means the lens is thin compared to distances
Thins Lens Assumption

- Recall Snell’s Law: \( n = \frac{\sin(\theta_1)}{\sin(\theta_2)} \)

- Expression to compute refraction much simpler if \( \theta \approx \sin(\theta) \) approximation used:

\[
\frac{1}{f} = \frac{1}{d_1} + \frac{1}{d_2}
\]
Aberrations

• Most common aberrations:
  – Spherical aberration
  – Coma
  – Astigmatism
  – Curvature of field
  – Chromatic aberration
  – Distortion
Spherical Aberration

- Deteriorates axial image
Coma

- Deteriorates off-axial bundles of rays
Astigmatism and Curvature of Field

- Produces multiple (two) images of a single object point
Chromatic Aberration

- Caused by wavelength dependent refraction
  - Apochromatic lenses (e.g., RGB) can help
Distortion

• Radial (and tangential) image distortions
Radial Distortion

before

after
Radial Distortion

• \((x, y)\) pixel before distortion correction
• \((x', y')\) pixel after distortion correction
• Let \(r = (x^2 + y^2)^{-1}\)

• Then
  – \(x' = x(1 - \Delta r/r)\)
  – \(y' = y(1 - \Delta r/r)\)
  – where \(\Delta r = k_0 r + k_1 r^3 + k_2 r^5 + \ldots\)

• Finally,
  – \(x' = x(1 - k_0 - k_1 r^2 - k_2 r^4 - \ldots)\)
  – \(y' = y(1 - k_0 - k_1 r^2 - k_2 r^4 - \ldots)\)
Camera Calibration

• Digital Cameras

• Perspective Projection

• Aberrations

• Calibration
Tsai’s Camera Calibration

• A widely used camera model to calibrate conventional cameras based on a pinhole camera

• Reference
Zhang’s Camera Calibration

• Another widely used camera model to calibrate conventional cameras based on a pinhole camera
• Many implementations are floating around!
• Reference
Bouguet’s Camera Calibration

• Another widely used camera model to calibrate conventional cameras based on a pinhole camera
• Many implementations are floating around!
• Reference: http://robots.stanford.edu/cs223b04/JeanYvesCalib/
Calibration Goal

• Determine the intrinsic and extrinsic parameters of a camera (with lens)
Camera Parameters

• Intrinsic/Internal
  – Focal length $f$
  – Principal point (center) $p_x, p_y$
  – Pixel size $s_x, s_y$
  – (Distortion coefficients) $k_1, \ldots$

• Extrinsic/External
  – Rotation $\phi, \varphi, \psi$
  – Translation $t_x, t_y, t_z$
Focal Length

\[
\begin{bmatrix}
x \\ y \\ fX \\ fY \\ fY/Z
\end{bmatrix}
= \begin{bmatrix}
X \\ Y \\ Z \\ 1
\end{bmatrix}
\]
Focal Length

\[
\begin{pmatrix}
  x \\
  y \\
  Z \\
\end{pmatrix}
= \begin{pmatrix}
  fX/Z \\
  fY/Z \\
  Z \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
  fX \\
  fY \\
  Z \\
\end{pmatrix}
= \begin{bmatrix}
  f & 0 & 0 & 0 \\
  0 & f & 0 & 0 \\
  0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{pmatrix}
  X \\
  Y \\
  Z \\
  1 \\
\end{pmatrix}
\]
Principal Point

\[
\begin{pmatrix}
  x_0 \\
  y_0 \\
\end{pmatrix}
= 
\begin{pmatrix}
  fX + Zp_x \\
  fY + Zp_y \\
  Z \\
\end{pmatrix}
= 
\begin{bmatrix}
  f & 0 & p_x & 0 \\
  0 & f & p_y & 0 \\
  0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{pmatrix}
  X \\
  Y \\
  Z \\
  1 \\
\end{pmatrix}
\]
CCD Camera: Pixel Size

\[
K = \begin{bmatrix}
    f & 0 & p_x & 0 \\
    0 & f & p_y & 0 \\
    0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
K = \begin{bmatrix}
    \alpha_x & 0 & p_x & 0 \\
    0 & \alpha_y & p_y & 0 \\
    0 & 0 & 1 & 0
\end{bmatrix}
\]

(intrinsic) calibration matrix
Translation & Rotation

\[ \tilde{x}_{\text{cam}} = R(\tilde{X} - C) \]
\[ \tilde{x}_{\text{cam}} = RX - RC \]

\( \tilde{x}_{\text{cam}} \) =

\[
\begin{bmatrix}
R & t \\
0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
\]

\( R = R_\phi R_\psi R_\chi \)

3x3 rotation matrices

\( t = \begin{bmatrix} t_x & t_y & t_z \end{bmatrix}^T \)
translation vector

(extrinsic) calibration matrix
Calibration Task

physical arrangement  
(calibration pad)

observation  
(camera with initial parameters)

Given $\tilde{X}_i \leftrightarrow \tilde{x}_i$  
What is $K$?  $P$?
Absolute Conic

• A conic section, conic or a quadratic curve is a curve obtained from a cone's surface intersecting a plane

• The absolute conic is a conic on the plane at infinity consisting of points $X$ such that
  - $X = [x \ y \ z \ t]$
  - Where $x^2 + y^2 + z^2 = 0$ and $t = 0$
  - Often $d = [x \ y \ z]^T$
  - Hence $d^T \cdot d = 0$
Absolute Conic

• The Absolute Conic $\Omega$ is invariant under Euclidean transformations and critical to camera calibration
  
  (e.g., like moon following you on straight road)
Absolute Conic

• Given point on Ω called \( x_\infty = [d^T \ 0]^T \), its image on a general camera is
  \[ v = KRD \]

• Combining the above with \( d^T \cdot d = 0 \)
  \[ v^T \cdot (KK^T)^{-1} \cdot v = 0 \]

• Thus the image of conic is represented by
  \[ \omega = (KK^T)^{-1} \]

• Since we know the general form of \( K \) (i.e., 5 unknowns), \( \omega \) is symmetric and defined up to a scale
Simple Internal Parameters
Calibration Device

- Observe these 3 planes, forming 3 homographies

- Each \( H = [h_1 \ h_2 \ h_3] \) gives constraints
  
  \[ h_1^T \omega h_2 = 0 \text{ and } h_1^T w h_1 = h_2^T h_2 \]

  Conic \( \omega \) is determined from 5 or more such equations, up to a scale (5 orthogonal line pairs)

- Compute \( K \) from \( \omega = (KK^T)^{-1} \) using Cholesky factorization, for example
Zhang’s Camera Calibration

• 1. Detect corners
• 2. Estimate matrix P
• 3. Recover intrinsic/extrinsic parameters
• 4. Refine: bundle adjustment
Let $M = KP$

$$\tilde{x}_{cam} = M\tilde{X}$$

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x' / w' \\ y' / w' \end{pmatrix}$$

$$x = (m_1 \cdot \tilde{X}) / (m_3 \cdot \tilde{X})$$

$$y = (m_2 \cdot \tilde{X}) / (m_3 \cdot \tilde{X})$$
A Linear Formulation

\[
x = \frac{(m_1 \cdot \tilde{X})}{(m_3 \cdot \tilde{X})}
\]
\[
y = \frac{(m_2 \cdot \tilde{X})}{(m_3 \cdot \tilde{X})}
\]

for \( i = 1 \ldots n \) observations

\[
(m_1 - x_i m_3) \cdot \tilde{X}_i = 0
\]
\[
(m_2 - y_i m_3) \cdot \tilde{X}_i = 0
\]

2\(n\) homogeneous linear equations and 12 unknowns (coefficients of \( M \))

Thus, given \( n \geq 6 \) can solve for \( M \); namely \( Qm = 0 \)

\[
Q = \begin{bmatrix}
\tilde{X}_1^T & 0^T & -x_1 \tilde{X}_1^T \\
0^T & \tilde{X}_1^T & -y_1 \tilde{X}_1^T \\
\vdots & \vdots & \vdots \\
\tilde{X}_n^T & 0^T & -x_n \tilde{X}_n^T \\
0^T & \tilde{X}_n^T & -y_n \tilde{X}_n^T
\end{bmatrix}
\]

\[
m = \begin{bmatrix}
m_1 \\
m_2 \\
m_3
\end{bmatrix}
\]
A Linear Formulation

• Goal: \( \min \| Qm \| \) subject to \( \| m \| = 1 \)
• Recall: normal equation \( Ax = b \rightarrow A^T Ax = A^T b \)
• Solution: so solve \( Q^T Qm = 0 \), e.g., use eigenvector of \( Q^T Q \) associated with the smallest eigenvalue. Use \( m \) to make matrix \( M \).
Decomposing M into Camera Parameters

\[
M = \rho [A \ b] = K [R \ t]
\]

\[
K = \begin{bmatrix}
\alpha & \gamma & u_0 \\
0 & \beta & v_0 \\
0 & 0 & 1
\end{bmatrix}
\]

(often \(\gamma = \pi/2\) which means no skew)
Decomposing $M$ into Camera Parameters

\[ M = \rho [A \ b] = K [R \ t] \]

\[ K = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & \nu_0 \\ 0 & 0 & 1 \end{bmatrix} \]

(often $\gamma = 0$ which means no skew)

\[ B = KR \text{ and } b = Kt \quad (\text{so } B \text{ is first 3x3 of } M) \]

Let \[ A = BB^T = (KR)(KR)^T = KK^T \]

\[ A = \begin{bmatrix} \alpha^2 + \gamma^2 + u_0^2 & u_0 \nu_0 + c\beta & u_0 \\ u_0 \nu_0 + c\alpha & \alpha^2_\nu + \nu_0^2 & \nu_0 \\ u_0 & \nu_0 & 1 \end{bmatrix} \]
Decomposing $M$ into Camera Parameters

\[
A = \begin{bmatrix}
\alpha^2 + \gamma^2 + u_0^2 & u_0 v_0 + c\beta & u_0 \\
u_0 v_0 + c\alpha & \alpha_v^2 + v_0^2 & v_0 \\
u_0 & v_0 & 1
\end{bmatrix}
\]

\[
= \begin{bmatrix}
k_u & k_c & u_0 \\
k_c & k_v & v_0 \\
u_0 & v_0 & 1
\end{bmatrix}
\]

(assumes square pixels and equal focal length in $x$ and $y$)
Decomposing M into Camera Parameters

\[ A = \begin{bmatrix} k_u & k_c & u_0 \\ k_c & k_v & v_0 \\ u_0 & v_0 & 1 \end{bmatrix} \]

\[ u_0 = A_{13} \]

\[ v_0 = A_{23} \]

\[ \beta = \sqrt{k_v - v_0^2} \]

\[ \gamma = \frac{k_c - u_0 v_0}{\beta} \]

\[ \alpha = \sqrt{k_u - u_0^2 - \gamma^2} \]

\[ K = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ R = K^{-1}B \]

\[ t = K^{-1}b \]
Zhang’s Camera Calibration

• 1. Detect corners
• 2. Estimate matrix P
• 3. Recover intrinsic/extrinsic parameters
• 4. Refine: bundle adjustment
Bundle Adjustment

• Given initial guesses, use nonlinear least squares to refine/compute the calibration parameters
• Simple but good convergence depends on accuracy of initial guess
Bundle Adjustment

Recall

\[ x = \frac{(m_1 \cdot \tilde{X})}{(m_3 \cdot \tilde{X})} \]
\[ y = \frac{(m_2 \cdot \tilde{X})}{(m_3 \cdot \tilde{X})} \]

\[ E = \frac{1}{mn} \sum_{ij} \left[ \left( x_{ij} - \frac{m_{i1} \cdot \tilde{X}_j}{m_{i3} \cdot \tilde{X}_j} \right)^2 + \left( y_{ij} - \frac{m_{i2} \cdot \tilde{X}_j}{m_{i3} \cdot \tilde{X}_j} \right)^2 \right] \]

Goal is \( E \to 0 \)
Bundle Adjustment

Option A:

Define $M$ as a matrix of 11 unknowns (i.e., $m_{34} = 1$)

And solve for $m_{ij}$

Can be made very efficient, especially for sparse matrices

Option B:

Define $M$ as function of intrinsic and extrinsic parameters so that it is “recomputed” during each loop of the optimization