(Geometric) Camera Calibration

CS635 Spring 2017

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Camera Calibration

- Cameras and CCDs
- Aberrations
- Perspective Projection
- Calibration
Cameras

• First photograph due to Niepce (1826)

http://www.hrc.utexas.edu/exhibitions/permanent/firstphotograph/process/#top
Digital Camera vs. “Film” Camera

• Charge-Coupled Device (CCD)
  – Image plane is a CCD array instead of film
  – CCD arrays are typically ¼ or ½ inch in size
  – CCD arrays have a pixel resolution (e.g., 640x480, 1920x1080, 10MP)
  – CCD Cameras have a maximum “frame rate”, usually determined by the hardware and bandwidth

• Number of CCDs
  – 3: each CCD captures only R, G, or B wavelengths
  – 1: the single CCD captures RGB simultaneously, reducing the resolution by 1/3 (kinda)

• Video
  – Interlaced: only “half” of the horizontal lines of pixels are present in each frame
  – Progressive scan: each frame has a full-set of pixels
The simplest 1-CCD camera in town

- **CCD array**
- **image plane**
- **object**
Exposures

- CCD array
- Object
- "Effective" shutter (e.g., often just turn CCD on/off)
- Image plane
- Aperture
Exposures

• An “exposure” is when the CCD is exposed to the scene, typically for a brief amount of time and with a particular set of camera parameters

• The characteristics of an “exposure” are determined by multiple factors, in particular:
  – Camera aperture
    • Determines amount of light that shines onto CCD
  – Camera shutter speed
    • Determines time during which aperture is “open” and light shines on CCD
Camera Calibration

- Digital Cameras and CCDs
- Aberrations
- Perspective Projection
- Calibration
Aberrations

• A “real” lens system does not produce a perfect image

• Aberrations are caused by imperfect manufacturing and by our approximate models
  – Lenses typically have a spherical surface
    • Aspherical lenses would better compensate for refraction but are more difficult to manufacture
  – Typically 1\textsuperscript{st} order approximations are used
    • Remember $\sin \Omega = \Omega - \Omega^3/3! + \Omega^5/5! - \ldots$
    • Thus, thin-lens equations only valid iff $\sin \Omega \approx \Omega$
Aberrations

• Most common aberrations:
  – Spherical aberration
  – Coma
  – Astigmatism
  – Curvature of field
  – Chromatic aberration
  – Distortion
Spherical Aberration

- Deteriorates axial image
Coma

- Deteriorates off-axial bundles of rays
Astigmatism and Curvature of Field

• Produces multiple (two) images of a single object point
Chromatic Aberration

• Caused by wavelength dependent refraction
  – Apochromatic lenses (e.g., RGB) can help
Distortion

• Radial (and tangential) image distortions

Orthoscopic  Pin–cushion distortion  Barrel distortion
Radial Distortion

before

after
Radial Distortion

• \((x, y)\) pixel before distortion correction
• \((x', y')\) pixel after distortion correction
• Let \(r = (x^2 + y^2)^{-1}\)

• Then
  – \(x' = x(1 - \Delta r/r)\)
  – \(y' = y(1 - \Delta r/r)\)
  – where \(\Delta r = k_0 r + k_1 r^3 + k_2 r^5 + ...\)

• Finally,
  – \(x' = x(1 - k_0 - k_1 r^2 - k_2 r^4 - ...)\)
  – \(y' = y(1 - k_0 - k_1 r^2 - k_2 r^4 - ...)\)
Camera Calibration

• Digital Cameras and CCDs

• Aberrations

• **Perspective Projection**

• Calibration
Perspective Projection

\[ x = f \frac{X}{\bar{Z}} \]
\[ y = f \frac{Y}{\bar{Z}} \]
Perspective Projection

\( y = \frac{Y}{Z} \) \( \frac{f}{Z} \)

\( x = f \frac{X}{Z} \) & \( y = f \frac{Y}{Z} \)

eye/viewpoint

optical axis

image plane

(X, Y, Z)
Camera Calibration

• Digital Cameras and CCDs
• Aberrations
• Perspective Projection
• Calibration
Tsai’s Camera Calibration

• A widely used camera model to calibrate conventional cameras based on a pinhole camera

• Reference
Zhang’s Camera Calibration

• Another widely used camera model to calibrate conventional cameras based on a pinhole camera
• Many implementations are floating around!
• Reference
Bouguet’s Camera Calibration

• Another widely used camera model to calibrate conventional cameras based on a pinhole camera
• Many implementations are floating around!
• Reference: http://www.vision.caltech.edu/bouguetj
Calibration Goal

• Determine the intrinsic and extrinsic parameters of a camera (with lens)
Camera Parameters

• Intrinsic/Internal
  – Focal length \( f \)
  – Principal point (center) \( p_x, p_y \)
  – Pixel size \( s_x, s_y \)
  – (Distortion coefficients) \( k_1, \ldots \)

• Extrinsic/External
  – Rotation \( \phi, \varphi, \psi \)
  – Translation \( t_x, t_y, t_z \)
Focal Length

\[
\begin{pmatrix}
x \\
y
\end{pmatrix} = \begin{pmatrix} fX / Z \\
fY / Z
\end{pmatrix}
\]

\[
\begin{pmatrix}
fX \\
fY \\
Z
\end{pmatrix} = \begin{pmatrix}
X \\
Y \\
Z \\
1
\end{pmatrix}
\]
Focal Length

\[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} = \begin{pmatrix} fX/Z \\
fY/Z
\end{pmatrix}
\]

\[
\begin{pmatrix} fX \\
fY \\
fY/Z
\end{pmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\
0 & f & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix} \begin{pmatrix} X \\
Y \\
Z \\
1
\end{pmatrix}
\]
Principal Point

\[
\begin{bmatrix}
  x_0 \\
  y_0
\end{bmatrix}
= \begin{bmatrix}
  fX + Zp_x \\
  fY + Zp_y \\
  Z
\end{bmatrix}
\begin{bmatrix}
  f & 0 & p_x & 0 \\
  0 & f & p_y & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  X \\
  Y \\
  Z \\
  1
\end{bmatrix}
\]

\( (x_0, y_0) \)
CCD Camera: Pixel Size

$K = \begin{bmatrix}
\alpha_x & 0 & p_x & 0 \\
0 & \alpha_y & p_y & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}$

(intrinsic) calibration matrix
Translation & Rotation

\[ \tilde{x}_{\text{cam}} = R(\tilde{X} - C) \]
\[ \tilde{x}_{\text{cam}} = R\tilde{X} - RC \]

\( \tilde{x}_{\text{cam}} = \) (extrinsic) calibration matrix

\[ R = R_\phi R_\psi R_\gamma \]
3x3 rotation matrices

\[ t = \begin{bmatrix} t_x & t_y & t_z \end{bmatrix}^T \]
translation vector
Calibration Task

physical arrangement
(calibration pad)

observation
(camera with initial parameters)

points in space

calibration result
(camera with calibrated parameters)

Given \( \tilde{X}_i \leftrightarrow \tilde{x}_i \)
What is \( K \), \( P \)?
Camera Calibration: Conics

Conic is degree 2 curve on a plane:

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

or

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} a & b/2 \\ b/2 & c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d & e \end{bmatrix} \begin{bmatrix} y \\ y \end{bmatrix} + f = 0$$
Conic is degree 2 curve on a plane:

\[ ax^2 + bxy + cy^2 + dx + ey + f = 0 \]

or

\[ \mathbf{x}^T \mathbf{C} \mathbf{x} = 0 \]

\[
\mathbf{C} = \begin{bmatrix}
    a & b/2 & d/2 \\
    b/2 & c & e/2 \\
    d/2 & e/2 & f \\
\end{bmatrix}, \mathbf{x} = \begin{bmatrix}
    x \\
    y \\
    1
\end{bmatrix}
\]
• A point transformation on an image is
  \[ x' = Hx \]
  (for homography H)

• A conic transformation on an image is
  \[ C' = H^{-T}CH^{-1} \]
Camera Calibration: Absolute Conic

• The Absolute Conic $\Omega$ is invariant under Euclidean transformations and critical to camera calibration
  (e.g., like moon following you on straight road)
Angle between two rays

\[
\cos \theta = \frac{d_1^T d_2}{\|d_1\| \|d_2\|} = \frac{(K^{-1} x_1)^T (K^{-1} x_2)}{\|K^{-1} x_1\| \|K^{-1} x_2\|} \\
= \frac{x_1^T (K^{-T} K^{-1}) x_2}{\|x_1^T (K^{-T} K^{-1}) x_1\| \|x_2^T (K^{-T} K^{-1}) x_2\|}
\]

\[x_i = K d_i\]
Absolute Conic

• Given point on $\Omega$ called $x_\infty = [d^T\ 0]^T$, its image on a general camera is
  $$x = K R d$$

• Recall $x' = H x$

• Thus image of the conic is
  $$\omega = H^{-T} C H^{-1} = (K R)^{-T} C (K R)^{-1} = (K K^T)^{-1}$$
  or
  $$\omega = K^{-T} K^{-1}$$
Angle between two rays

\[ \cos \theta = \frac{x_1^T \omega x_2}{\|x_1^T \omega x_1\| \|x_2^T \omega x_2\|} \]

\[ x_i = K d_i \]
Simple Calibration Device

- Observe these 3 planes, forming 3 homographies

- Each $H = [h_1 \ h_2 \ h_3]$ gives constraints
  
  $h_1^T \omega h_2 = 0$ and $h_1^T w h_1 = h_2^T h_2$

- Conic $\omega$ is determined from 5 or more such equations, up to a scale

- Compute $K$ from $\omega = (KK^T)^{-1}$ using Cholesky factorization, for example
Zhang’s Camera Calibration

1. Detect corners
2. Estimate matrix P
3. Recover intrinsic/extrinsic parameters
4. Refine: bundle adjustment
Let $M = KP$

$\tilde{x}_{cam} = M\tilde{X}$

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x'/w' \\ y'/w' \end{pmatrix}$$

$$x = (m_1 \cdot \tilde{X}) / (m_3 \cdot \tilde{X})$$

$$y = (m_2 \cdot \tilde{X}) / (m_3 \cdot \tilde{X})$$
A Linear Formulation

\[ x = (m_1 \cdot \tilde{X}) / (m_3 \cdot \tilde{X}) \]
\[ y = (m_2 \cdot \tilde{X}) / (m_3 \cdot \tilde{X}) \]

for \( i = 1 \ldots n \) observations

\[(m_1 - x_i m_3) \cdot \tilde{X}_i = 0 \]
\[(m_2 - y_i m_3) \cdot \tilde{X}_i = 0 \]

2n homogeneous linear equations and 12 unknowns (coefficients of \( M \))

Thus, given \( n \geq 6 \) can solve for \( M \); namely \( Qm = 0 \)

\[
Q = \begin{bmatrix}
\tilde{X}_1^T & 0^T & -x_1 \tilde{X}_1^T \\
0^T & \tilde{X}_1^T & -y_1 \tilde{X}_1^T \\
\vdots & \vdots & \vdots \\
\tilde{X}_n^T & 0^T & -x_n \tilde{X}_n^T \\
0^T & \tilde{X}_n^T & -y_n \tilde{X}_n^T
\end{bmatrix}
\]

\[
m = \begin{pmatrix}
m_1 \\
m_2 \\
m_3
\end{pmatrix}
\]
A Linear Formulation

- Goal: $\min \|Qm\| \text{ subject to } \|m\| = 1$
- Recall: normal equation $Ax = b \rightarrow A^T Ax = A^T b$
- Solution: so solve $Q^T Qm = 0$, e.g., use eigenvector of $Q^T Q$ associated with the smallest eigenvalue. Use $m$ to make matrix $M$. 
Decomposing M into Camera Parameters

\[ M = \rho [A \ b] = K [R \ t] \]

\[ K = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \]

(often \( \gamma = \pi/2 \) which means no skew)
Decomposing $M$ into Camera Parameters

\[ M = \rho [A \ b] = K [R \ t] \]

\[ K = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \]

(often $\gamma = 0$ which means no skew)

$B = KR$ and $b = Kt$  

(so $B$ is first 3x3 of $M$)

Let $A = BB^T = (KR)(KR)^T = KK^T$

\[ A = \begin{bmatrix} \alpha^2 + \gamma^2 + u_0^2 & u_0 v_0 + c \beta & u_0 \\ u_0 v_0 + c \alpha & \alpha_v^2 + v_0^2 & v_0 \\ u_0 & v_0 & 1 \end{bmatrix} \]
Decomposing $M$ into Camera Parameters

$$A = \begin{bmatrix} 
\alpha^2 + \gamma^2 + u_0^2 & u_0 v_0 + c\beta & u_0 \\
 u_0 v_0 + c\alpha & \alpha_v^2 + v_0^2 & v_0 \\
 u_0 & v_0 & 1 
\end{bmatrix}$$

$$= \begin{bmatrix} 
 k_u & k_c & u_0 \\
 k_c & k_v & v_0 \\
 u_0 & v_0 & 1 
\end{bmatrix}$$

(assumes square pixels and equal focal length in $x$ and $y$)
Decomposing M into Camera Parameters

\[ A = \begin{bmatrix} k_u & k_c & u_0 \\ k_c & k_v & v_0 \\ u_0 & v_0 & 1 \end{bmatrix} \]

\[ u_0 = A_{13} \]
\[ v_0 = A_{23} \]

\[ \beta = \sqrt{k_v - v_0^2} \]
\[ \gamma = \frac{k_c - u_0 v_0}{\beta} \]

\[ \alpha = \sqrt{k_u - u_0^2 - \gamma^2} \]

\[ K = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ R = K^{-1}B \]
\[ t = K^{-1}b \]
Zhang’s Camera Calibration

1. Detect corners
2. Estimate matrix P
3. Recover intrinsic/extrinsic parameters
4. Refine: bundle adjustment
Bundle Adjustment

- Given initial guesses, use nonlinear least squares to refine/compute the calibration parameters
- Simple but good convergence depends on accuracy of initial guess
Bundle Adjustment

Recall

\[ x = \frac{(m_1 \cdot \tilde{X})}{(m_3 \cdot \tilde{X})} \]

\[ y = \frac{(m_2 \cdot \tilde{X})}{(m_3 \cdot \tilde{X})} \]

\[
E = \frac{1}{mn} \sum_{ij} \left[ \left( x_{ij} - \frac{m_{i1} \cdot \tilde{X}_j}{m_{i3} \cdot \tilde{X}_j} \right)^2 + \left( y_{ij} - \frac{m_{i2} \cdot \tilde{X}_j}{m_{i3} \cdot \tilde{X}_j} \right)^2 \right]
\]

Goal is \( E \rightarrow 0 \)
**Bundle Adjustment**

Option A:

Define $M$ as a matrix of 11 unknowns (i.e., $m_{34} = 1$)

And solve for $m_{ij}$

Can be made very efficient, especially for sparse matrices

Option B:

Define $M$ as a function of intrinsic and extrinsic parameters so that it is “recomputed” during each loop of the optimization