Light Transport

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Topics

• Local and Global Illumination Models
• Helmholtz Reciprocity
• Dual Photography/Light Transport in Real-World
Diffuse Lighting

• A.k.a. Lambertian illumination
• A fraction of light is radiated in every direction
• Intensity varies with cosine of the angle with normal
Diffuse Lighting

\[ I_{\text{diff}} = I_{\text{Light}} \rho_{\text{diff}} (N \cdot L) \]
Specular Lighting

• Shiny surfaces reflect predominantly in a particular direction, creating *highlights*
• Where the highlights appear depends on the viewer’s position
Specular Lighting

• The most common lighting model was suggested by Phong

\[ I_{\text{spec}} = \rho_{\text{spec}} I_{\text{Light}} (\cos \phi)^{n_{\text{shiny}}} \]

• The \( n_{\text{shiny}} \) term is an empirical constant to model the rate of falloff

• The model has no physical basis, but it sort of works
Example
Inter-reflections
Scattering
Scattering

Without (subsurface) scattering  With (subsurface) scattering
Scattering

Without (subsurface) scattering

With (subsurface) scattering
Scattering through participating media with volume caustics…

Hu et al. 2010
Rendering Equation  
(also known as the light-transport equation)  

- Illumination can be generalized to

\[ I(x, x') = g(x, x')[\varepsilon(x, x') + \int_{s} \rho(x, x', x'')I(x', x'')dx''] \]

\( I \): illumination at first point from second  
\( g \): geometry term for visibility  
\( \varepsilon \): emitted light from second point to first  
\( \rho \): reflectivity of light from \( x'' \) to \( x \) via \( x' \)  

(note: equation is recursive)

...but it does not model all illumination effects
Global Illumination

- Ray tracing
  - Rays bounce off potentially all objects
  - Good for specular scenes
Global Illumination

• Ray tracing
Global Illumination

- Ray tracing
Global Illumination

- Radiosity
  - All diffuse surfaces are influenced by potentially all other diffuse surfaces
  - Good for diffuse scenes
Global Illumination

• Radiosity
Global Illumination

• Radiosity
Global Illumination

- Radiosity
Radiosity

• All energy emitted or reflected by every surface is accounted for by its reflection or absorption by other surfaces

\[ B_i = E_i + \rho_i \sum_{i \leq j \leq n} B_j F_{j-i} \frac{A_j}{A_i} \]

Reflected

\[ A_i F_{j-i} = A_j F_{i-j} \]

Emitted

Radiosity equation:

\[ B_i - \rho_i \sum_{1 \leq j \leq n} B_j F_{i-j} = E_i \]
Radiosity

• All energy emitted or reflected by every surface is accounted for by its reflection or absorption by other surfaces

\[ B_i - \rho_i \sum_{1 \leq j \leq n} B_j F_{i-j} = E_i \]

\[
\begin{bmatrix}
1 - \rho_1 F_{1-1} & -\rho_1 F_{1-2} & \cdots & -\rho_1 F_{1-n} \\
-\rho_2 F_{2-1} & 1 - \rho_2 F_{2-2} & \cdots & -\rho_2 F_{2-n} \\
\cdots & \cdots & \cdots & \cdots \\
-\rho_n F_{n-1} & -\rho_n F_{n-2} & \cdots & -\rho_n F_{n-n}
\end{bmatrix}
\begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{bmatrix}
= 
\begin{bmatrix}
E_1 \\
E_2 \\
\vdots \\
E_n
\end{bmatrix}
\]

Solve for \( B \), etc…
• Modeling illumination is hard
• “Undoing” physically-observed illumination in order to discover the underlying geometry is even harder
• Insight: let’s sample it and “re-apply” it!
Dual Photography

Sen et al., SIGGRAPH 2005

(slides courtesy of M. Levoy)
Helmholtz Reciprocity

camera

light

scene
Helmholtz Reciprocity

light

scene
camera
Measuring transport along a set of paths.
Reversing the paths

camera

point light

scene
Forming a dual photograph

“dual” camera
projector

“dual” light

scene
Forming a dual photograph

“dual” camera

“dual” light

image of scene

scene
Physical demonstration

- light replaced with projector
- camera replaced with photocell
- projector scanned across the scene

conventional photograph, with light coming from right as seen from projector’s position
dual photograph, and as illuminated from photocell’s position
Related imaging methods

• time-of-flight scanner
  – if they return reflectance as well as range
  – but their light source and sensor are typically coaxial

• scanning electron microscope

Velcro® at 35x magnification, Museum of Science, Boston
The 4D transport matrix
The 4D transport matrix
The 4D transport matrix

\[
C = \begin{bmatrix} \text{mn x 1} \end{bmatrix} = \begin{bmatrix} \text{mn x pq} \end{bmatrix} \begin{bmatrix} \text{pq x 1} \end{bmatrix}
\]
The 4D transport matrix

\[
\begin{bmatrix}
C
\end{bmatrix}_{mn \times 1} =
\begin{bmatrix}
T
\end{bmatrix}_{mn \times pq} \cdot
\begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix}_{pq \times 1}
\]
The 4D transport matrix

\[
\begin{bmatrix}
C \\
\text{mn x 1}
\end{bmatrix}
= 
\begin{bmatrix}
\text{mn x pq} \\
\text{T}
\end{bmatrix}
\begin{bmatrix}
0 \\
1 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\text{pq x 1}
\end{bmatrix}
\]
The 4D transport matrix

\[
\begin{align*}
C_{mn \times pq} &= 100^{\text{mn \times 1}} \quad \text{T}^{\text{pq \times 1}}
\end{align*}
\]
The 4D transport matrix

\[
\begin{bmatrix}
C
\end{bmatrix}
= \begin{bmatrix}
\text{mn x pq}
\end{bmatrix}
\begin{bmatrix}
T
\end{bmatrix}
\begin{bmatrix}
P
\end{bmatrix}
\]

\[
\begin{bmatrix}
\text{mn x 1}
\end{bmatrix}
= \begin{bmatrix}
\text{pq x 1}
\end{bmatrix}
\]
The 4D transport matrix

\[
C = \begin{bmatrix}
T
\end{bmatrix}
\begin{bmatrix}
P
\end{bmatrix}
\]

applying Helmholtz reciprocity...

\[
C' = \begin{bmatrix}
T^T
\end{bmatrix}
\begin{bmatrix}
P'
\end{bmatrix}
\]
Example

conventional photograph with light coming from right

dual photograph as seen from projector’s position
Properties of the transport matrix

- little inter-reflection
  - → sparse matrix
- many inter-reflections
  - → dense matrix
- convex object
  - → diagonal matrix
- convex object
  - → full matrix

Can we create a dual photograph entirely from diffuse reflections?
Dual photography from diffuse reflections

the camera’s view
Relighting

Paul Debevec's Light Stage 3

- subject captured under multiple lights
- one light at a time, so subject must hold still
- point lights are used, so can’t relight with cast shadows
Relighting

With Dual Photography…
Relighting

With Dual Photography…
Relighting

With Dual Photography…
The 6D transport matrix
The 6D transport matrix
The advantage of dual photography

- capture of a scene as illuminated by different lights cannot be parallelized

- capture of a scene as viewed by different cameras can be parallelized
Measuring the 6D transport matrix
Relighting with complex illumination

- step 1: measure 6D transport matrix $T$
- step 2: capture a 4D light field
- step 3: relight scene using captured light field
Running time

• the different rays within a projector can in fact be parallelized to some extent

• this parallelism can be discovered using a coarse-to-fine adaptive scan

• can measure a 6D transport matrix in 5 minutes
Can we measure an 8D transport matrix?

- Projector array
- Camera array
- Scene