



Surface Triangulation

CS535

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[help from Michael Kazhdan @ JHU and

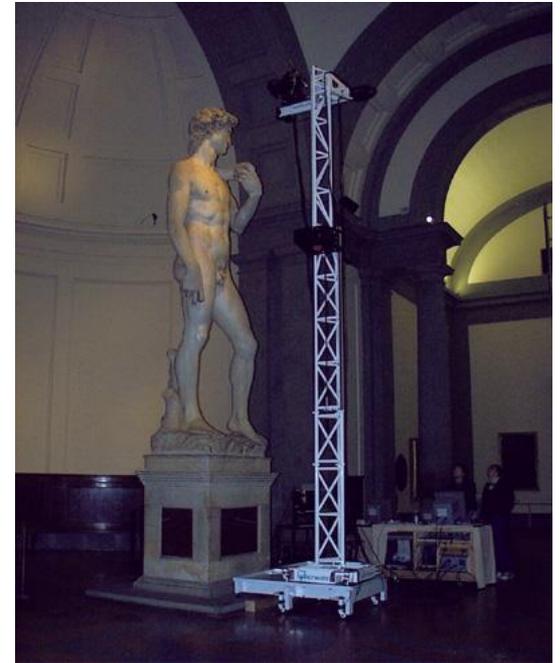
Ioannis Stamos @ CUNY]



Motivation

- Time of flight
- Structured light
- Stereo images
- Shape from shading
- Etc.

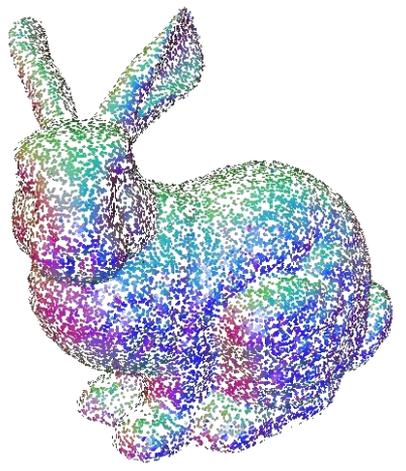
■ <http://graphics.stanford.edu/projects/mich/>



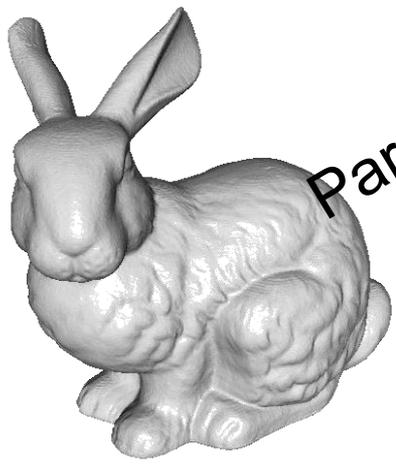


Motivation

Surface reconstruction



Geometry processing

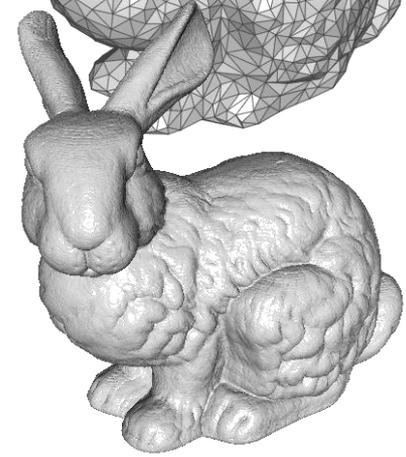
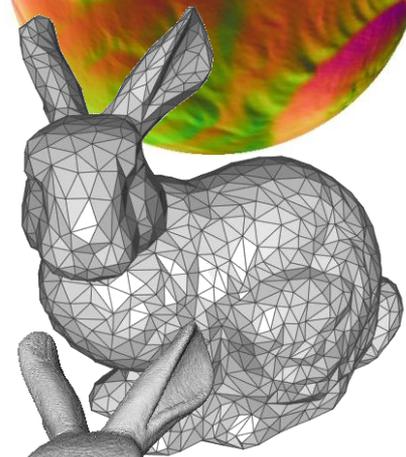
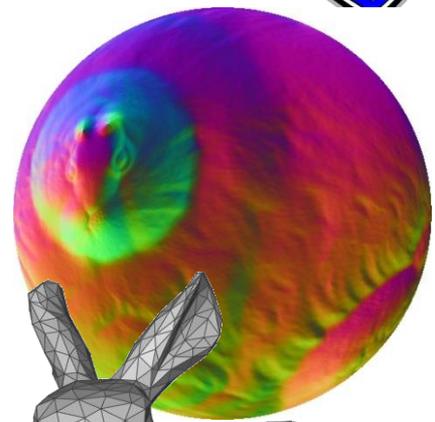


Parameterization

Decimation

Filtering

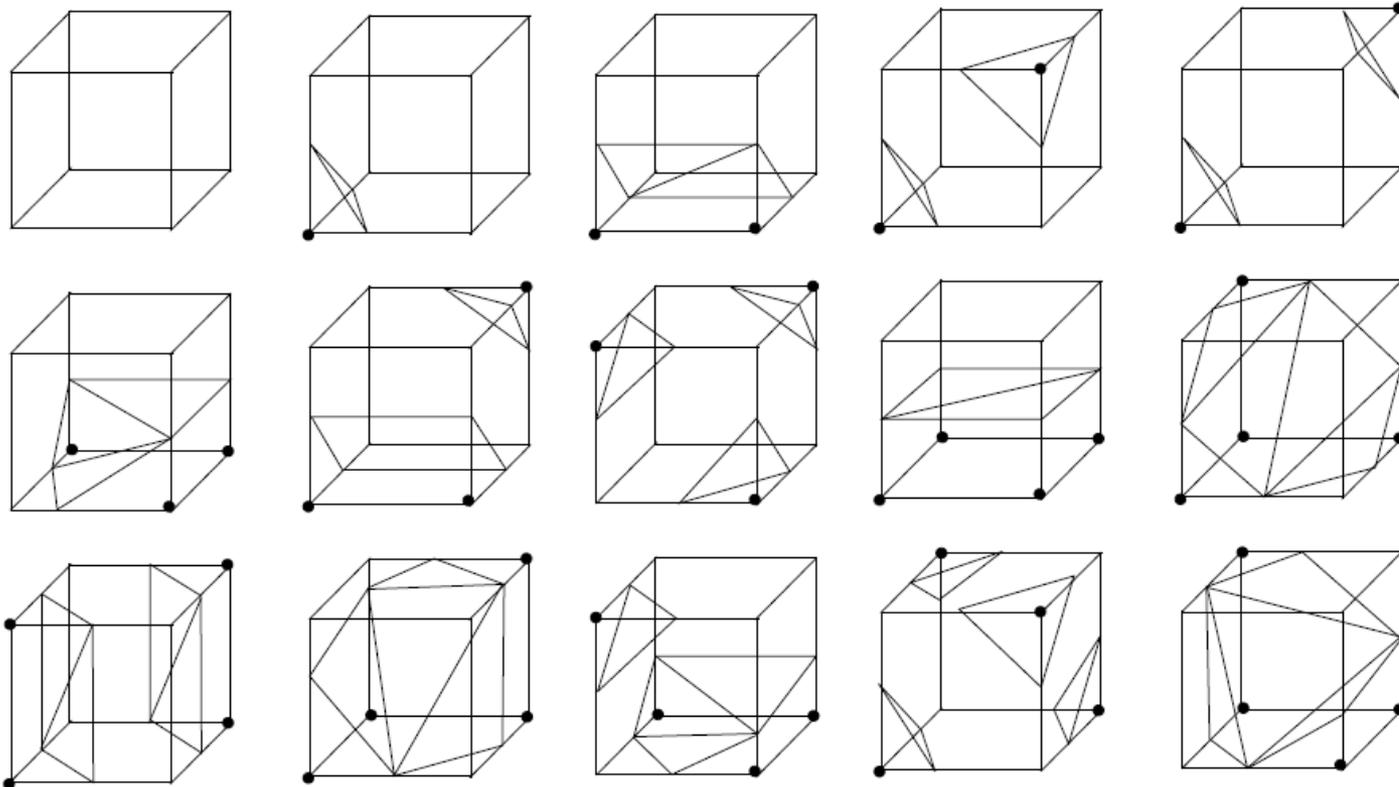
etc.





One Option: Marching Cubes

If the function is sampled on a regular voxel grid, we can independently triangulate each voxel.



One Option: Marching Cubes

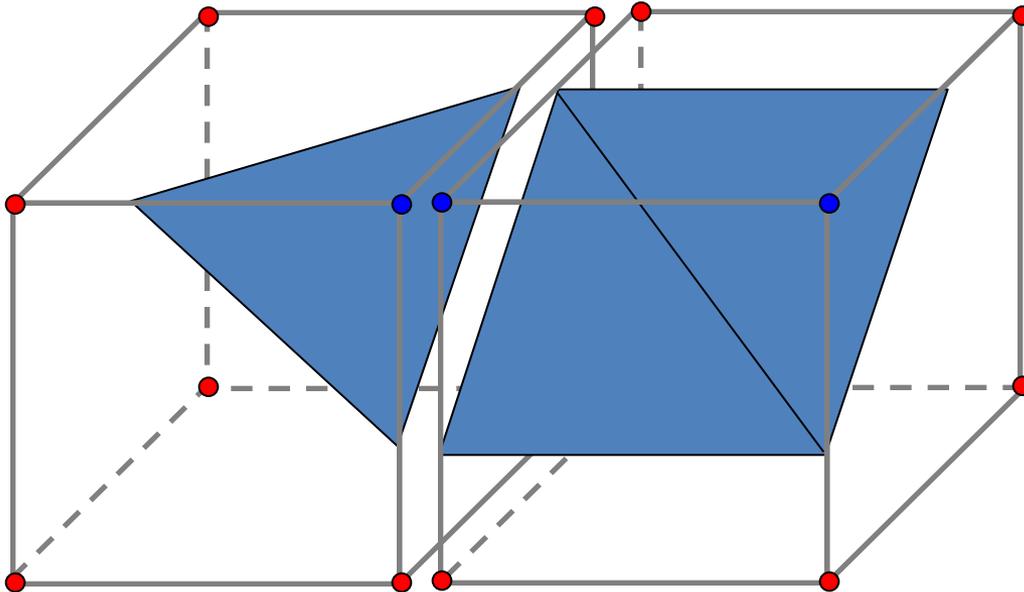


See blackboard...



Marching Cubes

Although each of the voxels is triangulated independently, the mesh is usually water-tight.

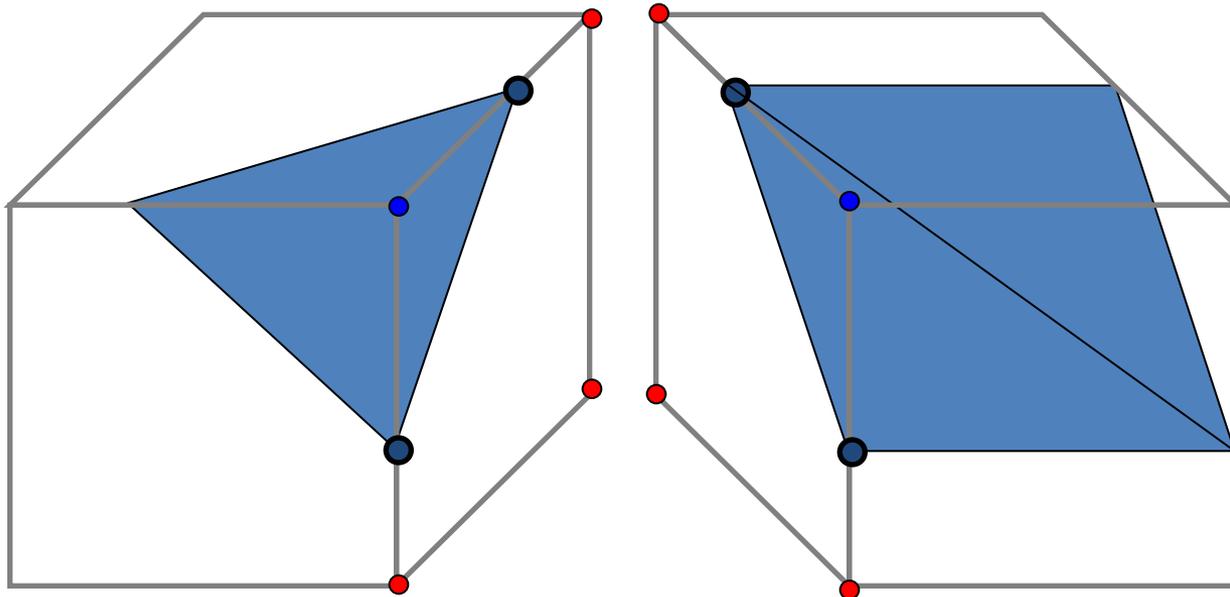




Marching Cubes

Iso-vertices on an edge are only determined by the values on the corner of the edge:

⇒ Iso-vertices are consistent across voxels.

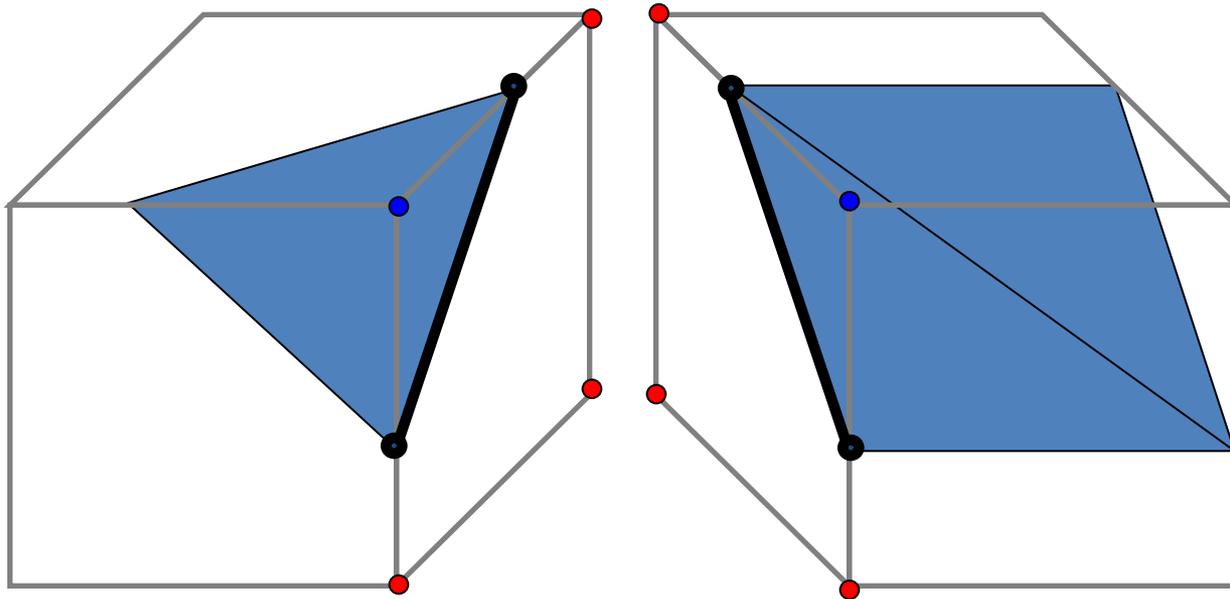




Marching Cubes

Iso-edges on a face are only determined by the values on the face:

⇒ Each iso-edge is shared by two triangles so the mesh is water-tight.

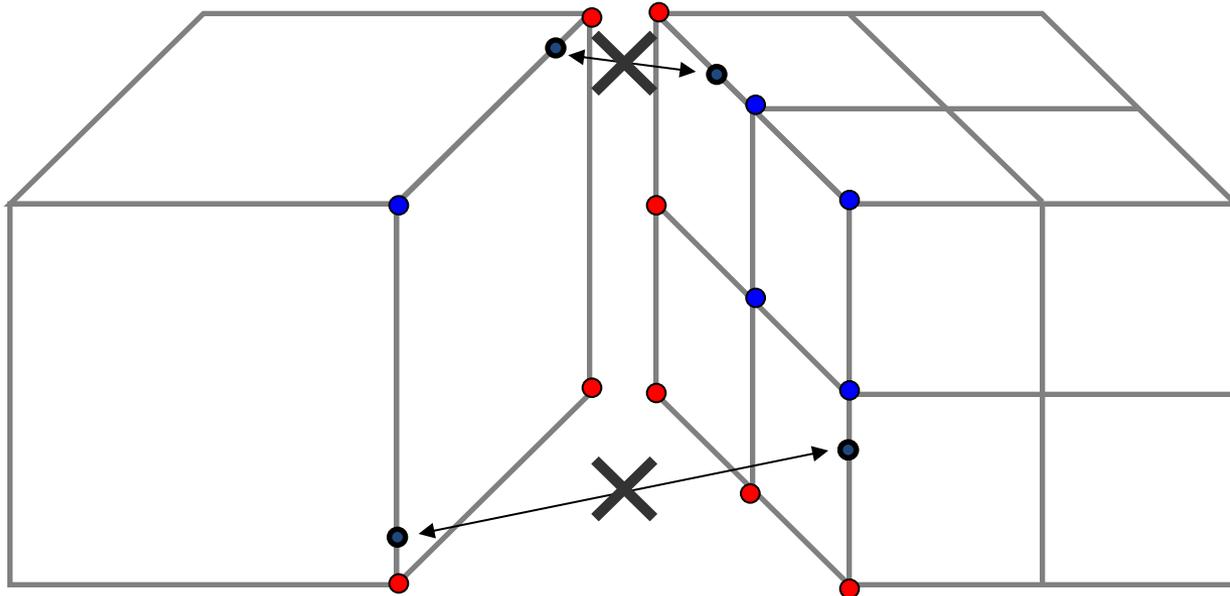




Challenges

Extracting a surface by independently triangulating the leaf octants, depth-disparities can cause:

- Inconsistent extrapolation to edges
⇒ Inconsistent iso-vertex positions

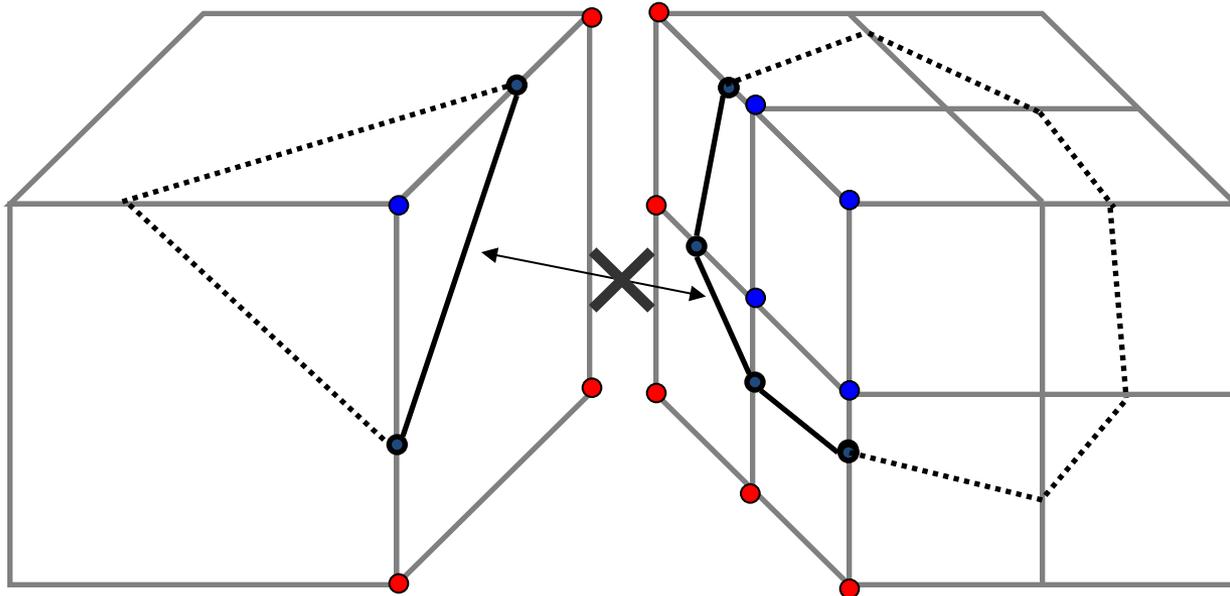




Challenges

Extracting a surface by independently triangulating the leaf octants, depth-disparities can cause:

- Inconsistent extrapolation to faces
⇒ Inconsistent iso-edges

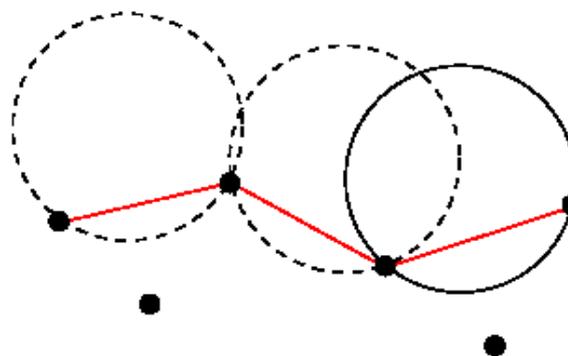




Ball-pivoting



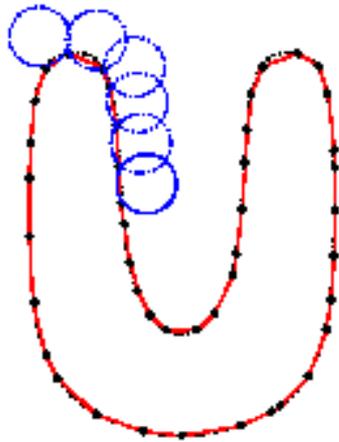
Bernardini et al., IBM



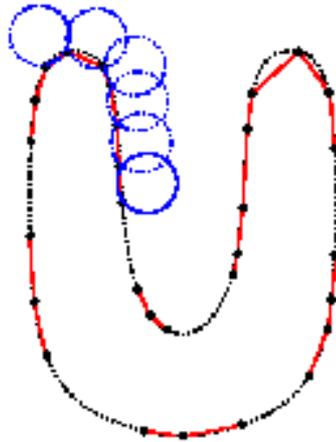
Fixed-radius ball “rolling”
over points selects subset of
alpha-shape.



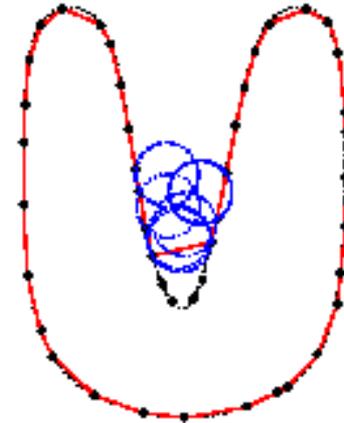
Pivoting in 2D



(a)



(b)



(c)

(a) Circle of radius ρ pivots from point to point, connecting them with edges.

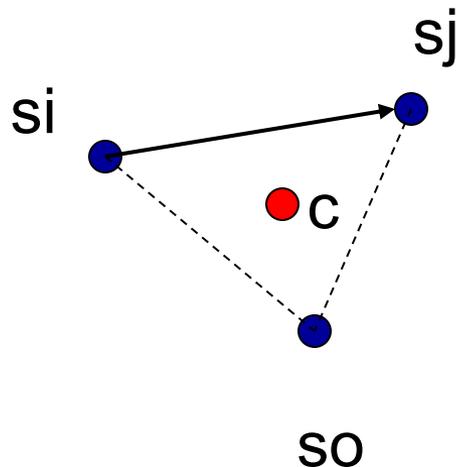
(b) When sampling density is low, some of the edges will not be created, leaving holes.

■ (c) When the curvature of the manifold is larger than $1/\rho$, some of the points will not be reached by the pivoting ball, and features will be missed.



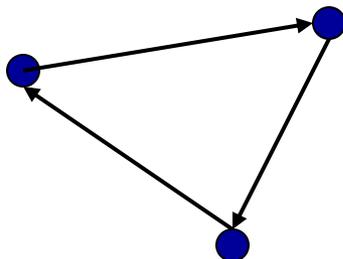
The algorithm [Edge representation]

- Edge (s_i, s_j)
 - Opposite point s_o , center of empty ball c
 - Edge: “Active”, “Boundary”, or “Frozen”





Pivoting example



Initial seed triangle:

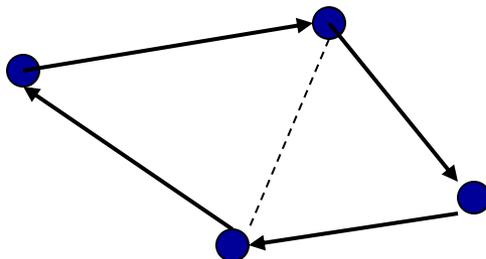
Empty ball of radius ρ passes through the three
points

Active edge
→

● Point on front



Pivoting example



Ball pivoting around active edge

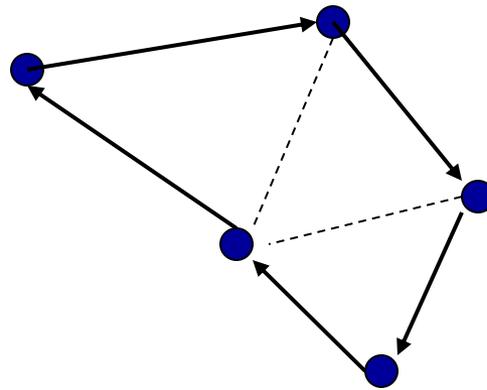
Active edge



● Point on front



Pivoting example



Ball pivoting around active edge

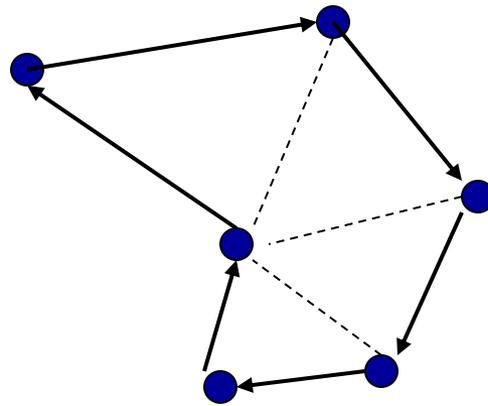
Active edge



● Point on front



Pivoting example



Ball pivoting around active edge

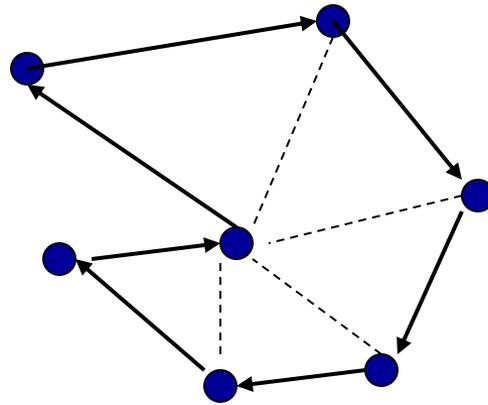
Active edge



● Point on front



Pivoting example



Ball pivoting around active edge

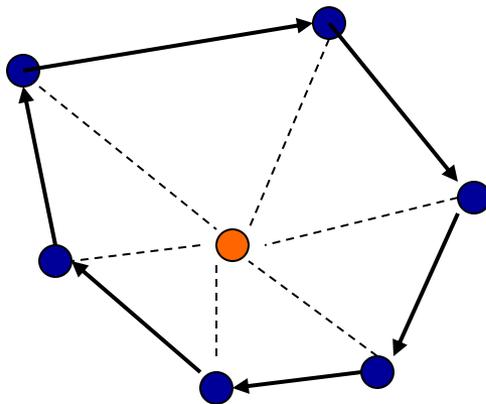
Active edge



● Point on front



Pivoting example



Ball pivoting around active edge

Active edge



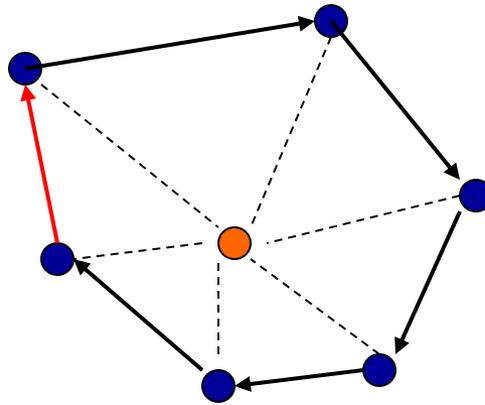
● Point on front

● Internal point



Pivoting example

Boundary edge



Ball pivoting around active edge

No pivot found

Active edge



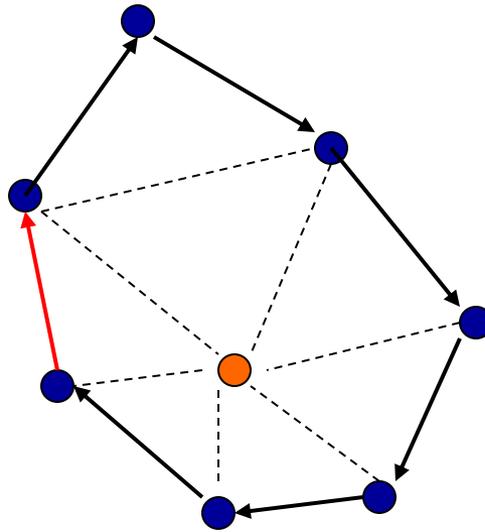
● Point on front

● Internal point



Pivoting example

Boundary edge



Ball pivoting around active edge

Active edge



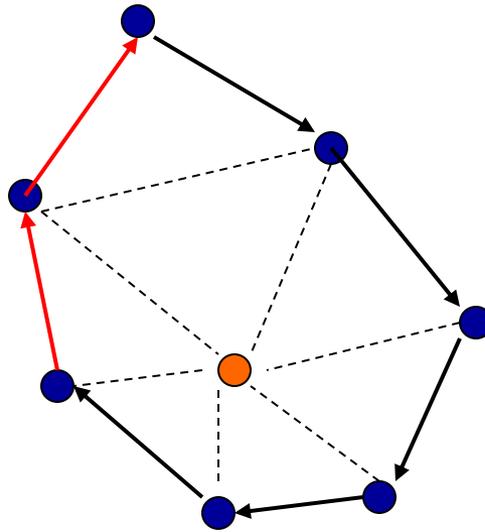
● Point on front

● Internal point



Pivoting example

Boundary edge



Ball pivoting around active edge

No pivot found

Active edge



● Point on front

● Internal point

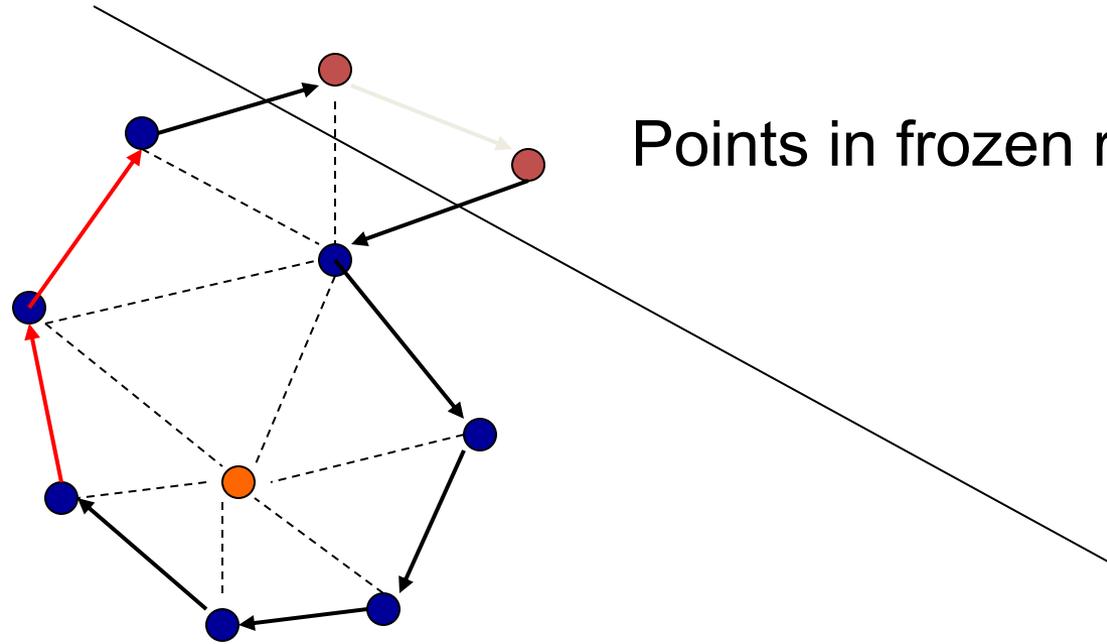


Pivoting example

Boundary edge



Frozen edge



Points in frozen region

Active edge



● Point on front

● Internal point

Ball pivoting around active edge

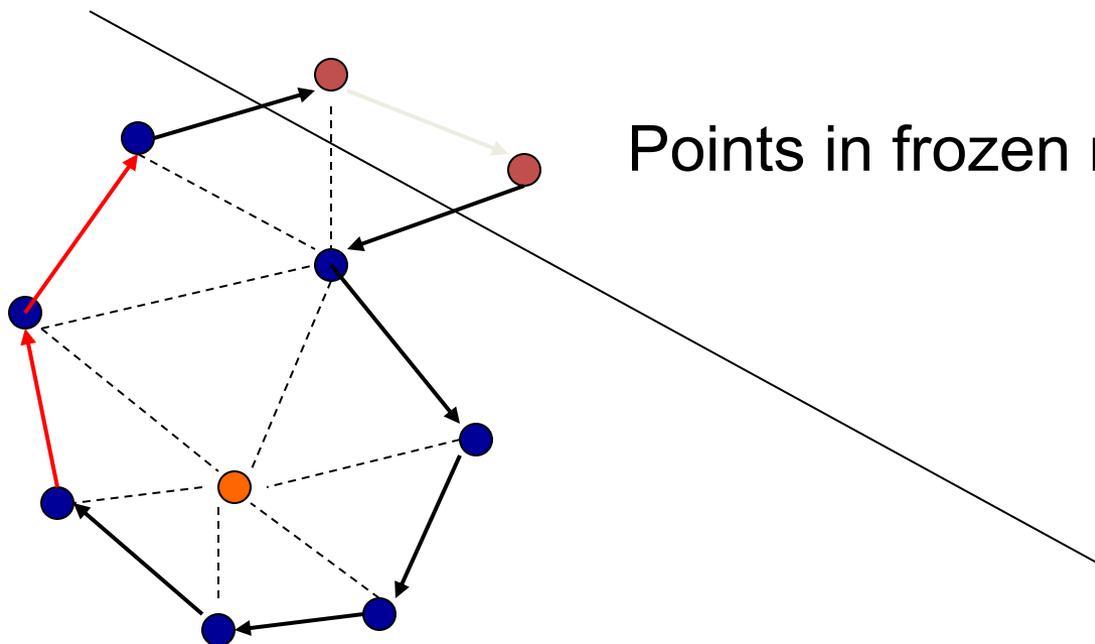


Pivoting example

Boundary edge



Frozen edge



Points in frozen region

Active edge



● Point on front

● Internal point

Ball pivoting around active edge

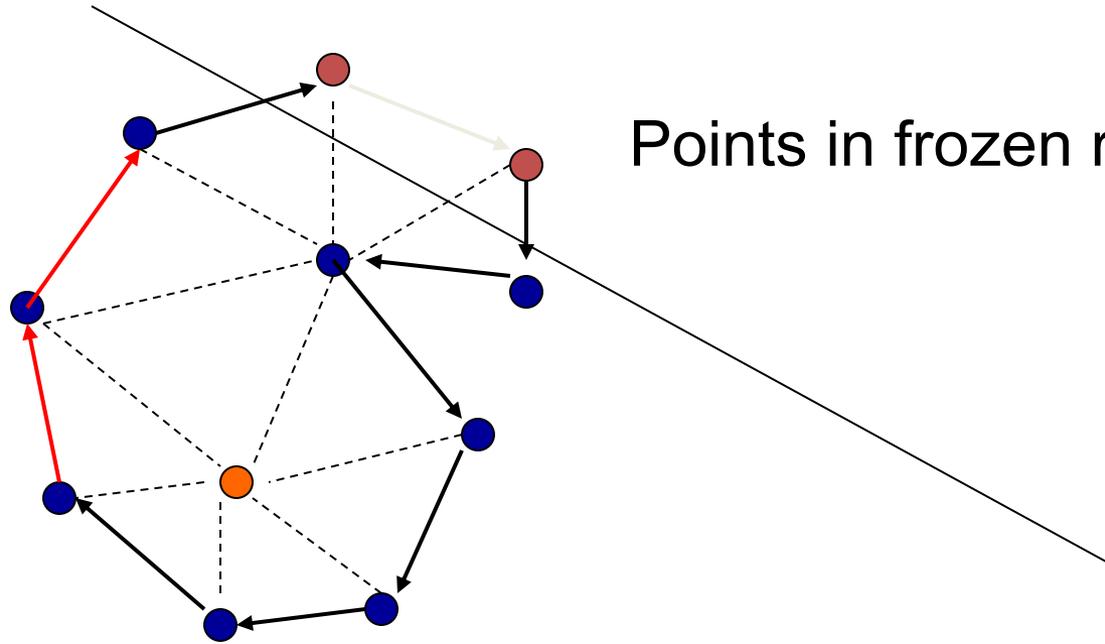


Pivoting example

Boundary edge



Frozen edge



Points in frozen region

Active edge



● Point on front

● Internal point

Ball pivoting around active edge

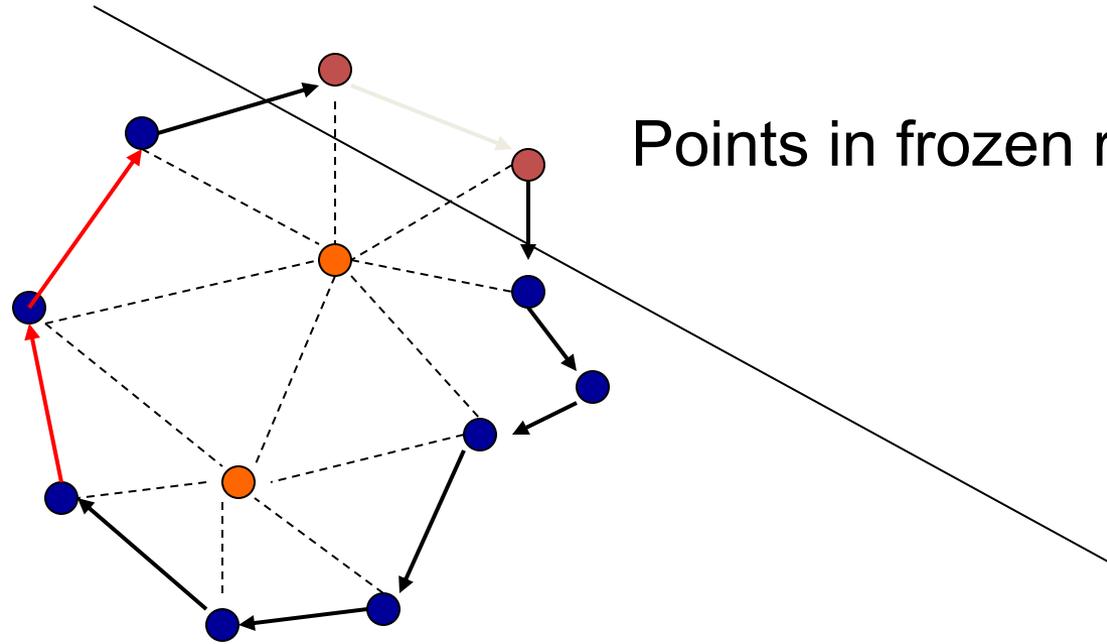


Pivoting example

Boundary edge



Frozen edge



Points in frozen region

Ball pivoting around active edge

Active edge

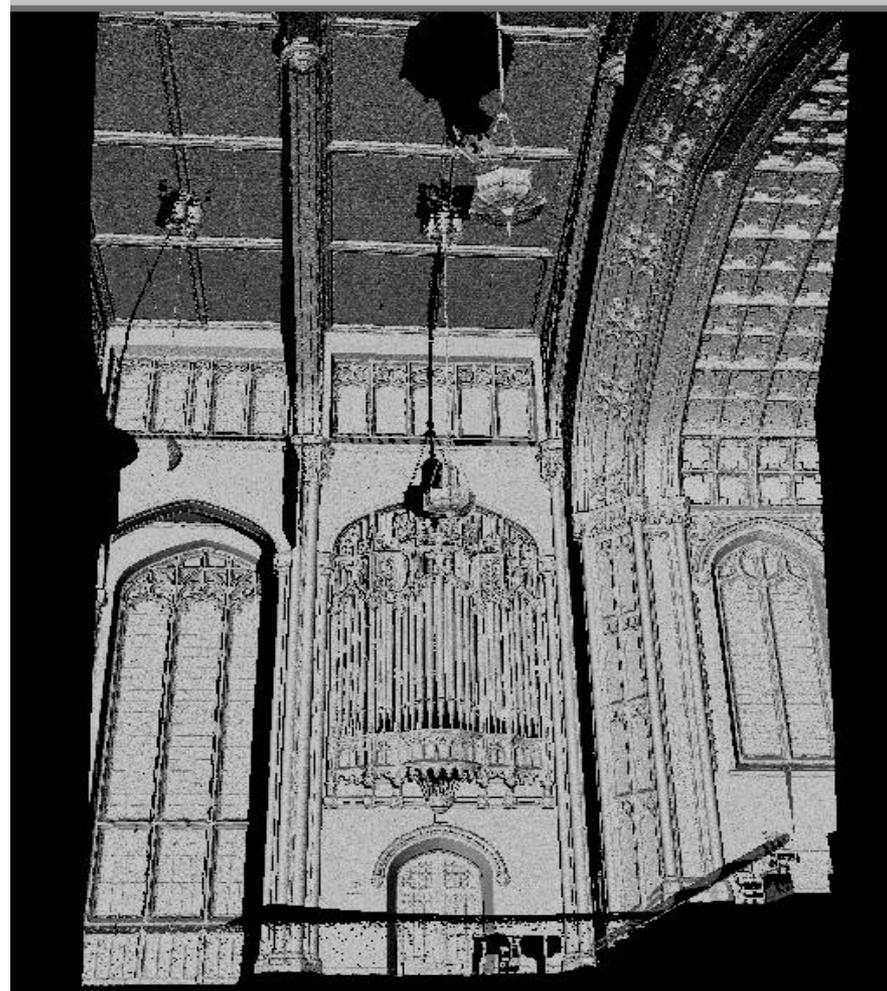
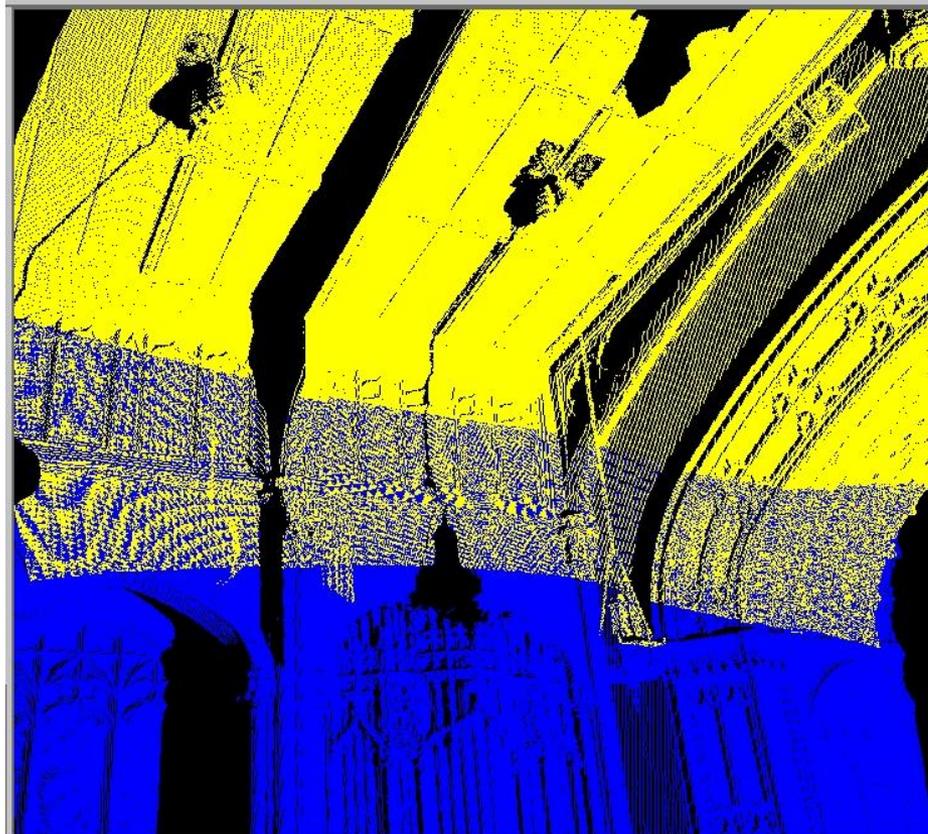


● Point on front

● Internal point

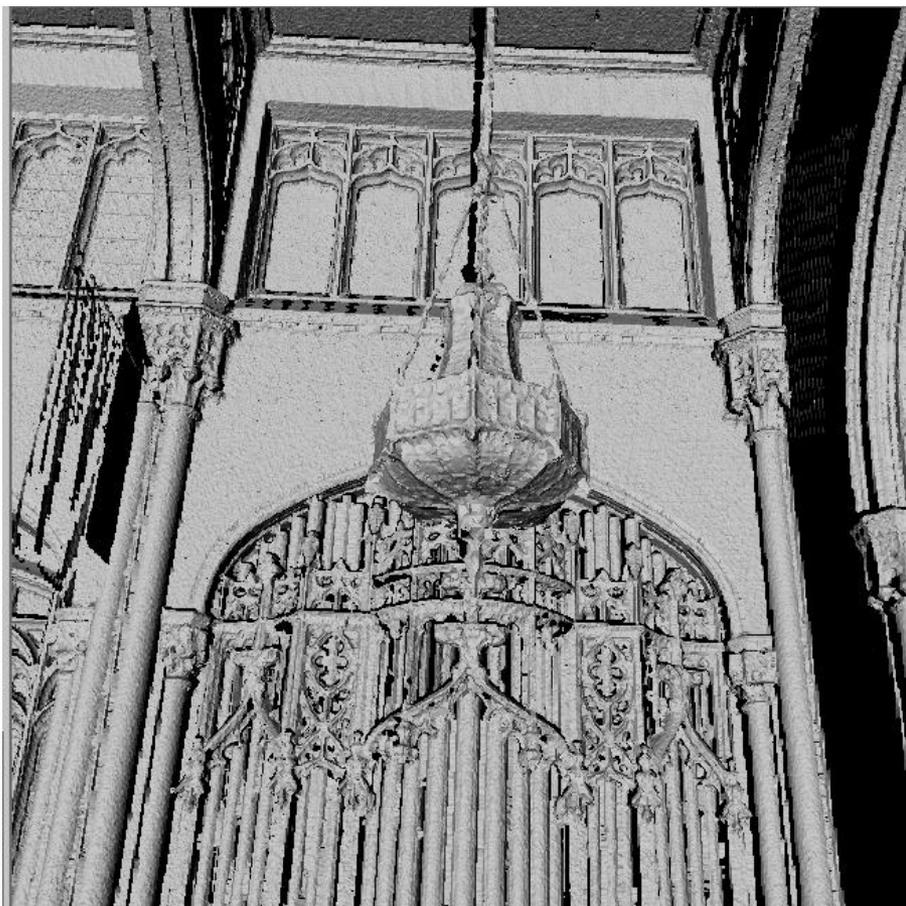


Ball Pivoting Algorithm





Ball Pivoting Algorithm

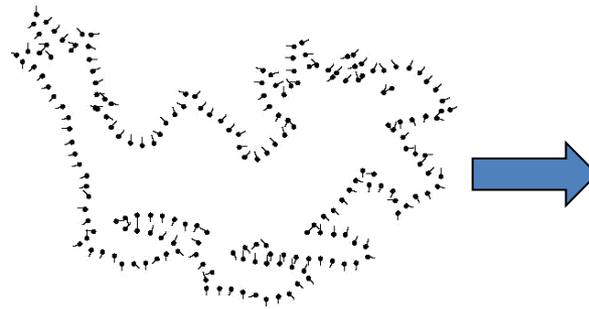




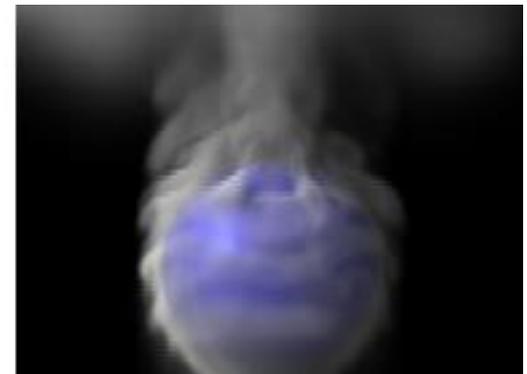
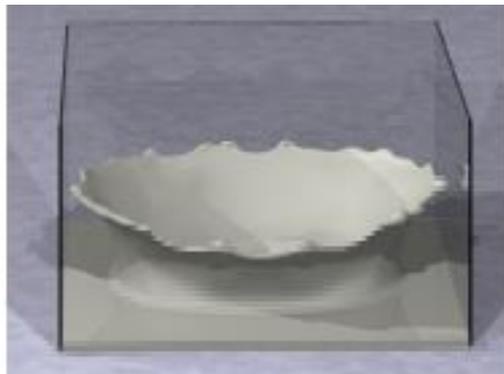
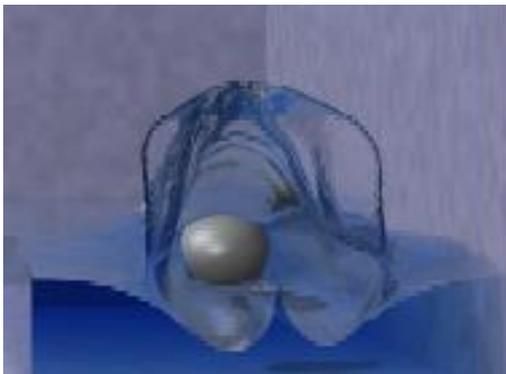
Implicit Representation

Another option is representing a 3D model by an implicit function for:

- Reconstruction
- Fluid Dynamics
- 3D Texturing



Kazhdan 2005



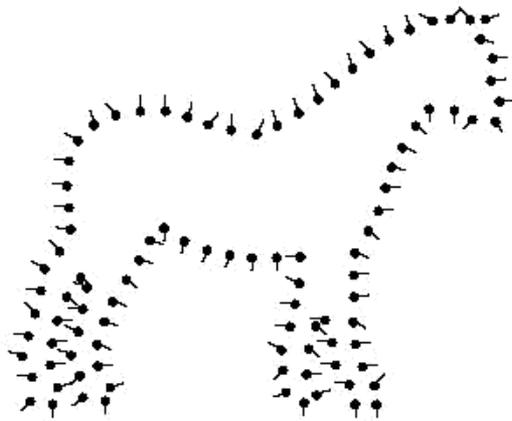
Losasso et al. 2004



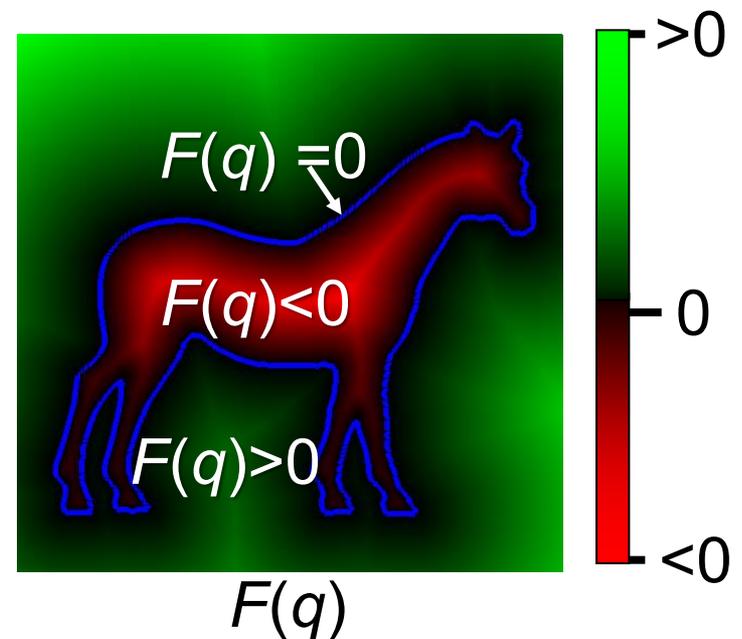
Implicit Function Fitting

Given point samples:

- Define a function with value zero at the points.
- Extract the zero isosurface.



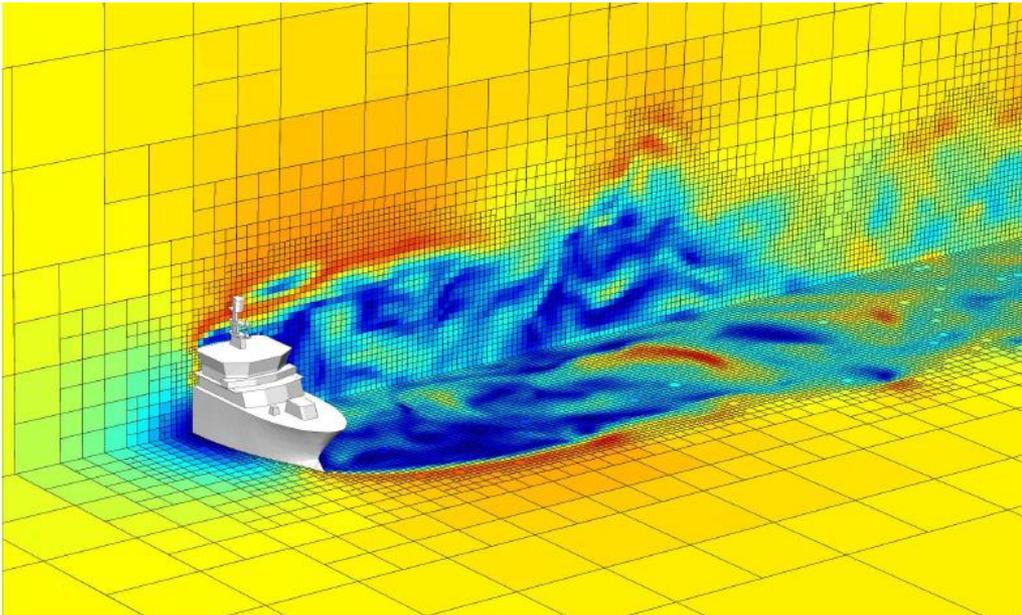
Sample points



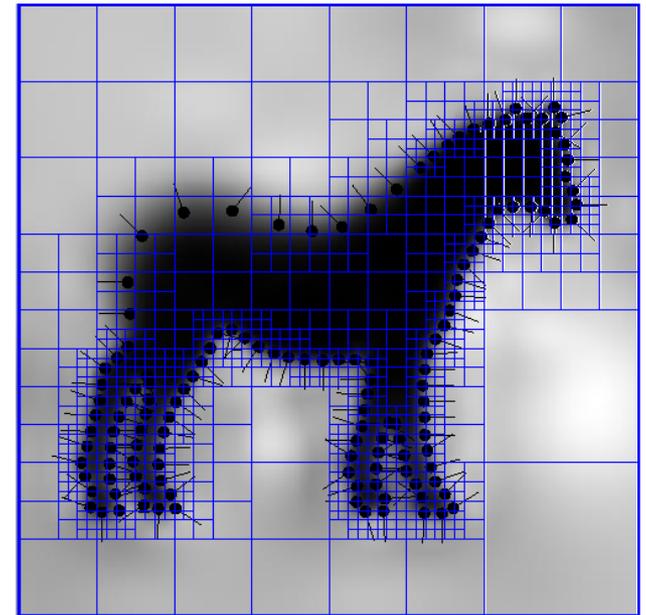


Octree Representation

An option is to sample a function on an adaptive grid



■ Popinet 2003

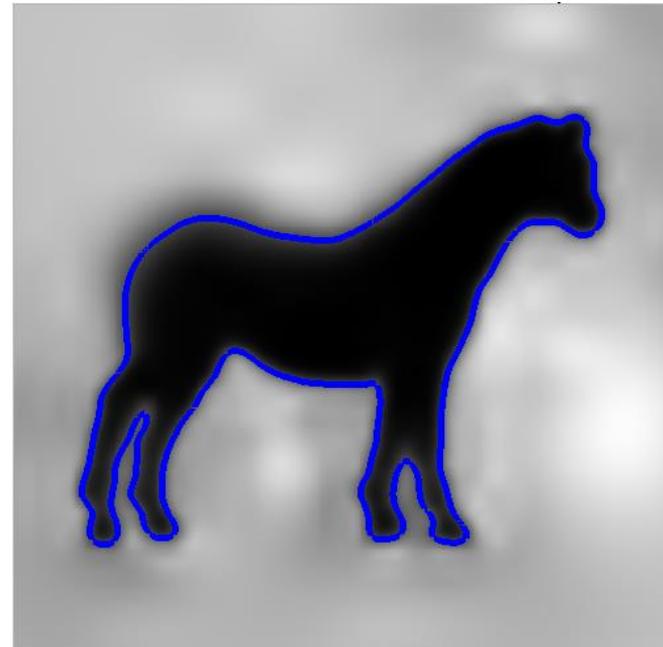
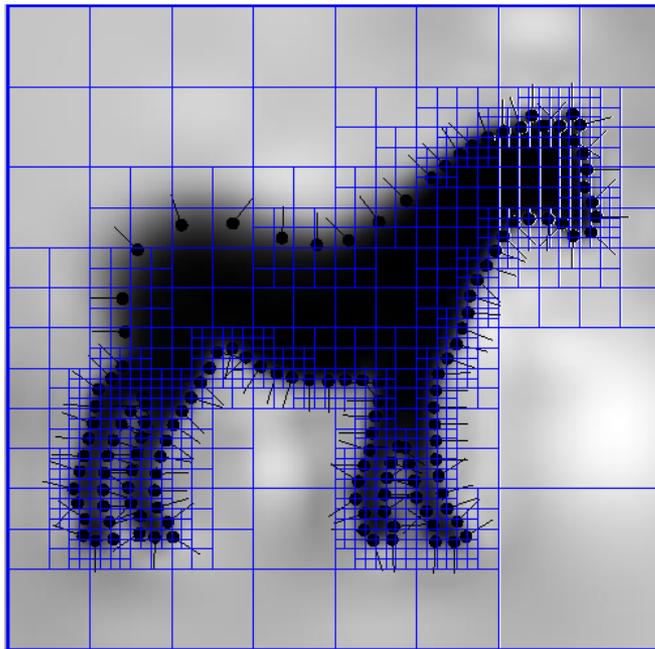


■ Kazhdan *et al.* 2006



Octree Extraction

From the sampled function, we would like to extract an implicit surface.

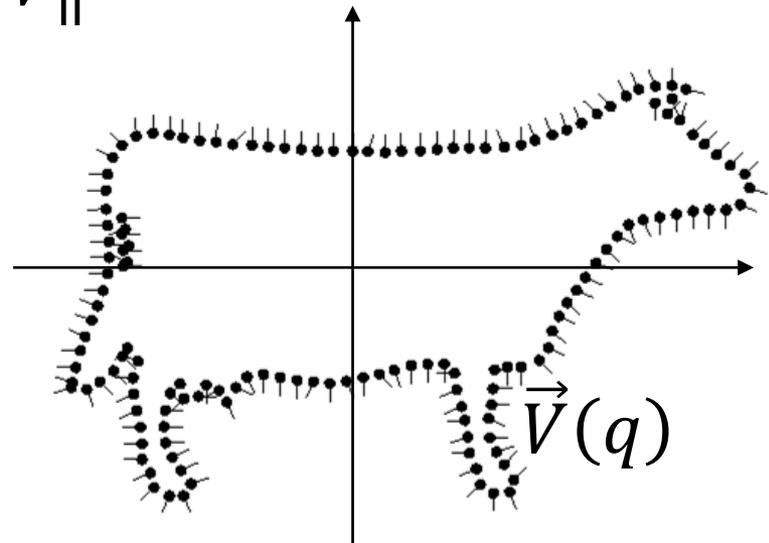
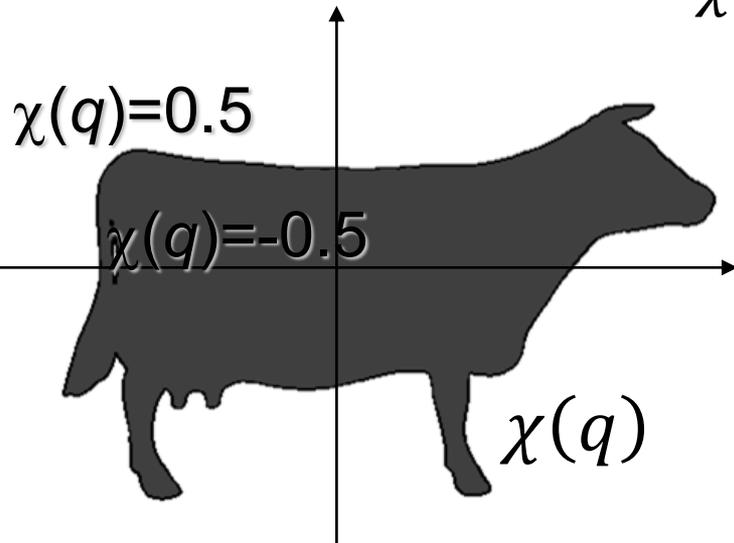




Poisson Surface Reconstruction

- Oriented points \approx samples of indicator gradient.
- Fit a scalar field to the gradients.

$$\chi = \min_{\chi} \|\nabla \chi - \vec{V}\|^2$$



Poisson Surface Reconstruction

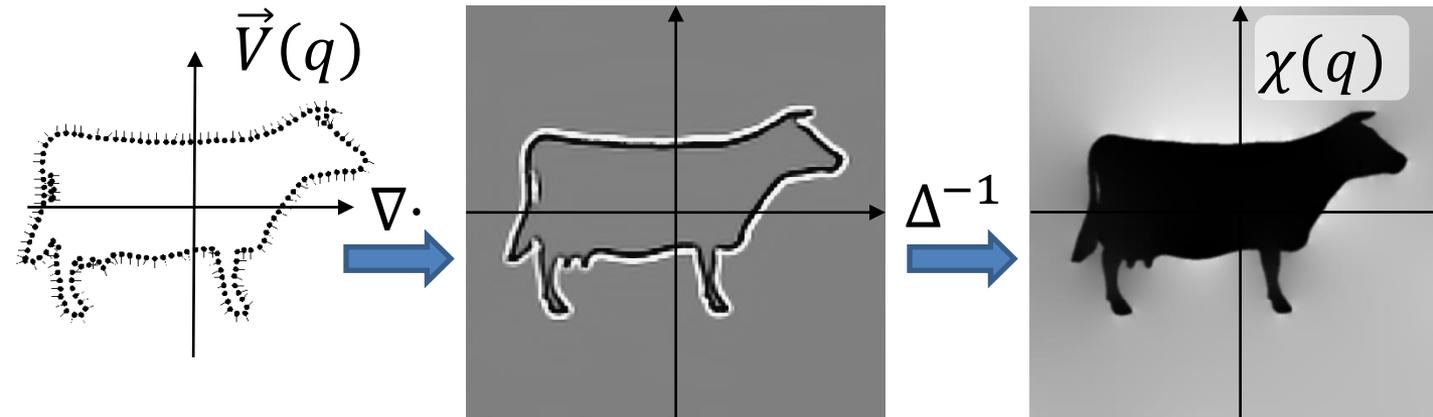


- But in general \vec{V} is not integrable (e.g., is a set of samples of an unknown function)
- So we apply a divergence operator so that can solve in best least squares way...

Poisson Surface Reconstruction



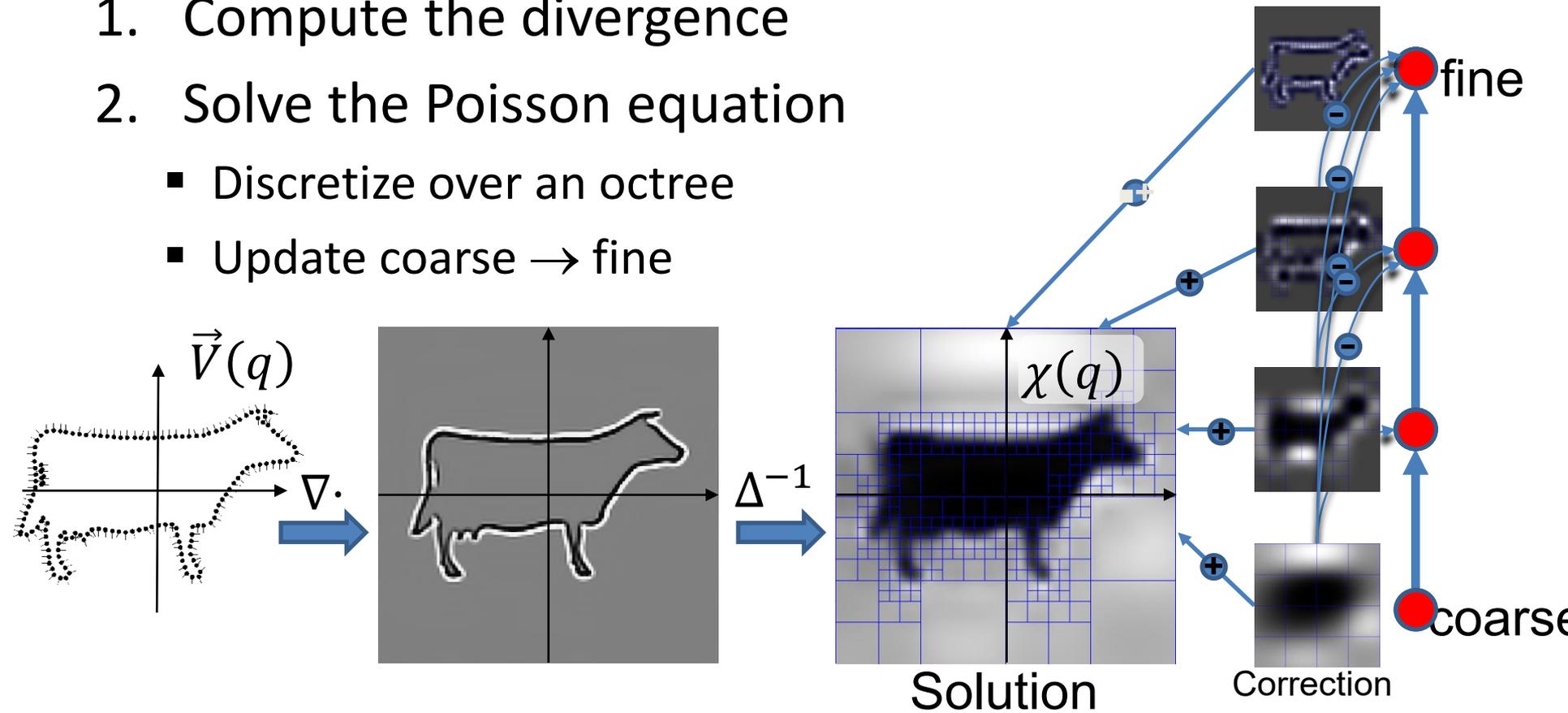
1. Compute the divergence
2. Solve the Poisson equation $\Delta \chi = \nabla \cdot \vec{V}$
or written as $\nabla^2 \chi = \nabla \cdot \vec{V}$



Poisson Surface Reconstruction



1. Compute the divergence
2. Solve the Poisson equation
 - Discretize over an octree
 - Update coarse \rightarrow fine



Poisson Surface Reconstruction



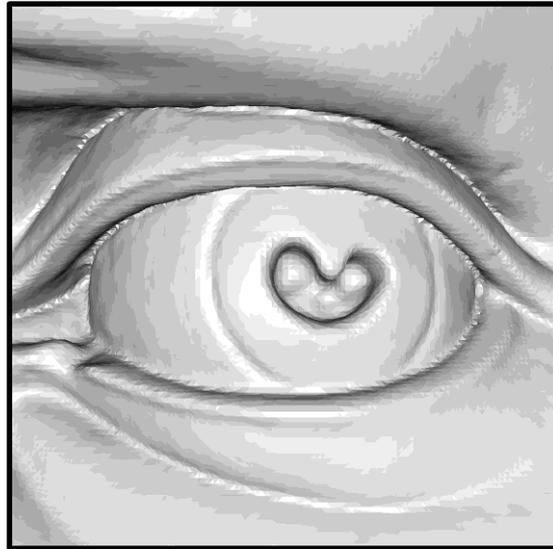
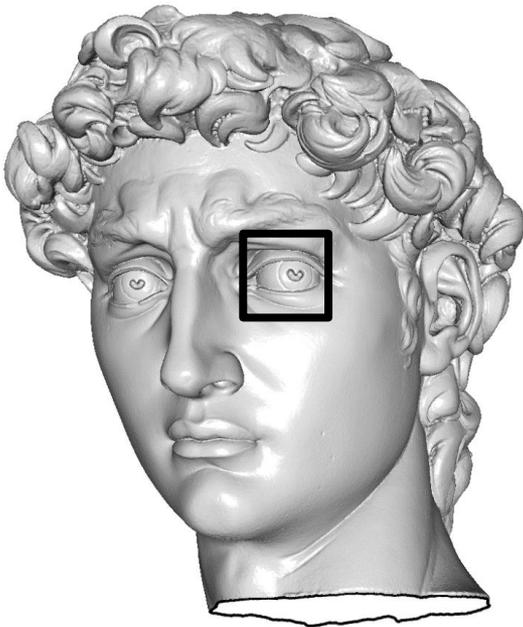
Properties:

- ✓ Supports noisy, non-uniform data
- ✗ Over-smoothes
- ✗ Solver time is super-linear

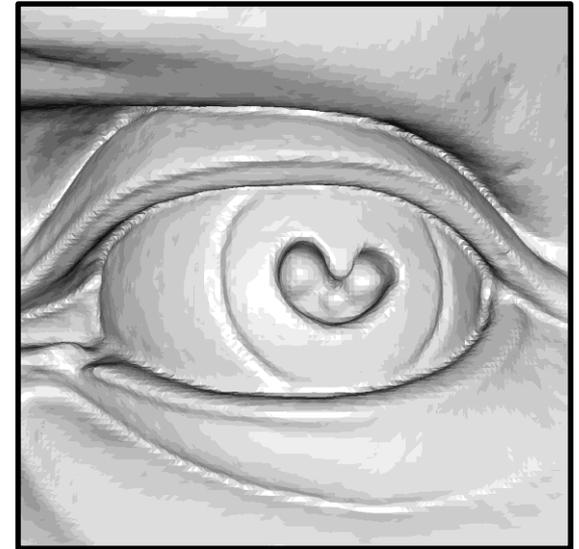


Screened Poisson Reconstruction

- Higher fidelity – at same triangle count
- Faster – solver time is linear



Poisson



Screened
Poisson