Spatial Data Structures and Hierarchies

CS535

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Spatial Data Structures

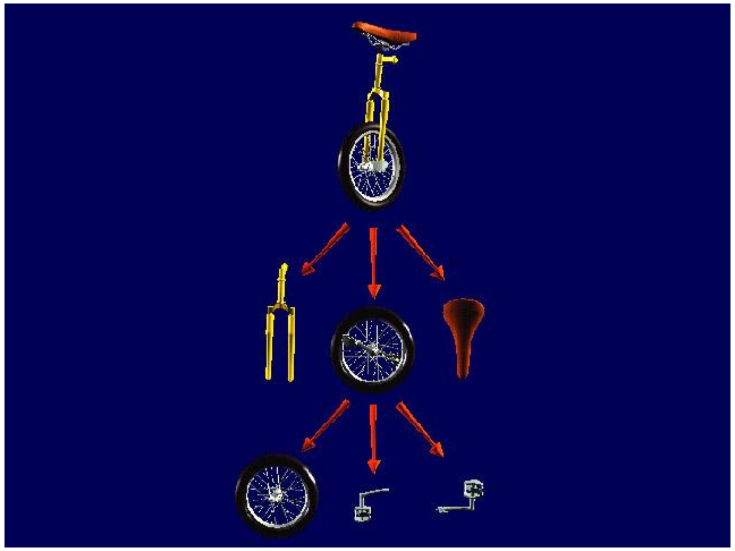
- Store geometric information
- Organize geometric information
- Permit fast access to/of geometric information
- Applications
  - Heightfields
  - Collision detection (core to *many* uses)
  - Simulations (e.g., surgery, games)
  - Rendering (e.g., need to render fast!)

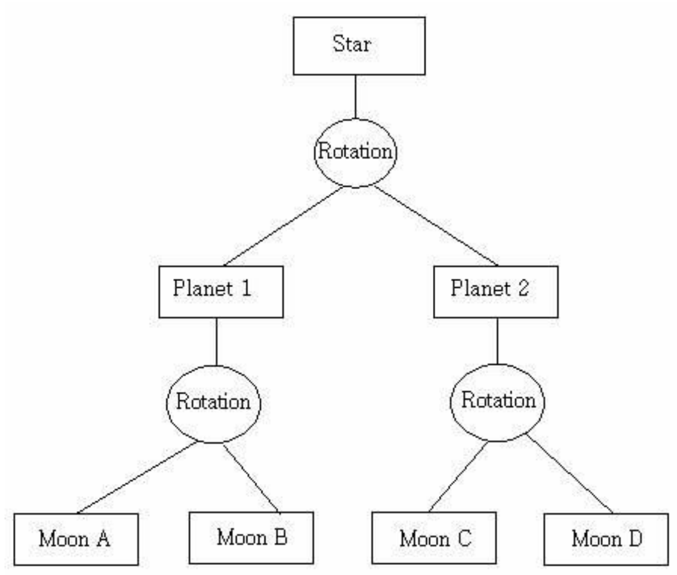
Hierarchical Modeling

- Concept is old but fundamental
  - "Hierarchical geometric models for visible surface algorithms", James Clark - 1976

Hierarchical Modeling

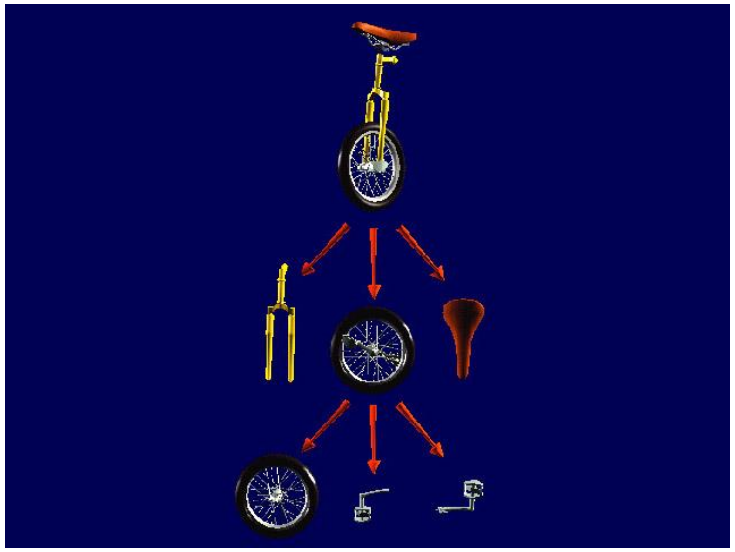
- Trees and Scene Graphs

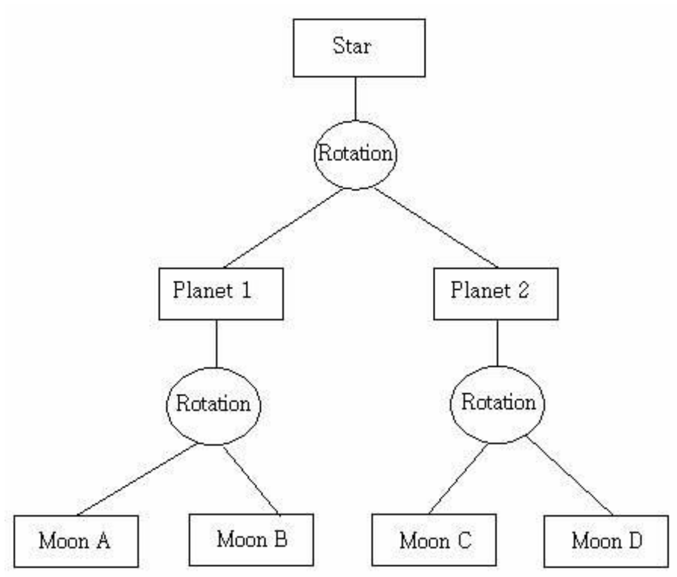




Hierarchical Modeling

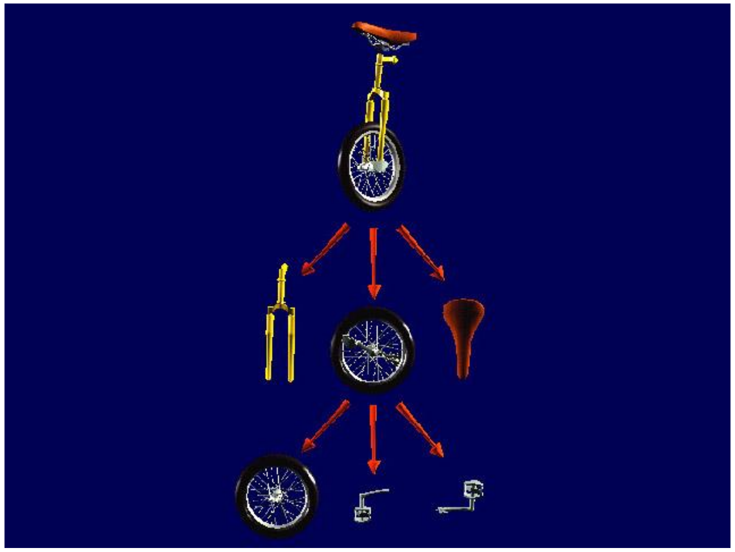
- Trees and Scene Graphs

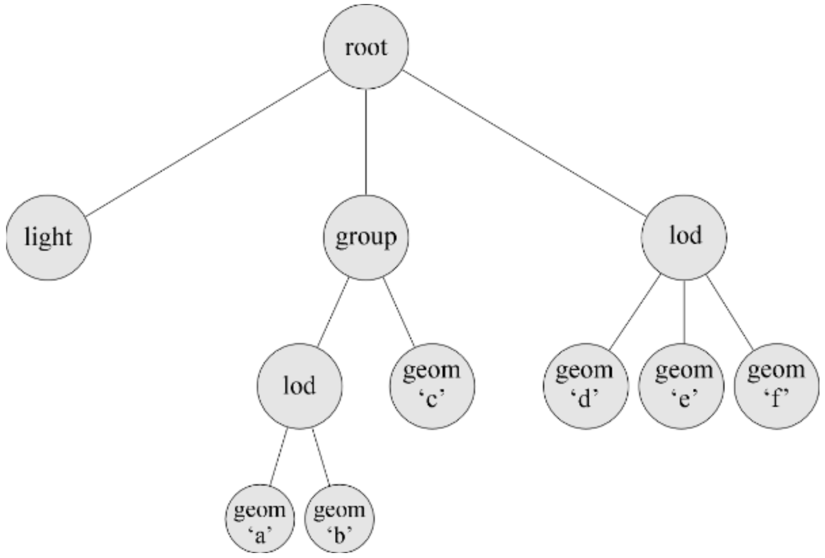




Hierarchical Modeling

- Trees and Scene Graphs





Bounding Volumes

- Problem:
  - Suppose you need to intersect rays with a scene…
  - Suppose you have a scene divided into objects…
- Solution: bottom-up
  - Wrap complex objects into simple ones
    - Boxes, spheres, other shapes…
  - Organize into a tree

Bounding Volume Hierarchy (BVH)

- How to building an axis aligned bounding box (AABB) BVH?
- How to intersect?
- Complexity? Problem cases?

AABB BVH

- Example construction
  - Given M 2D points, use k-means clustering to determine clusters
  - Then group nearby clusters (e.g., use Voronoi diagram or Delaunay triangulation)
  - And iteratively form a tree from the bottom-up
  - In each node, approximate the contained points using an axis-aligned bounding box
    - e.g., box = [min of all contained pts, max of all contained pts]

Bounding Volume Hierarchy (BVH)

- How to building an oriented bounding box (OBB) BVH?
- How to intersect?
- Complexity? Problem cases? Advantages over axis-aligned?

OBB BVH

- Example construction
  - Similar to AABB BVH but “fit” an oriented box to the points within each cluster/node of the tree
  - Methods:
    - Sample possible rotations and sizes in order to pick the best box
    - Compute distance of points to a line and optimize the line equation parameters until finding the line that best approximates all points
    - Then compute a box width – consider the benefit/cost of the box size
      - e.g., totally containing all points might make the box very large; could also choose to mostly contain the points – however, what does this mean with regards to operations using the BVH?

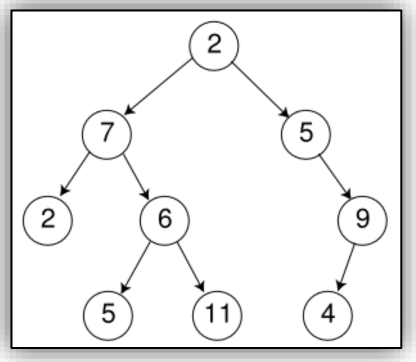
Space Subdivision

- Binary tree / Quadtree / Octree
- k-D tree
- Binary Space Partitioning (BSP) Tree

Binary Tree

- A directed edge refers to the link from the parent to the child (the arrows in the picture of the tree).
- The root node of a tree is the node with no parents; there is at most one root node in a rooted tree.
- A leaf is a node that has no children.
- The depth of a node is the length of the path from the root to the node. The root node is at depth zero.
- The height of a tree is the depth of its furthest leaf. A tree with only a root node has a height of zero.
- Siblings are nodes that share the same parent node

Size = 9
Height = 3
Root node = 2

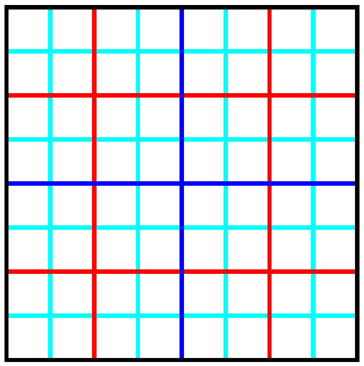


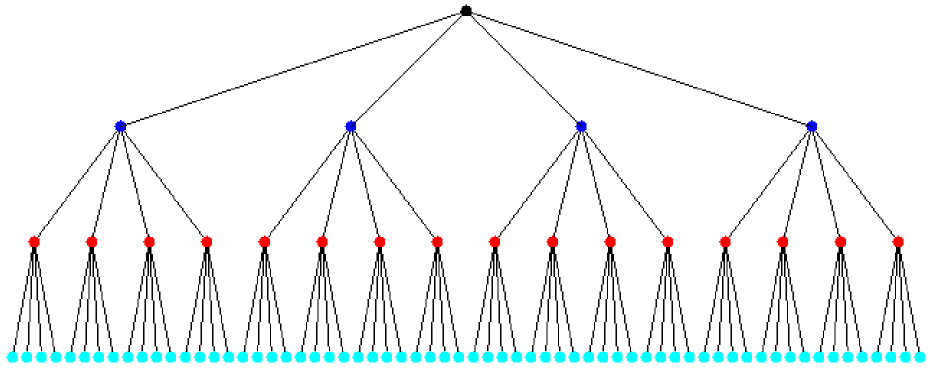
Binary Tree

- Operations
  - Search
  - Insert
  - Delete

Quadtree

- Similar to binary-tree, but have 4 children per node
- Each node corresponds to one of four rectangular regions of the current quad





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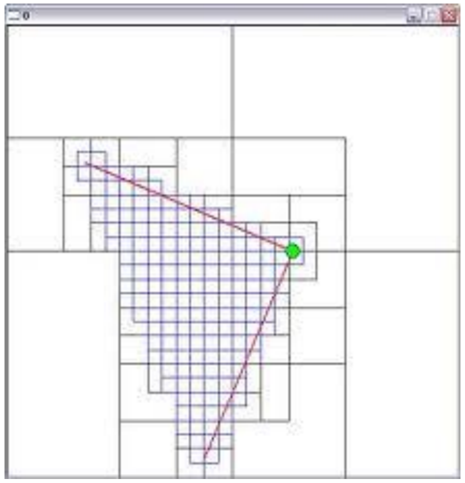
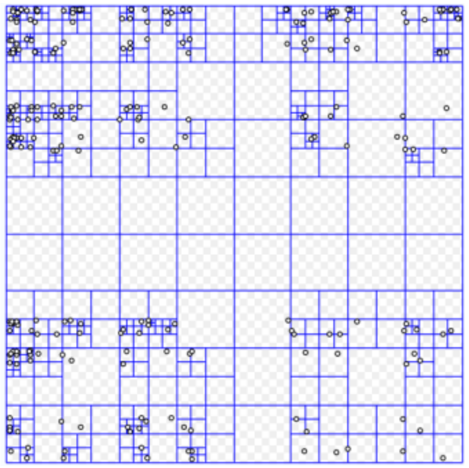
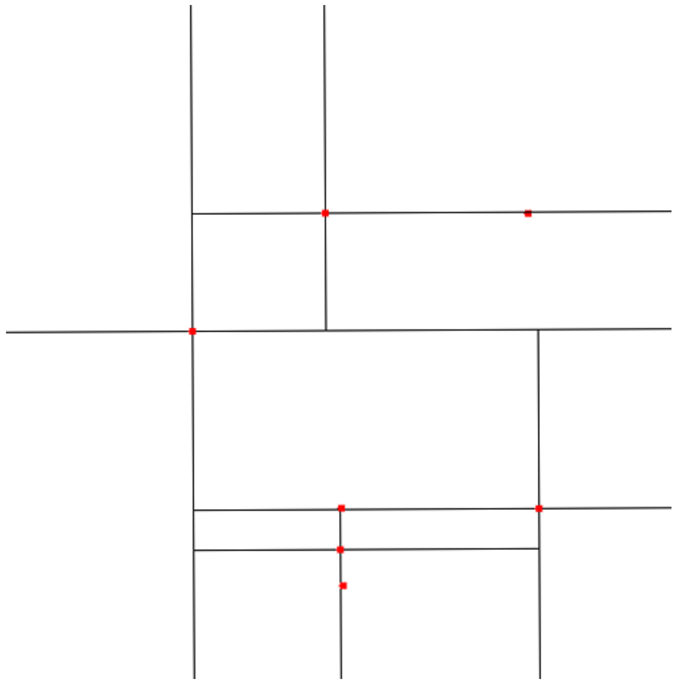



Quadtree

- Point quadtree
  - Partitions depend on the data
  - The quad is divided using the previous point within it

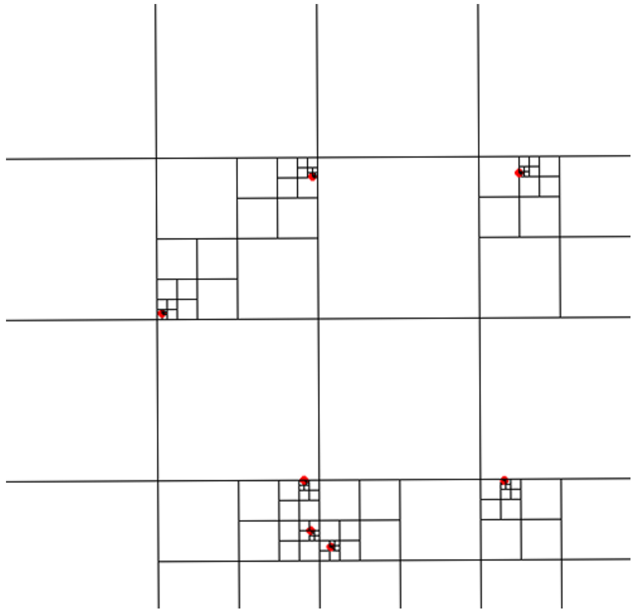


Quadtree

- Point quadtree
  - Partitions depend on the data
  - The quad is divided using the previous point within it
- Advantage
  - Data dependent subdivision reduces (unnecessary) number of quads
- Disadvantage
  - Quads do not tightly approximate region surrounding the point

Quadtree

- Matrix (MX) quadtree
  - Location of partition lines independent of the data
  - The occupied nodes are all subdivided until a tight fitting box

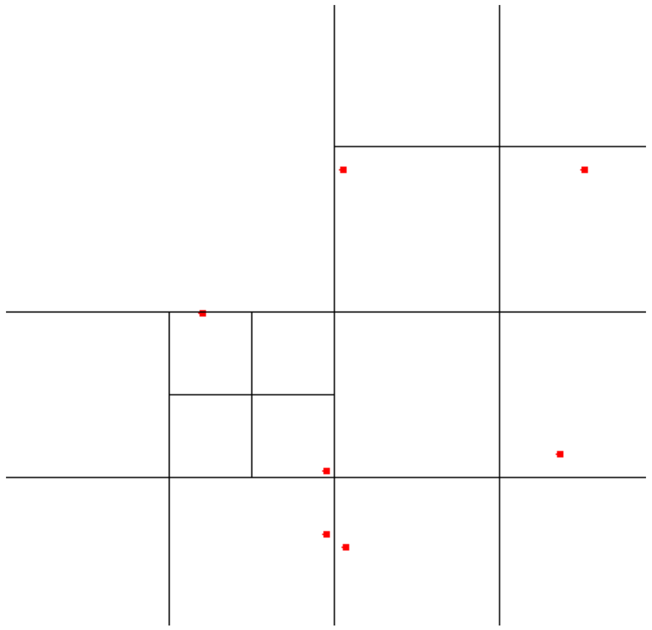


Quadtree

- MX quadtree
  - Location of partition lines independent of the data
  - The occupied nodes are all subdivided until a tight fitting box
- Advantage
  - Quads leaf nodes always tightly approximate region surrounding the point
- Disadvantage
  - Potentially lots of levels from root to a point

Quadtree

- Point Region (PR) quadtree
  - Location of partition lines independent of the data
  - The nodes are all subdivided until p or less points per node (e.g., p=1)

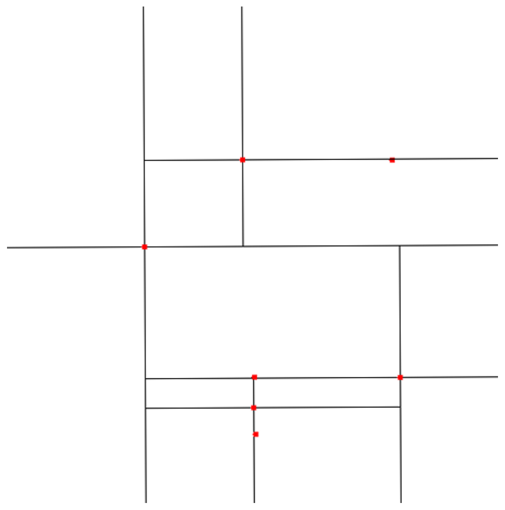


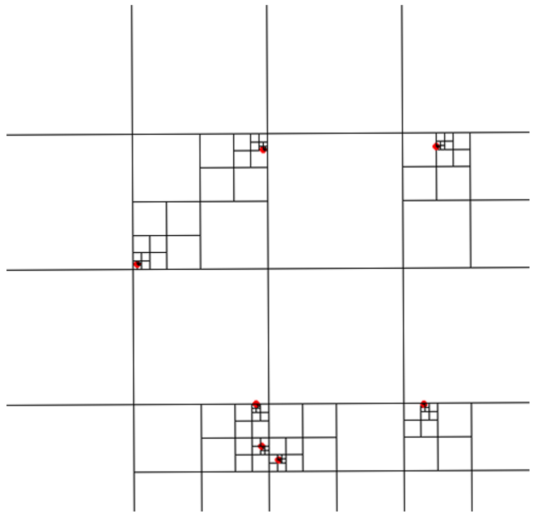
Quadtree

- PR quadtree
  - Location of partition lines independent of the data
  - The nodes are all subdivided until p or less points per node (e.g., p=1)
- Advantage
  - Partition lines are known and paths from root to point is only as long as needs to be
- Disadvantage
  - Quads do not tightly approximate region surrounding the point

Quadtree

- Comparison


Point QT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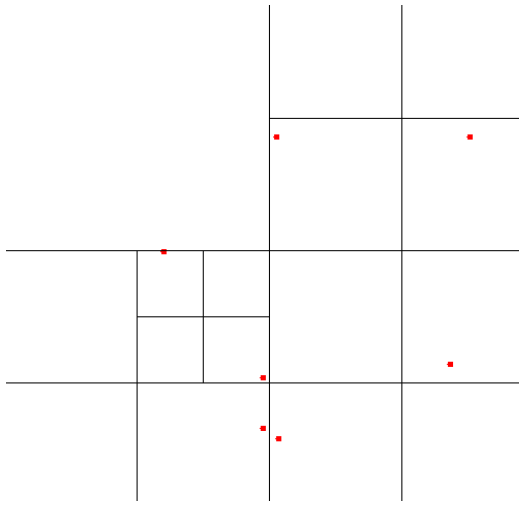
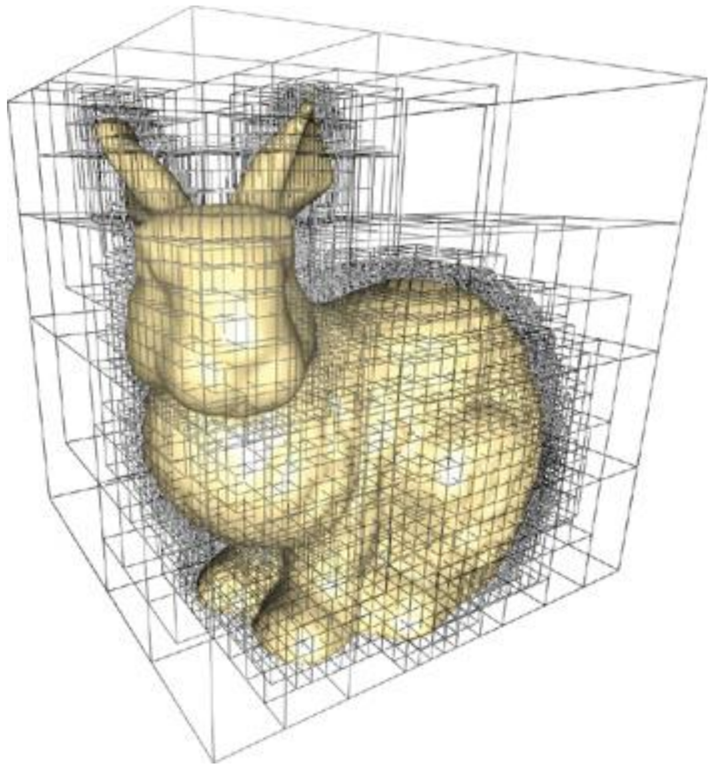
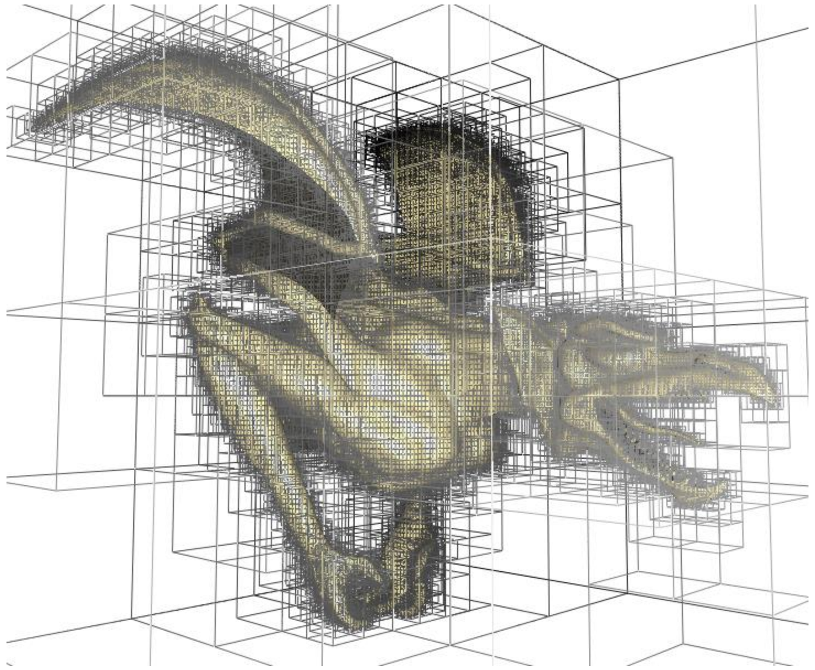

PR QT

Demo

- http://donar.umiacs.umd.edu/quadtree/

Octree

- Analogous to Quadtree but extended to 3D
- Each node is divided into eight subboxes

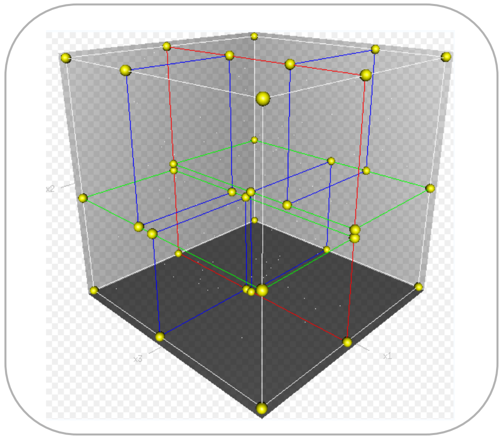
Octree

- Analogous to Quadtree but extended to 3D
- Each node is divided into eight subboxes
- Similar, there are
  - Point octrees
  - MX octrees
  - PR octrees

K-D tree

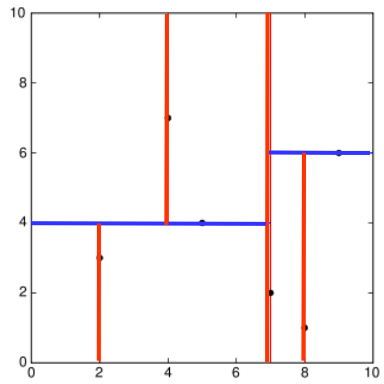
- Partition each dimension in a cyclical fashion
  - Thus, can be applied to 2D, 3D, or higher dimensions
- Each node stores a next partitioned “half-space” of data points (or of the data space)

k-D tree

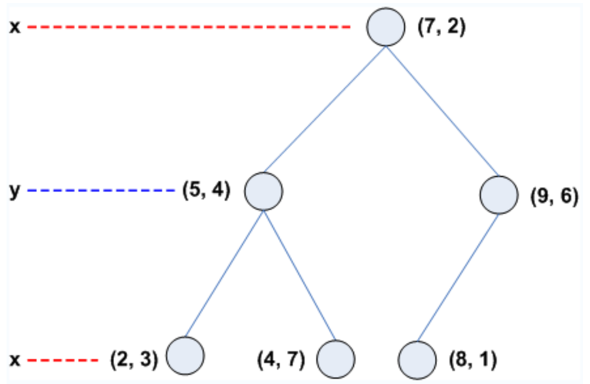


A 3-dimensional kd-tree

- The first split (red) cuts the root cell (white) into two
- Each of which is then split (green) into two subcells
- Each of those four is split (blue) into two subcells
- The final eight called leaf cells
- The yellow spheres represent the tree vertices



The resulting kd-tree decomposition



The resulting kd-tree

Demo

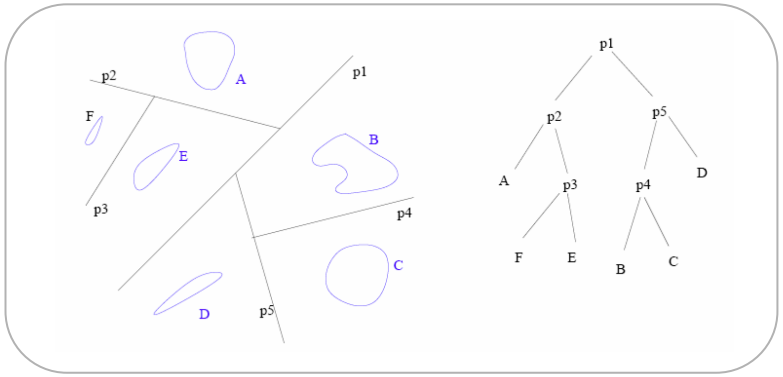
- http://donar.umiacs.umd.edu/quadtree/

Binary Space Partitioning (BSP)

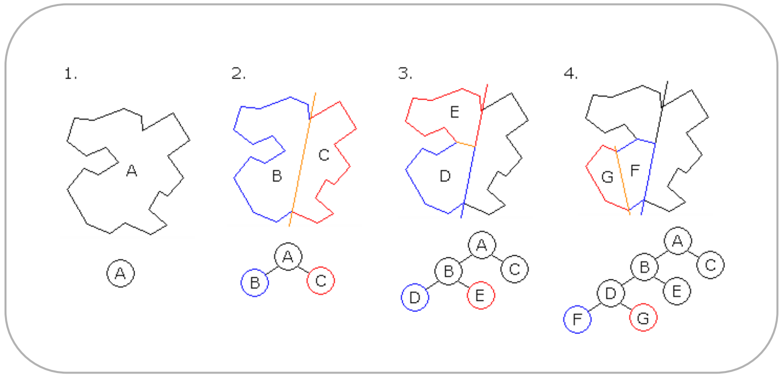
- Similar to k-D tree but splitting lines/planes are not necessarily axis-aligned
- Can adapt better to data
- Was algorithm used for visibility sorting…

Binary Space Partitioning (BSP)

- Suitable for any number of dimensions



Separating planes are shown in black and objects in blue)



BSP trees

Demo

- More stuff at
  - http://donar.umiacs.umd.edu/quadtree
  - R-tree?

- See
  - H. Samet, *Foundations of Multidimensional and Metric Data Structures*, Morgan-Kaufmann, San Francisco, 2006