Local and Global Illumination Models

CS535

Daniel G. Aliaga
Department of Computer Science
Purdue University

Lighting Models

- Light sources
  - Point light
    - Models an omnidirectional light source (e.g., a bulb)
  - Directional light
    - Models an omnidirectional light source at infinity
  - Spot light
    - Models a point light with direction

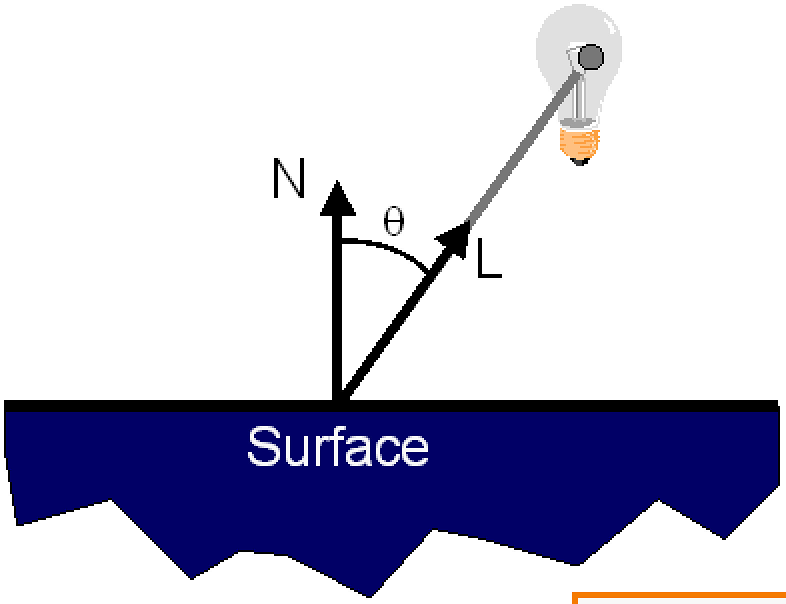
- Basic Direct Lighting Models
  - Ambient light
  - Diffuse reflection
  - Specular reflection

Lighting Models

- Light sources
  - Point light
    - Models an omnidirectional light source (e.g., a bulb)
  - Directional light
    - Models an omnidirectional light source at infinity
  - Spot light
    - Models a point light with direction
- Basic Direct Lighting Models
  - **Ambient light: just a "constant", e.g., 0.1 of background color**
  - Diffuse reflection
  - Specular reflection

Diffuse Reflection

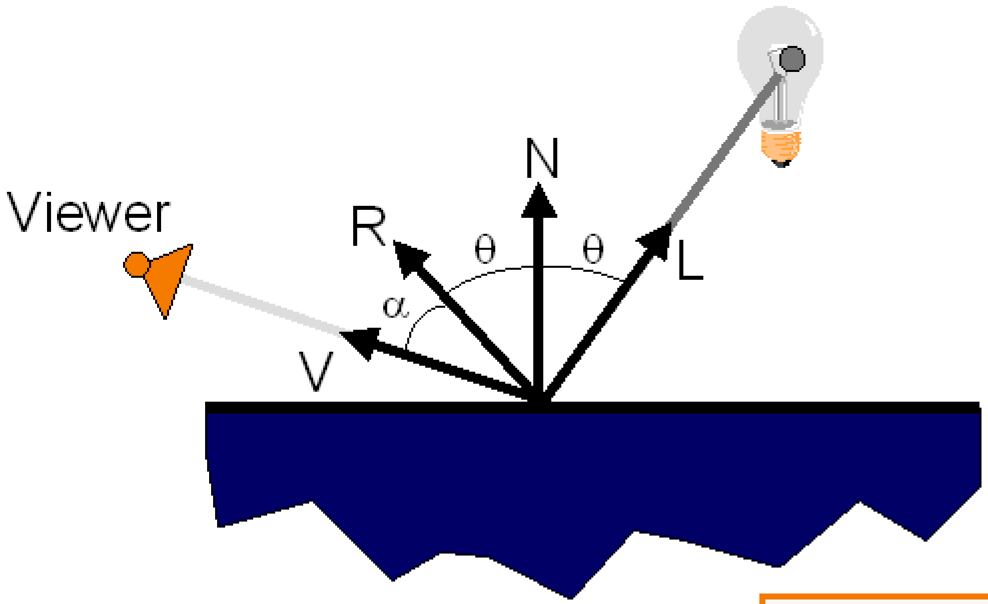
- Lambertian model



$$I_D = K_D(N \bullet L)I_L$$

Specular Reflection

- Phong model



$$I_S = K_S (V \bullet R)^n I_L$$

Specular Reflection

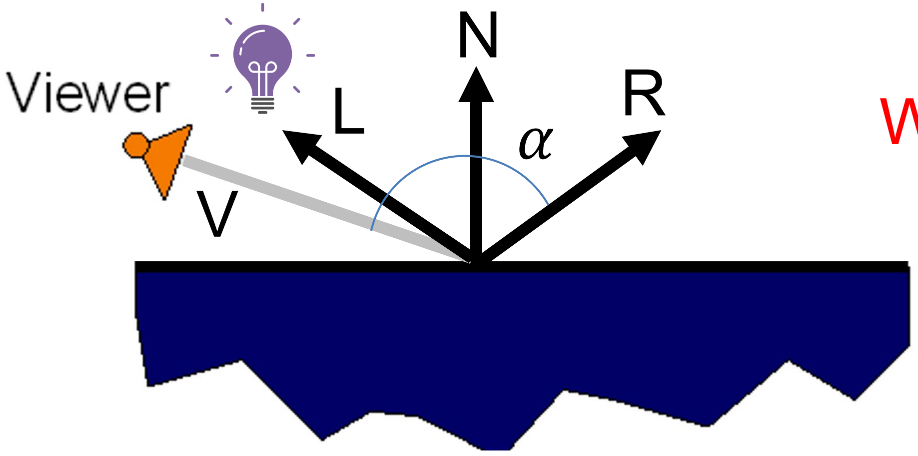
- Problem:

α is ?

$\alpha > 90$

$V \cdot R < 1$

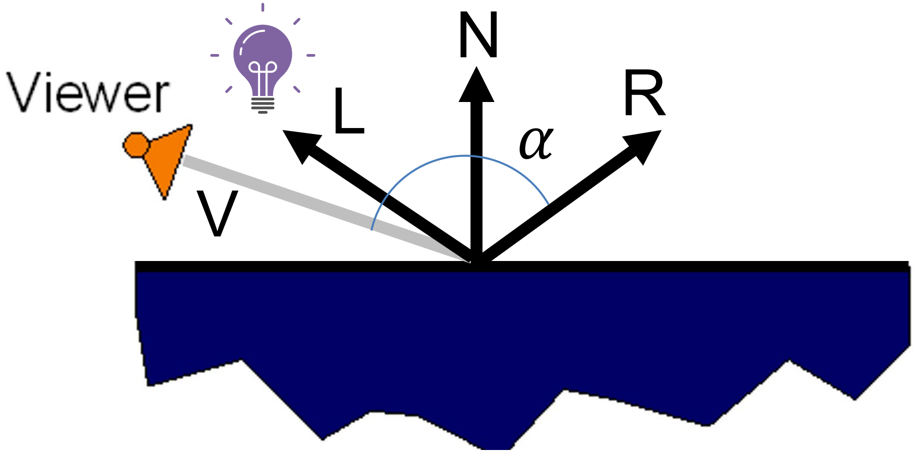
$K_S(V \cdot R)^n I_L < 0$



What can be done?

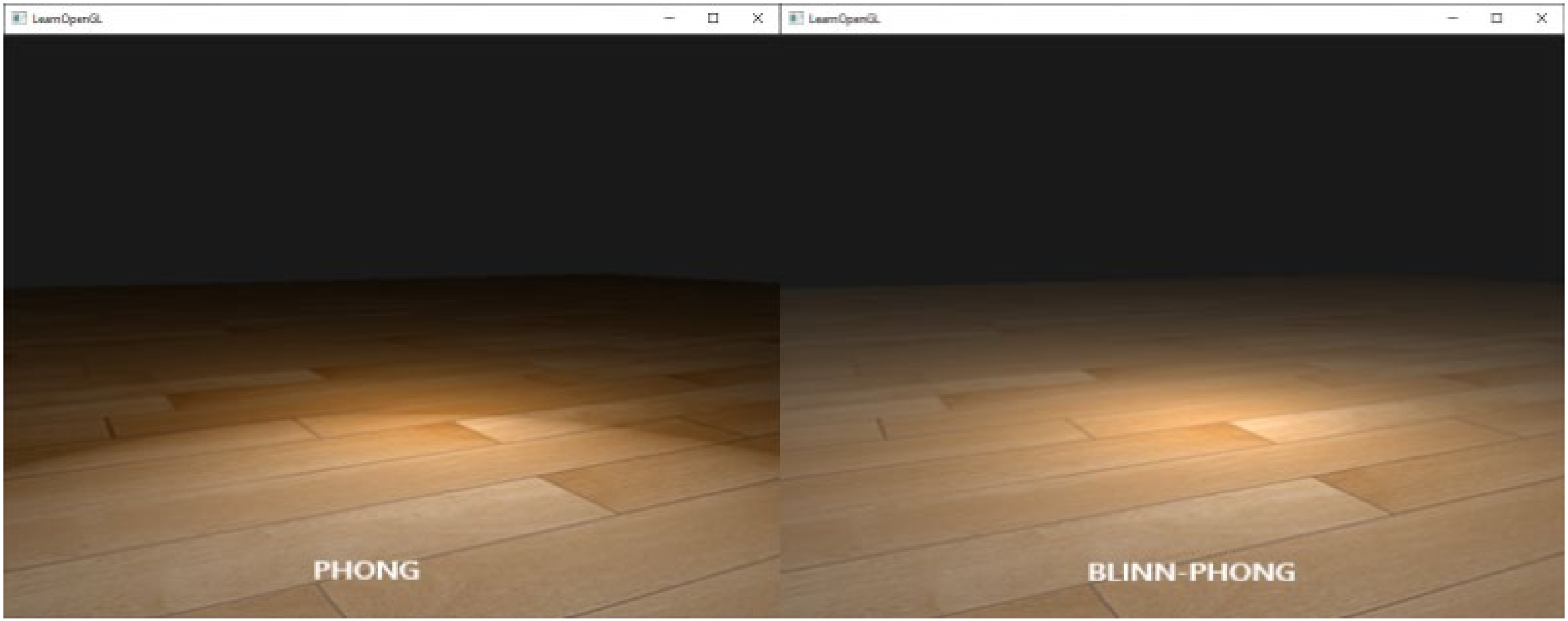
Specular Reflection

- Blinn-Phong Model (or Blinn Model)



$$H = (L + V)/\|L + V\|$$

$$I_{bp} = K_s (H \cdot N)^n I_L$$

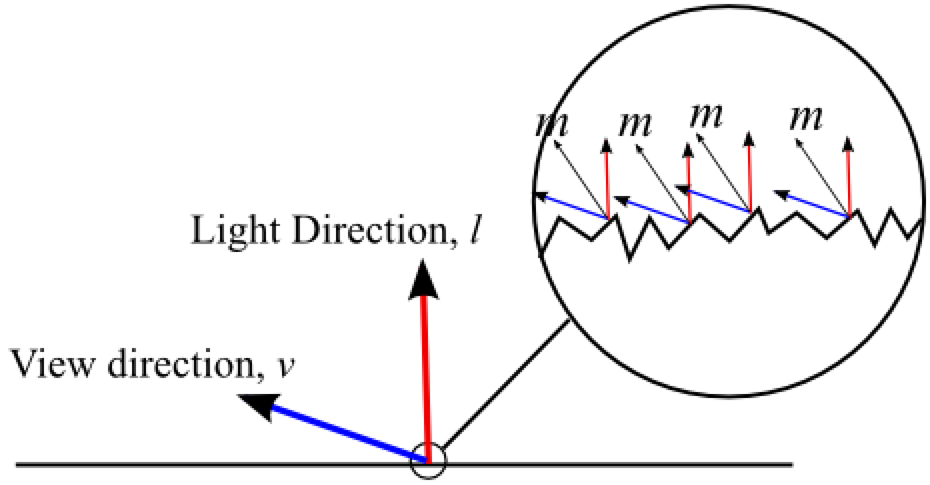

LearnOpenGL
PHONG
LearnOpenGL
BLINN-PHONG

Physically-Based Illumination Models

- Consider
  - Radiance
  - Irradiance
  - Microfacets
  - And more…

Microfacets

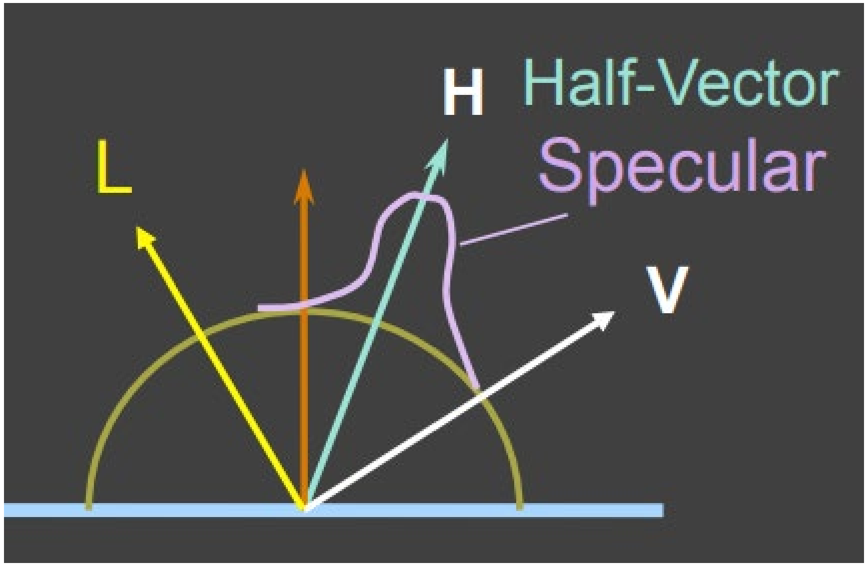
- Key question:
  - What proportion of the microfacets of the surface are oriented in such a way as to specularly reflect light towards the viewer?



Microfacets

- Key question:
  – What proportion of the microfacets of the surface are oriented in such a way as to specularly reflect light towards the viewer?

- If "very specular", how does shape change?
- If "not very specular", how does shape change?

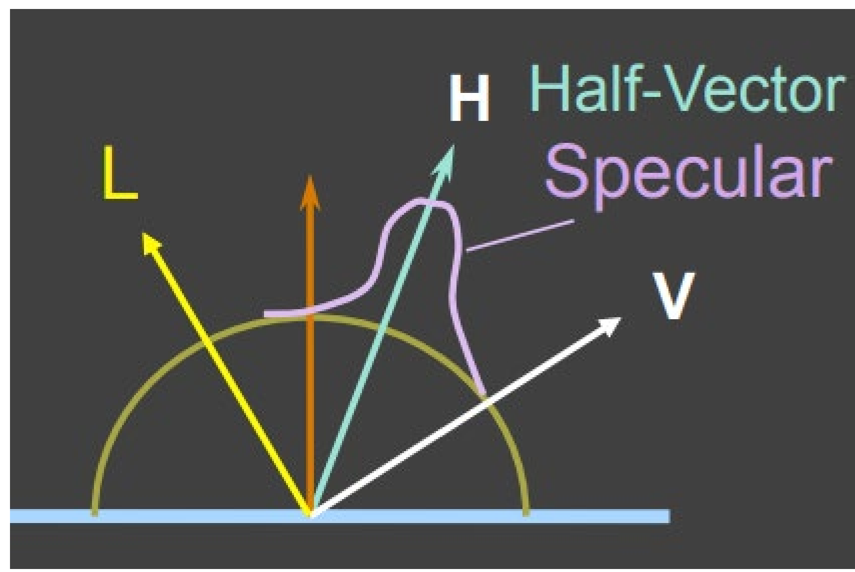


[courtesy Kavita Bala]

Microfacets

- Key question:
  – What proportion of the microfacets of the surface are oriented in such a way as to specularly reflect light towards the viewer?

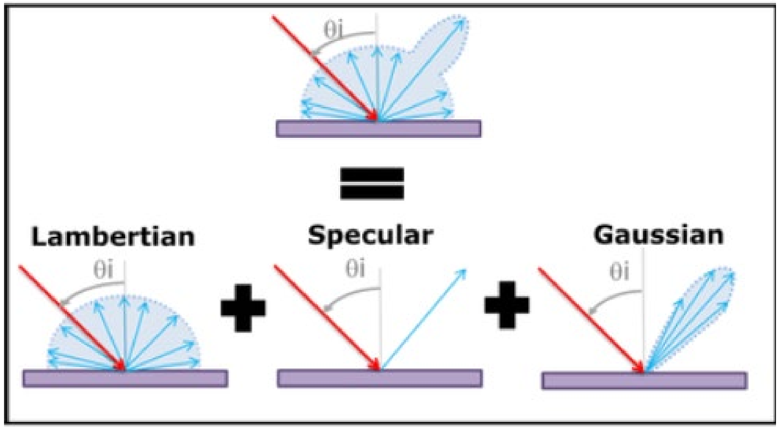
- How else can change shape?
  – Hint: a varying amount of microfacets reflect light in different directions…



[courtesy Kavita Bala]

Microfacets

- Key question:
  - What proportion of the microfacets of the surface are oriented in such a way as to specularly reflect light towards the viewer?
- An approximation:
  - Instead of $H \cdot N$ or $V \cdot R$, use Gaussian…



[courtesy L. Marot]

Microfacets

- Key question:
  - What proportion of the microfacets of the surface are oriented in such a way as to specularly reflect light towards the viewer?
- An approximation:
  - Instead of $H \cdot N$ or $V \cdot R$, use $e^{-\left(\frac{\alpha}{m}\right)^2}$ where $m \in [0,1]$ where larger m means rougher surface (=less specular)

$$I_{bp} = K_s e^{-\left(\frac{\operatorname{acos}(H \cdot N)}{m}\right)^2} I_L$$

Microfacets

- Torrance-Sparrow Model
  - Surface reflection model developed by physicists, assumes isotropic collection of planar microfacets

$$I_s = \frac{F_k}{\pi} \frac{DG}{(N \cdot V)(N \cdot L)}$$

where

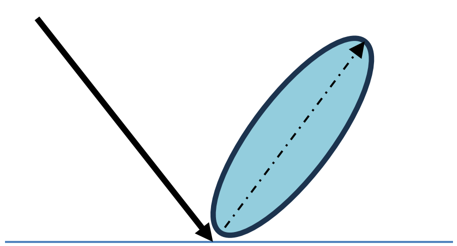
  - D is distribution function of microfacet orientations
  - G is geometrical attenuation factor (e.g., occlusion, shadowing)
  - F_k is the Fresnel term describing the reflection and transmission of light incident on different media

Microfacets

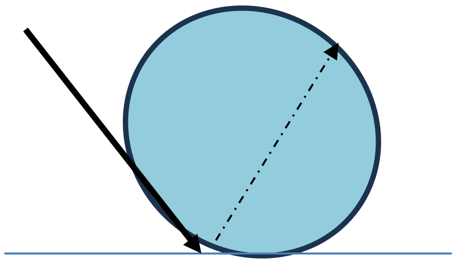
- Torrance-Sparrow Model

$$D = \frac{1}{4m^2 \cos^4 \beta} e^{-\left(\frac{\tan \beta}{m}\right)^2}$$

  - β is angle between N and H
  - m is root-mean-square slope of the microfacets
    - small m = facets similar to normal; highly directional
    - large m = facets slopes are steep; rough surface



m=0.2



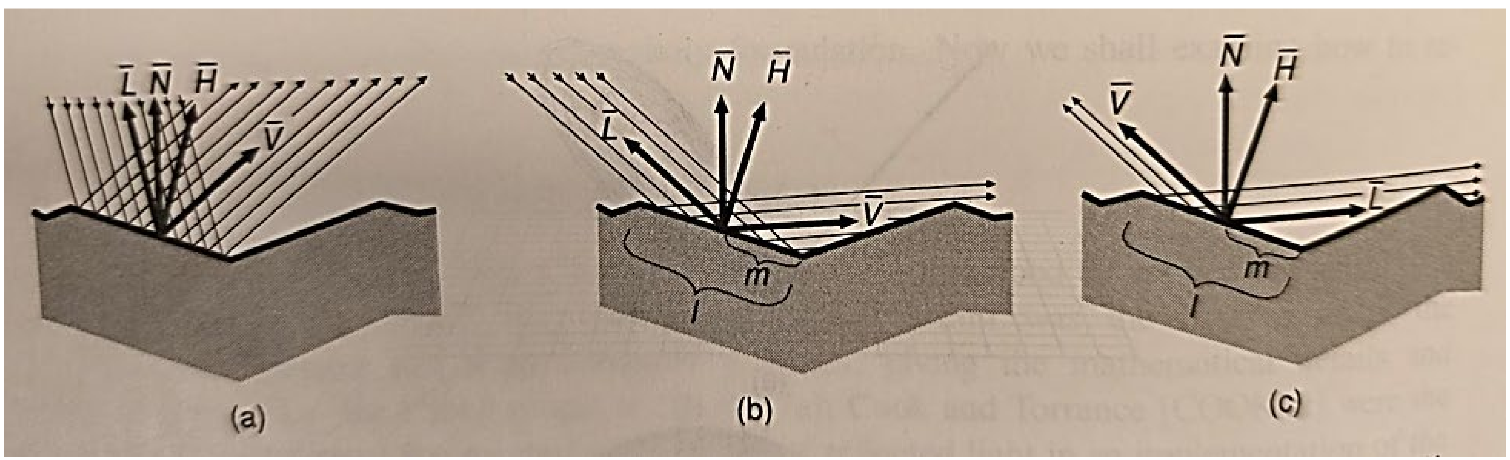
m=0.6

Microfacets

- Torrance-Sparrow Model

$$G = \min\{1, \frac{2(N\cdot H)(N\cdot V)}{V\cdot H}, \frac{2(N\cdot H)(N\cdot L)}{V\cdot H}\}$$

…quantifies how much microfacets shadow each other



Microfacets

- Torrance-Sparrow Model

$$F_k = \frac{1}{2}\frac{(g-c)^2}{(g+c)^2}\left(1 + \frac{[c(g+c)-1]^2}{[c(g-c)+1]^2}\right)$$

where $c = \mathrm{L} \cdot H$, $g^2 = \eta_\lambda^2 + c^2 - 1$, $\eta_\lambda = \frac{\eta_{t\lambda}}{\eta_{i\lambda}}$

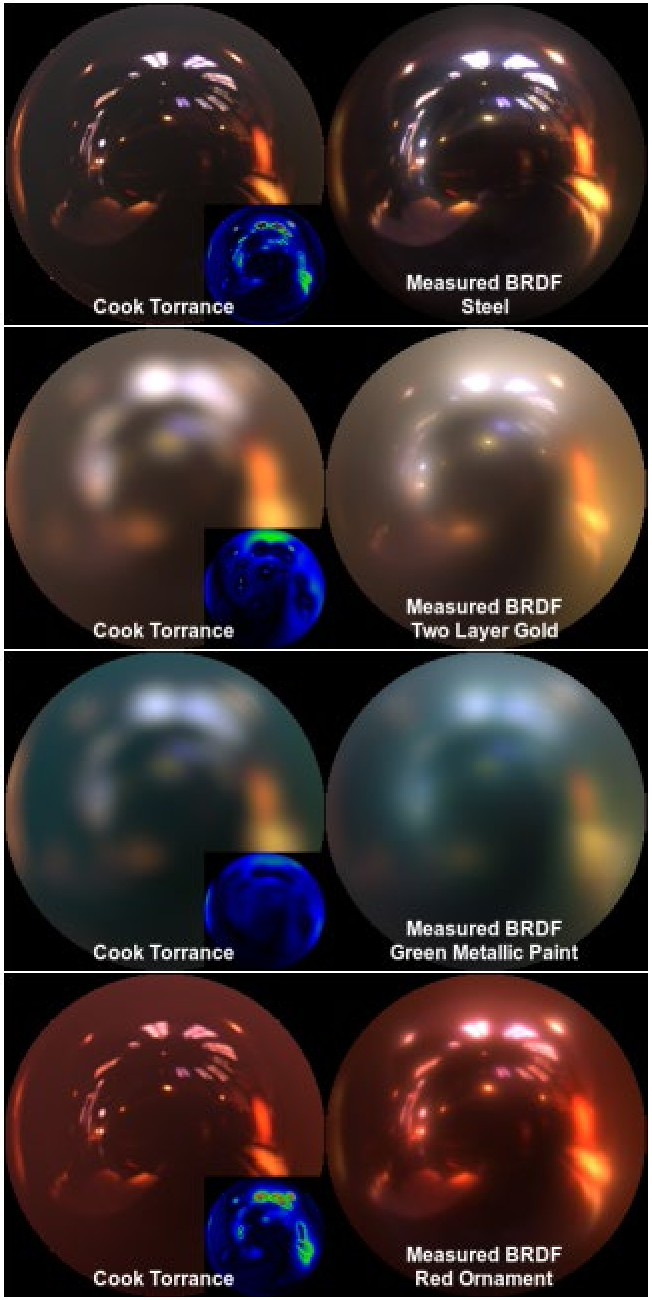
or, simplified to

$F_k \approx F_0 + (1 - F_0)(1 - (V \cdot H))^5$ [Schlick's approximation]

where F_0 is reflectance at normal incidence (e.g., RGB color)

Microfacets

- Cook-Torrance Model
  - Similar, with some improvements
  - D is a weighted sum of distribution functions to model surfaces of multiple scales of roughness



[Brady et al., TOG 2014]

What about rough (diffuse) surfaces?

- Must consider occlusion, shadowing, and diffuse interreflection – thoughts?
- $I_d = N \cdot L$
- Is that it?

What about rough (diffuse) surfaces?

- Must consider occlusion, shadowing, and diffuse interreflection – thoughts?
- Derived from microfacets model
  - Oren-Nayar Model:

$$L_r = L_1 + L_2$$

$$L_1 = \frac{\rho}{\pi} E_0 \cos\theta_i \left(C_1 + C_2 \cos(\phi_i - \phi_r) \tan\beta + C_3 (1 - |\cos(\phi_i - \phi_r)|) \tan\frac{\alpha+\beta}{2} \right),$$

$$L_2 = 0.17 \frac{\rho^2}{\pi} E_0 \cos\theta_i \frac{\sigma^2}{\sigma^2 + 0.13} \left[1 - \cos(\phi_i - \phi_r) \left(\frac{2\beta}{\pi} \right)^2 \right],$$

where

$$C_1 = 1 - 0.5 \frac{\sigma^2}{\sigma^2 + 0.33},$$

$$C_2 = \begin{cases} 0.45 \frac{\sigma^2}{\sigma^2+0.09} \sin\alpha & \text{if } \cos(\phi_i - \phi_r) \geq 0, \\ 0.45 \frac{\sigma^2}{\sigma^2+0.09} \left(\sin\alpha - \left(\frac{2\beta}{\pi} \right)^3 \right) & \text{otherwise,} \end{cases}$$

$$C_3 = 0.125 \frac{\sigma^2}{\sigma^2 + 0.09} \left(\frac{4\alpha\beta}{\pi^2} \right)^2,$$

$$\alpha = \max(\theta_i, \theta_r),$$

$$\beta = \min(\theta_i, \theta_r),$$

What about rough (diffuse) surfaces?

- Oren-Nayar Model (simplified)

$$L_r = \cos\theta_i \, [A + B \max(0, \cos(\theta_r - \theta_i)) \sin\alpha \tan\beta]$$

where using numerical experiments:

$$A \approx \rho(\frac{1}{\pi} - 0.09 \frac{\sigma^2}{\sigma^2 + 0.4})$$

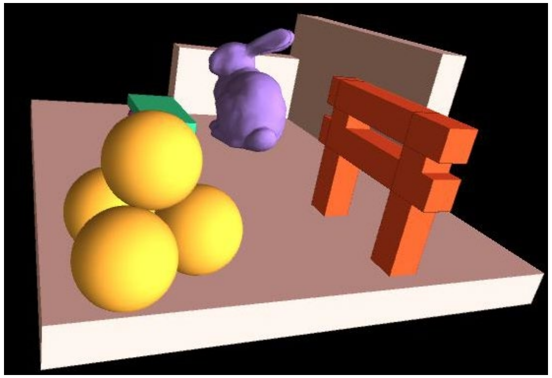
$$B \approx \rho(0.125 \frac{\sigma^2}{\sigma^2 + 0.18})$$

Illumination Models

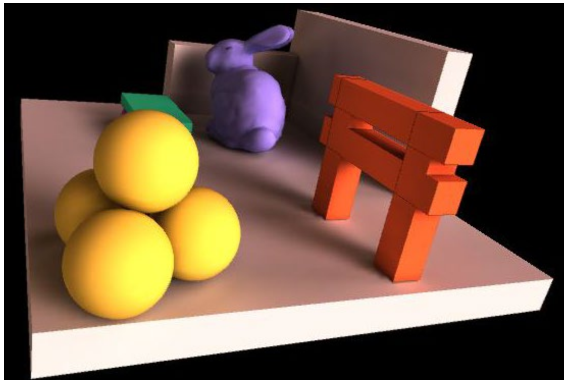
- So far, you considered mostly local (direct) illumination
  - Light directly from light sources to surface
  - No shadows (actually is a global effect)
- Global (indirect) illumination: multiple bounces of light
  - Hard and soft shadows
  - Reflections/refractions (you kinda saw already)
  - Diffuse and specular interreflections
  - Subsurface scattering

Global Illumination

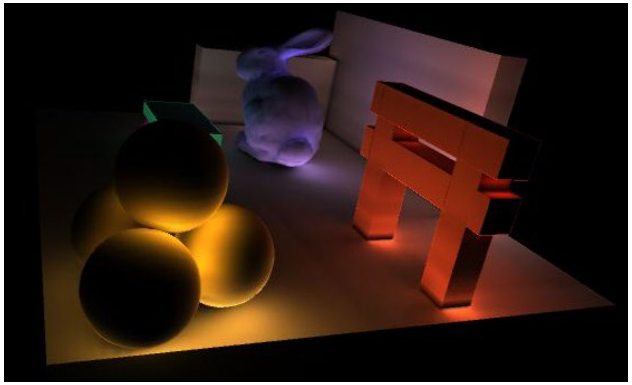
- *Direct illumination* + *indirect illumination*; e.g.
  - Direct = reflections, refractions, shadows, …
  - Indirect = diffuse and specular inter-reflection, subsurface scattering, participating media…



direct illumination



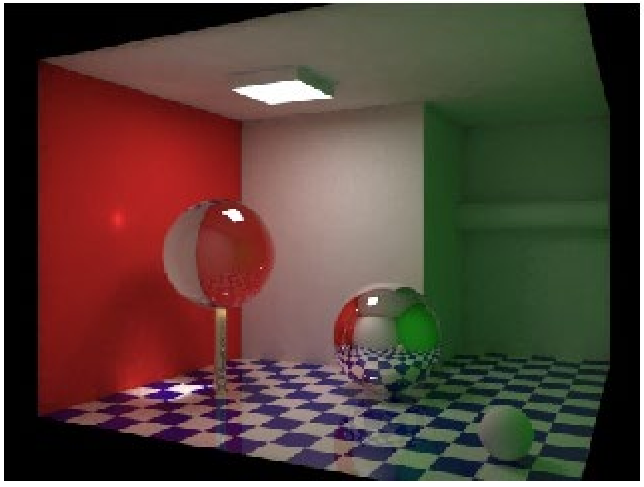
with global illumination

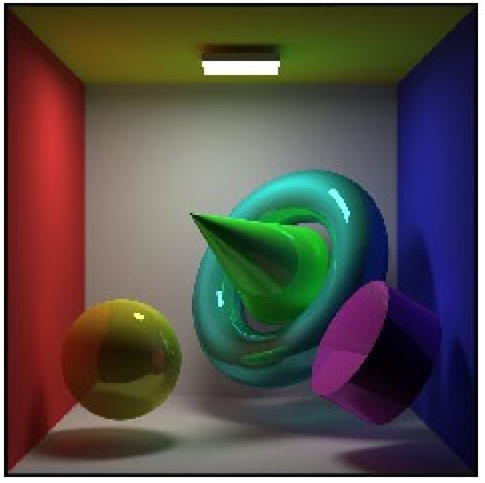


only diffuse inter-reflection

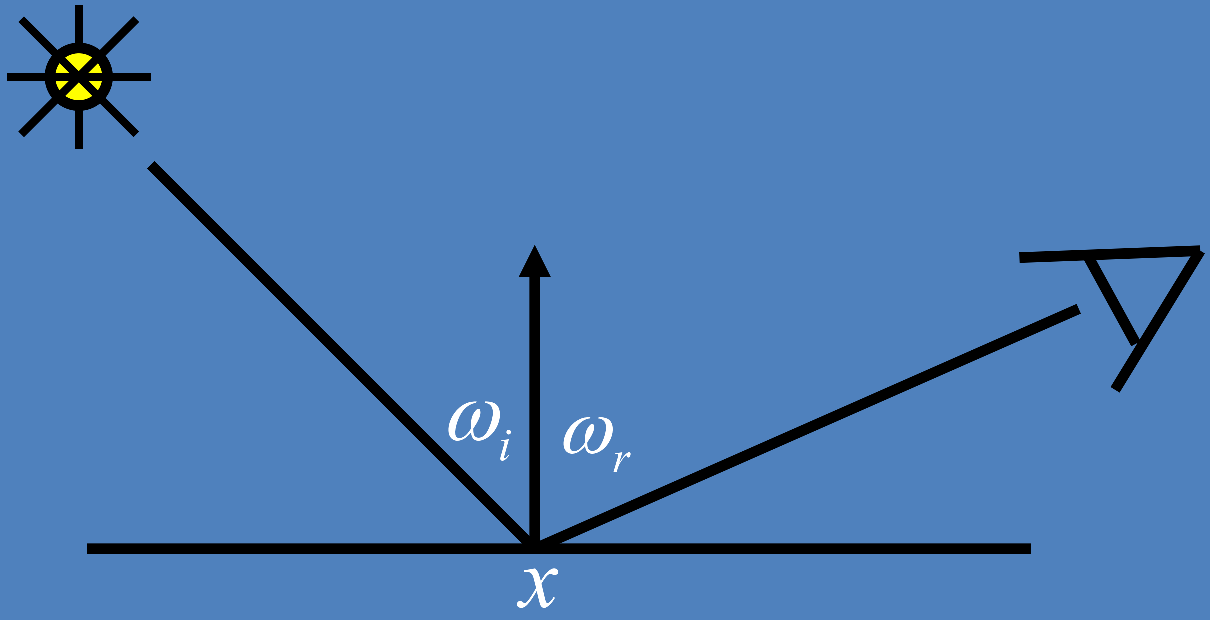
Global Illumination

- *Direct illumination* + *indirect illumination*; e.g.
  - Direct = reflections, refractions, shadows, …
  - Indirect = diffuse and specular inter-reflection, subsurface scattering, participating media…





Reflectance Equation

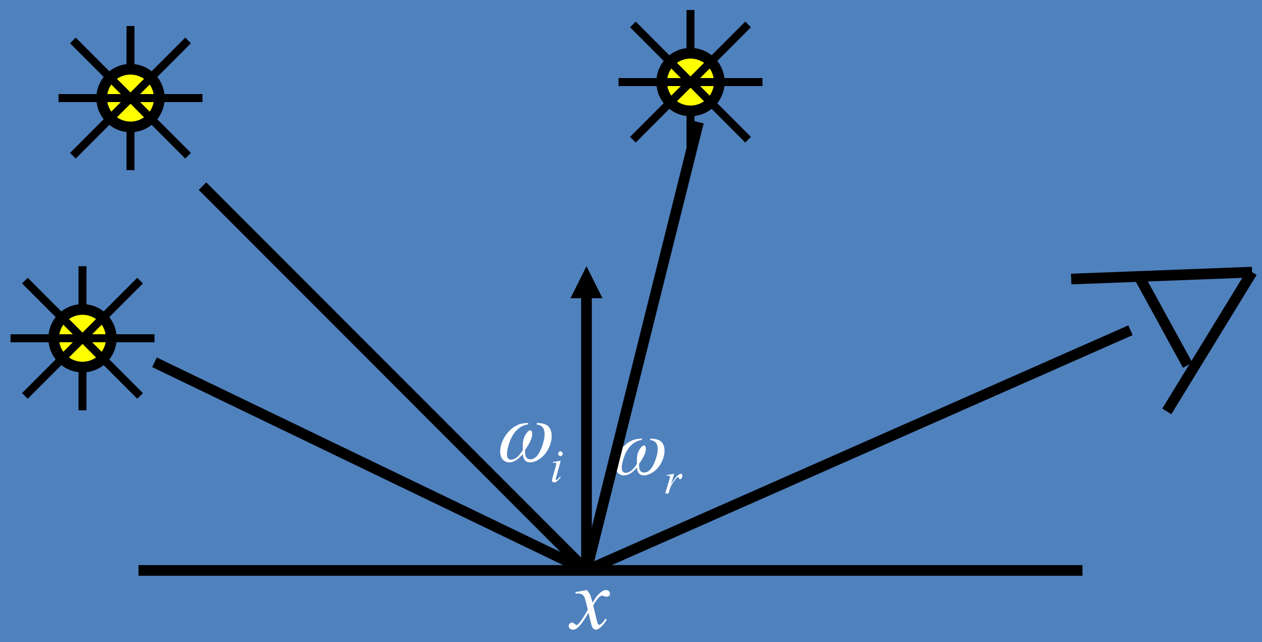


$$L_r(x,\omega_r) = L_e(x,\omega_r) + L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

[Slides with help from Pat Hanrahan and Henrik Jensen]

Reflectance Equation

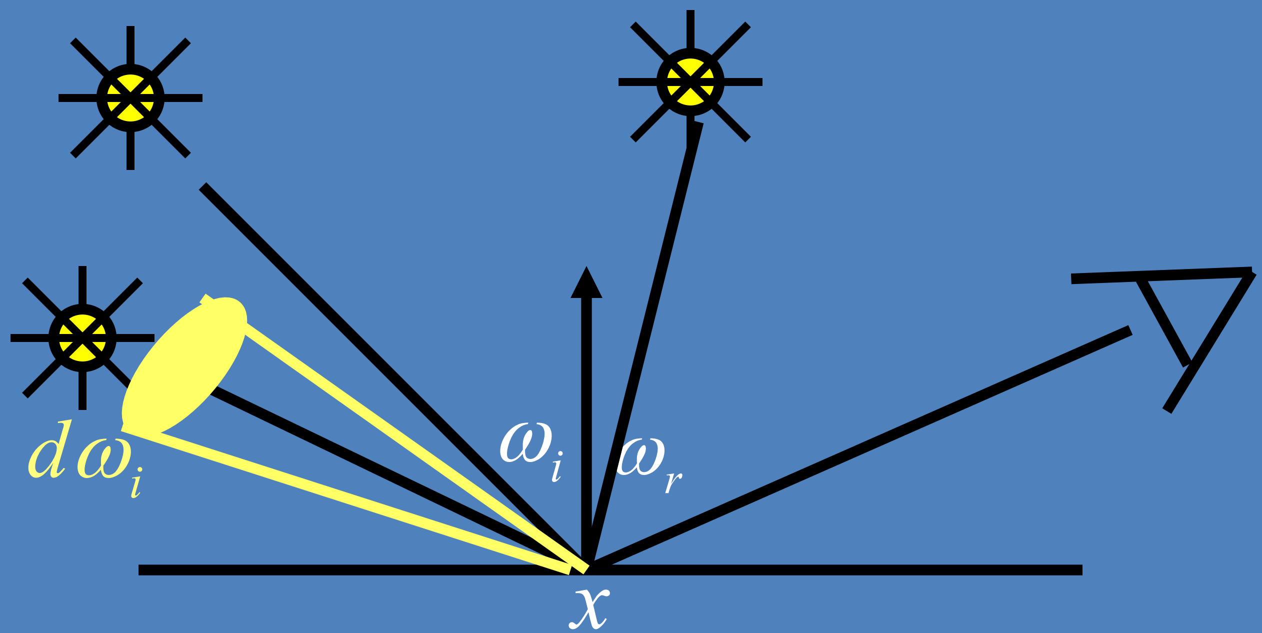


Sum over all light sources

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \sum L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle

Reflectance Equation

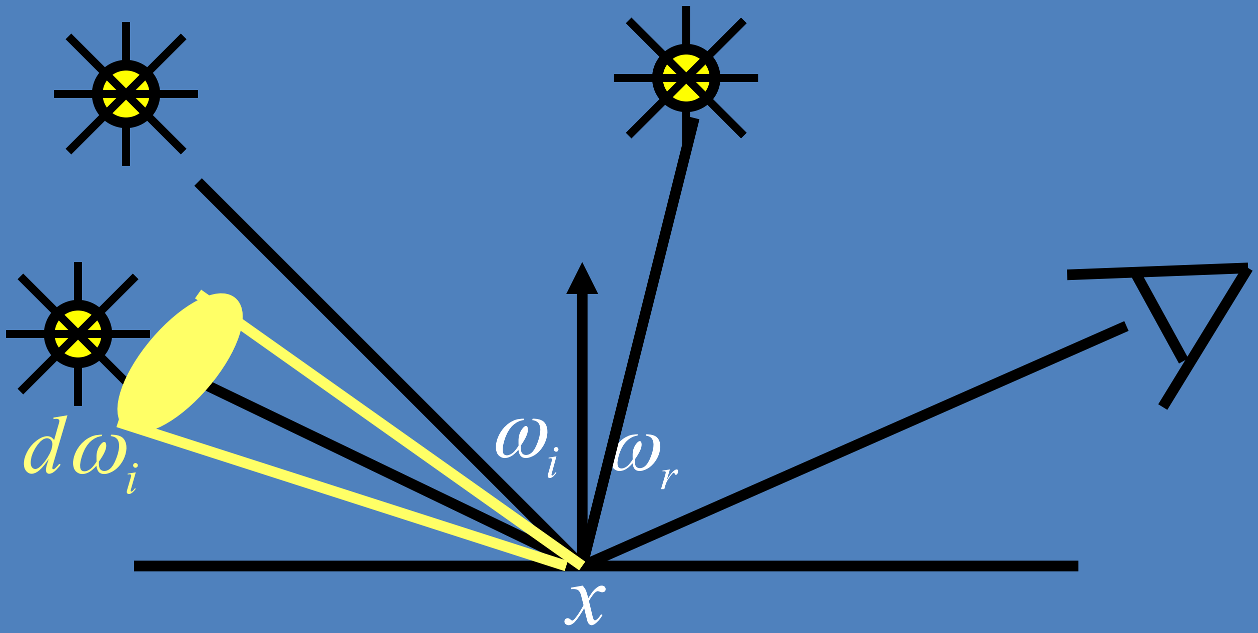


Replace sum with integral

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_i(x,\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle

Reflectance Equation



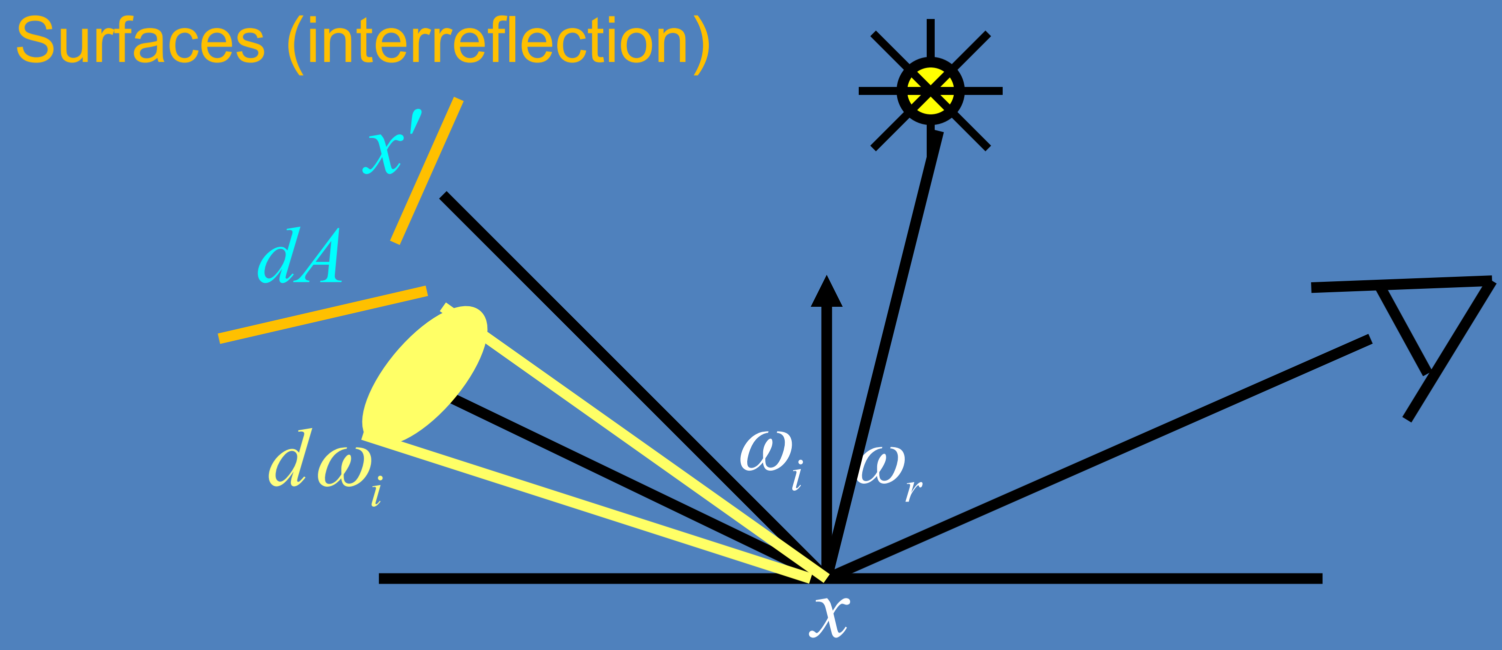
$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_i(x,\omega_i) f(x,\omega_i,\omega_r)\cos\theta_i d\omega_i$$

The Challenge

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_i(x,\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i d\omega_i$$

- Computing reflectance equation requires knowing the incoming radiance from surfaces
- …But determining incoming radiance requires knowing the reflected radiance from surfaces

Global Illumination

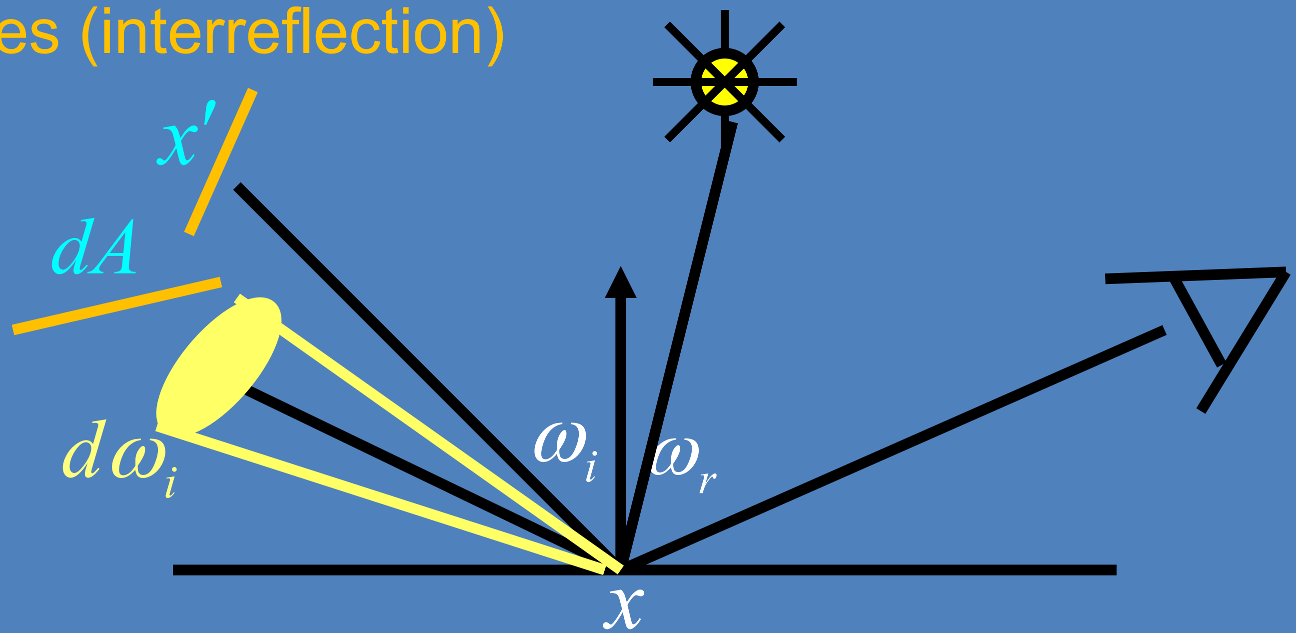


$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i)\, f(x,\omega_i,\omega_r)\cos\theta_i\, d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light (from prev surface)	BRDF	Cosine of Incident angle

Rendering Equation

Surfaces (interreflection)



$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i)\, f(x,\omega_i,\omega_r)\cos\theta_i\, d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Rendering Equation (Jim Kajiya)

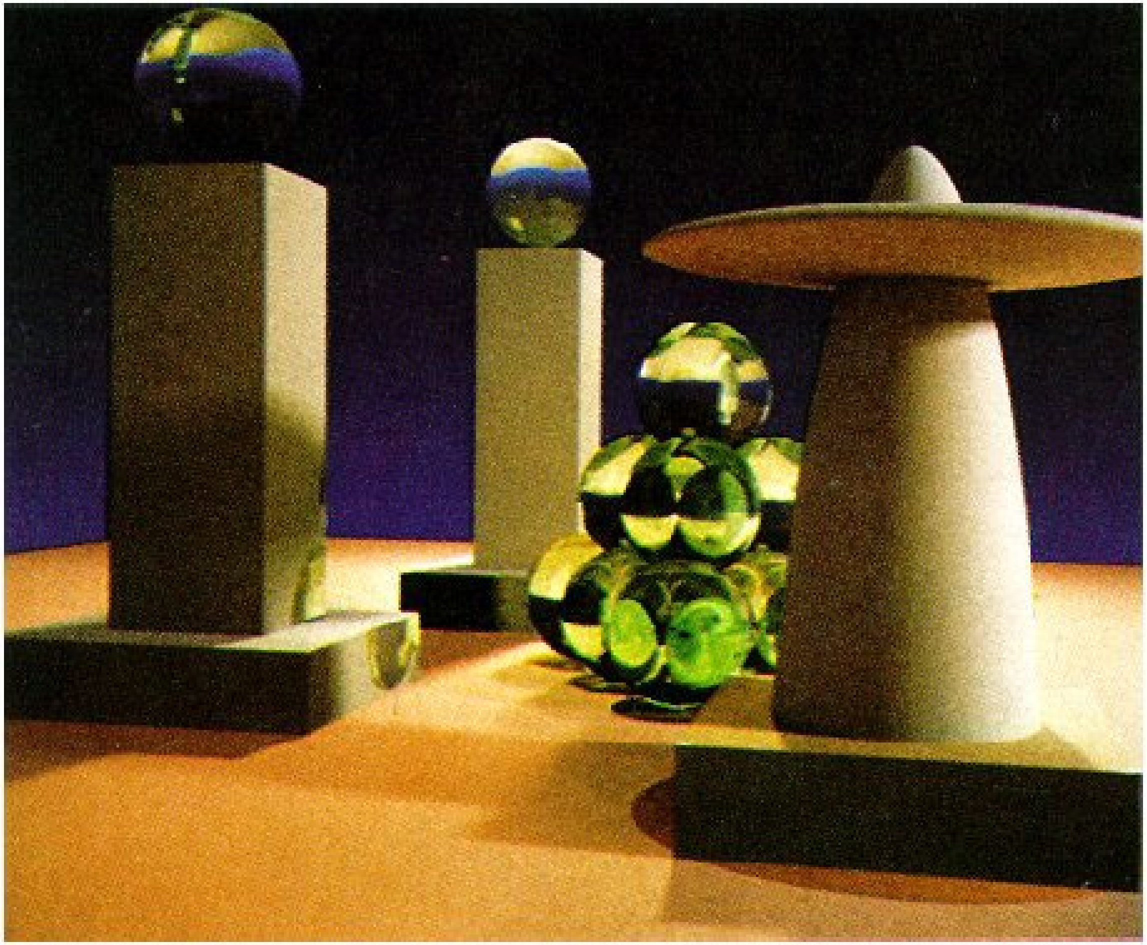


Figure 6. A sample image. All objects are neutral grey. Color on the objects is due to caustics from the green glass balls and color bleeding from the base polygon.

Can we simplify to linear operators?

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i)\, f(x,\omega_i,\omega_r) \cos\theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Rendering Equation as Integral Equation

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i)\, f(x,\omega_i,\omega_r)\cos\theta_i\, d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

$$l(u) = e(u) + \int l(v)\, K(u,v)\, dv$$

Kernel of equation

Linear Operator Theory 101

Linear operators act on functions like matrices act on vectors or discrete representations

$$h(u) = \left(M \circ f\right)(u)$$

M is a linear operator.

f and h are functions of u

$$M \circ \left(af + bg\right) = a\left(M \circ f\right) + b\left(M \circ g\right)$$

a and b are scalars

f and g are functions

Basic linearity relations hold

$$\left(K \circ f\right)(u) = \int k(u,v) f(v)\, dv$$

$$\left(D \circ f\right)(u) = \frac{\partial f}{\partial u}(u)$$

(e.g., integration and differentiation)

Linear Operator Equation

$$l(u) = e(u) + \int l(v) \boxed{K(u,v)dv}$$

Kernel of equation

$$L = E + KL$$

which is effectively a simple matrix equation (or system of simultaneous linear equations) where

L, E are vectors,

K is the light transport matrix (more on this later!)

Solving the Rendering Equation (=how to compute L?)

- In general, too hard for analytic solution
- But there are iterative approximations and some nice observations…

Solving the Rendering Equation (=how to compute L?)

$$L = E + KL$$

$$IL - KL = E$$

$$(I - K)L = E$$

$$L = (I - K)^{-1} E$$

(using Binomial Theorem)

$$L = (I + K + K^2 + K^3 + \ldots)E$$

$$L = E + KE + K^2 E + K^3 E + \ldots$$

where term n corresponds to n-th bounces of light

Tracing…

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly
From light sources

Direct Illumination
on surfaces

Global Illumination
(One bounce indirect)
[Mirrors, Refraction]

(Two bounce indirect)
[Caustics, etc…]

Tracing…

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly
From light sources

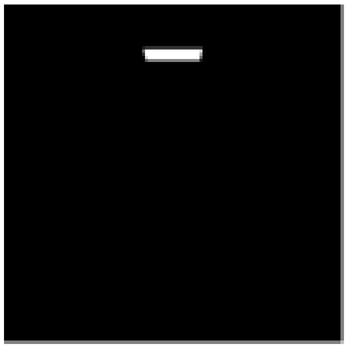
Direct Illumination
on surfaces

Global Illumination
(One bounce indirect)
[Mirrors, Refraction]

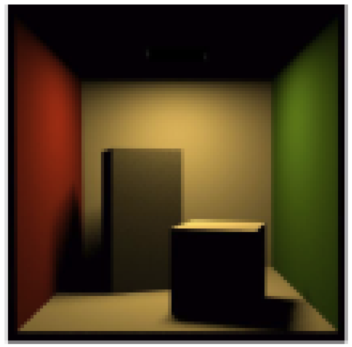
(Two bounce indirect)
[Caustics, etc…]

OpenGL
Shading

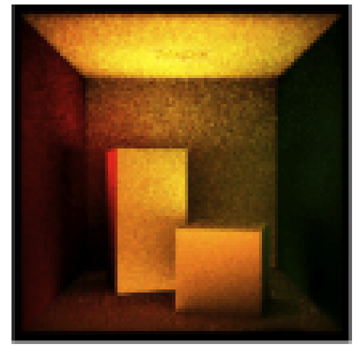
Successive Approximation



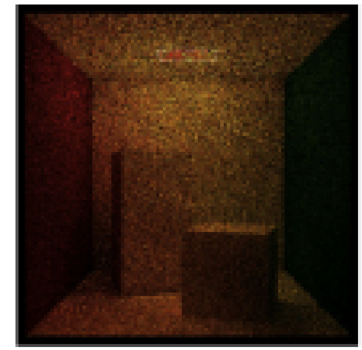
L_e



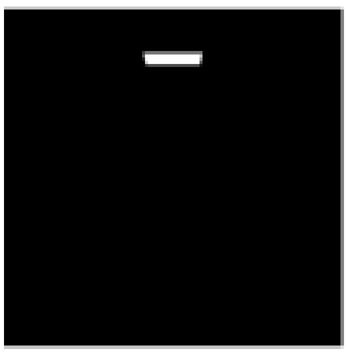
$K \circ L_e$



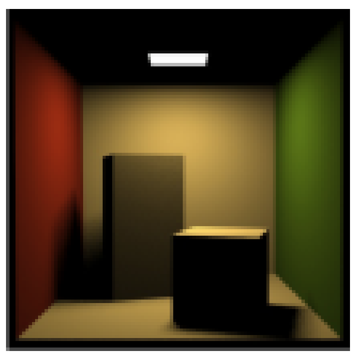
$K \circ K \circ L_e$



$K \circ K \circ K \circ L_e$

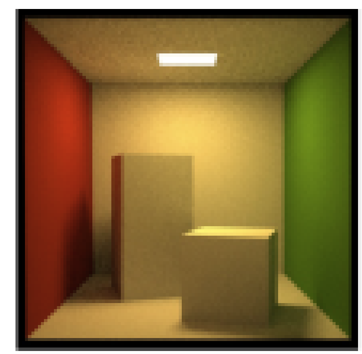


L_e



$L_e + K \circ L_e$

$L_e + \cdots K^2 \circ L_e$



$L_e + \cdots K^3 \circ L_e$

Global Illumination Concepts

- (Reflectance and Rendering Equations)
- Colors and Perception
  - Color models
- BRDFs
  - Surface-level behaviors
- Light Transport
  - The “K” matrix of linearization of rendering equation
- Illumination:
  - (Ray Tracing)
  - Path Tracing (single and bidirectional)
  - Photon Mapping
  - Radiosity
- Making it faster:
  - Ambient occlusion