Graphics Pipeline: Modeling, Illumination, and Rendering

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Computer Graphics Pipeline

Geometry

Modeling Transformation	Transform into 3D *world* coordinate system
Lighting	Simulate illumination and reflectance
Viewing Transformation	Transform into 3D *camera* coordinate system
Clipping	Clip primitives outside camera's view
Projection	Transform into 2D camera coordinate system
Scan Conversion	Draw pixels (incl. texturing, hidden surface…)

Image

Computer Graphics Pipeline

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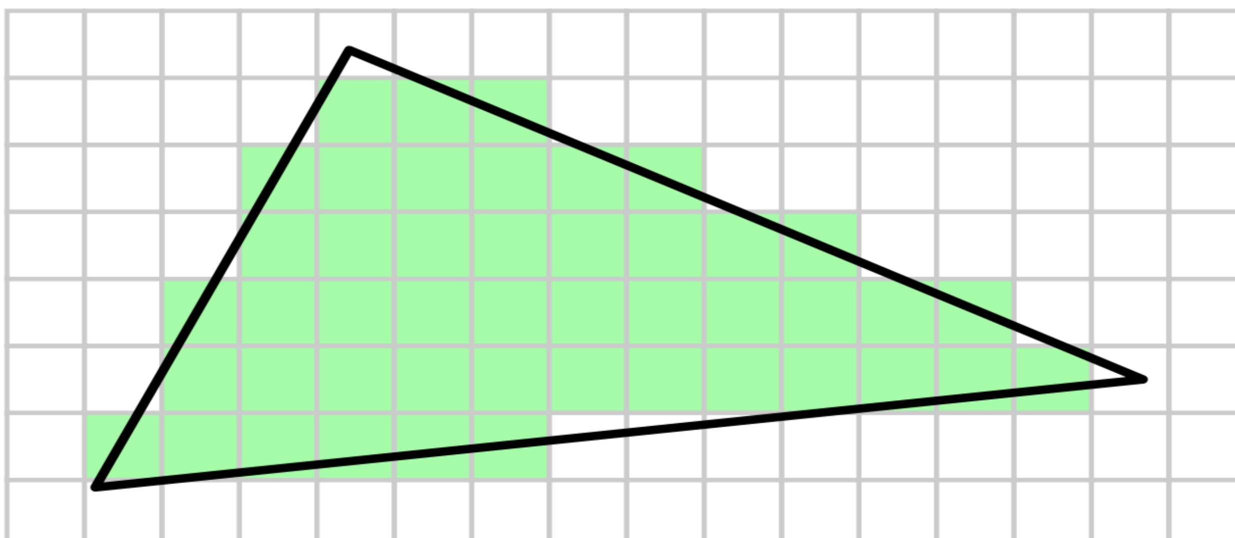
Image

Scan Conversion/Rasterization

- Determine which fragments get generated
- Interpolate parameters (colors, textures, normals, etc.)



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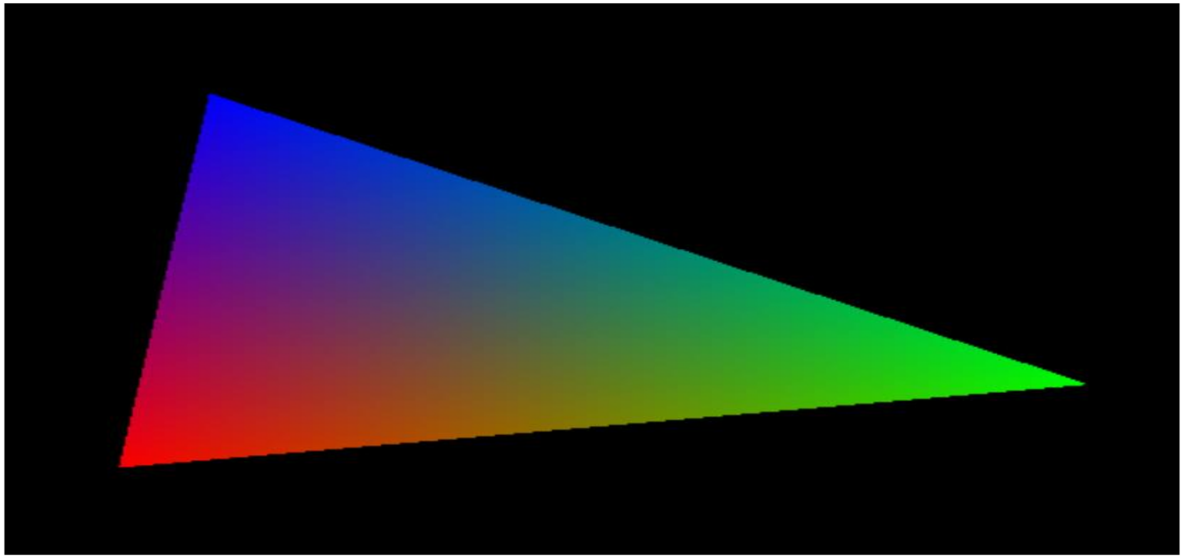

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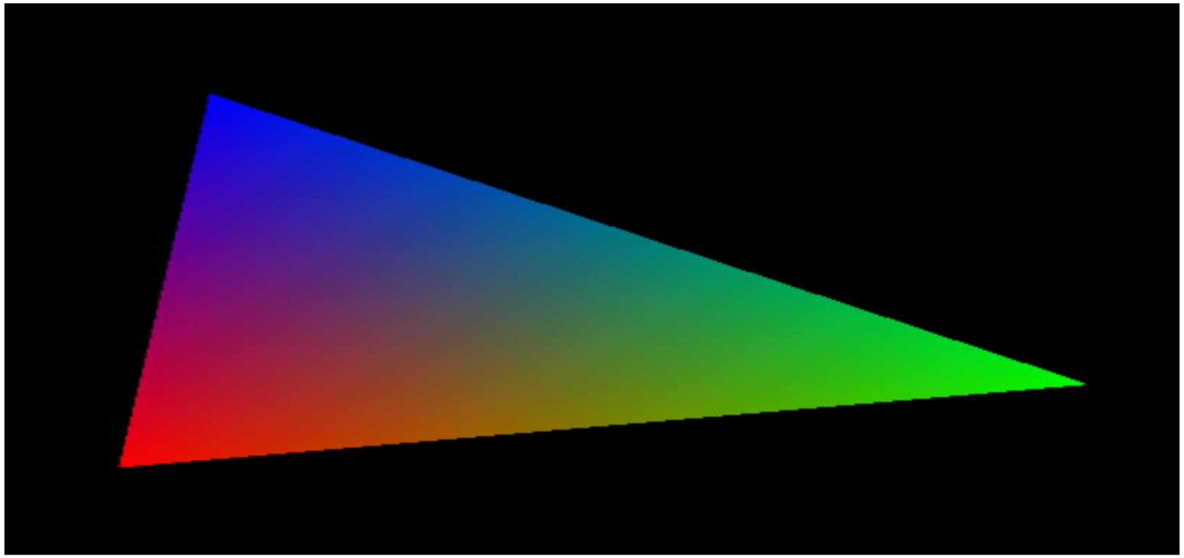

- How?

Scan Conversion/Rasterization

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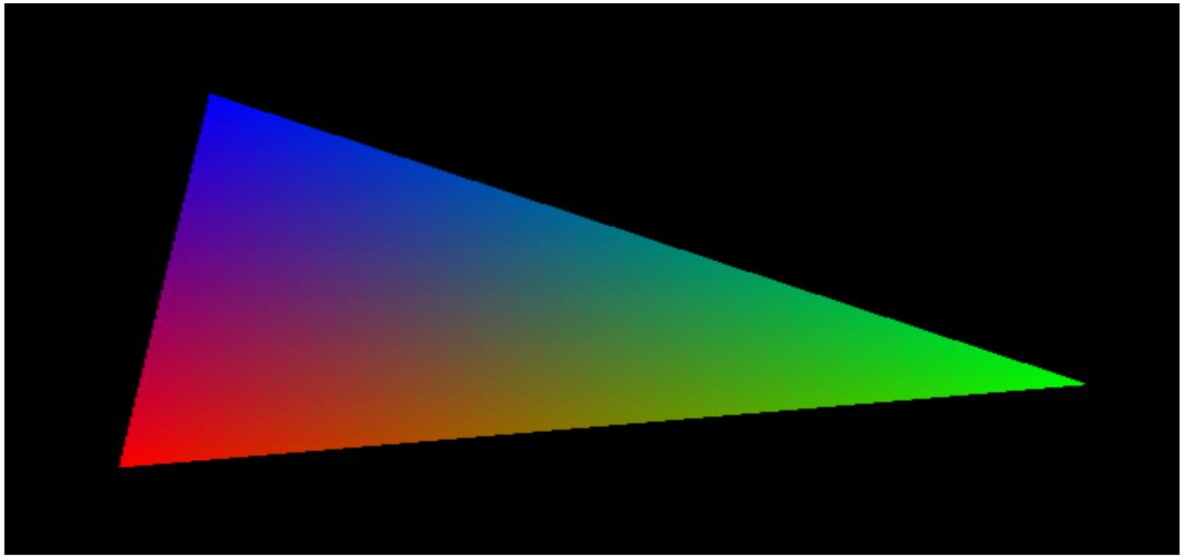
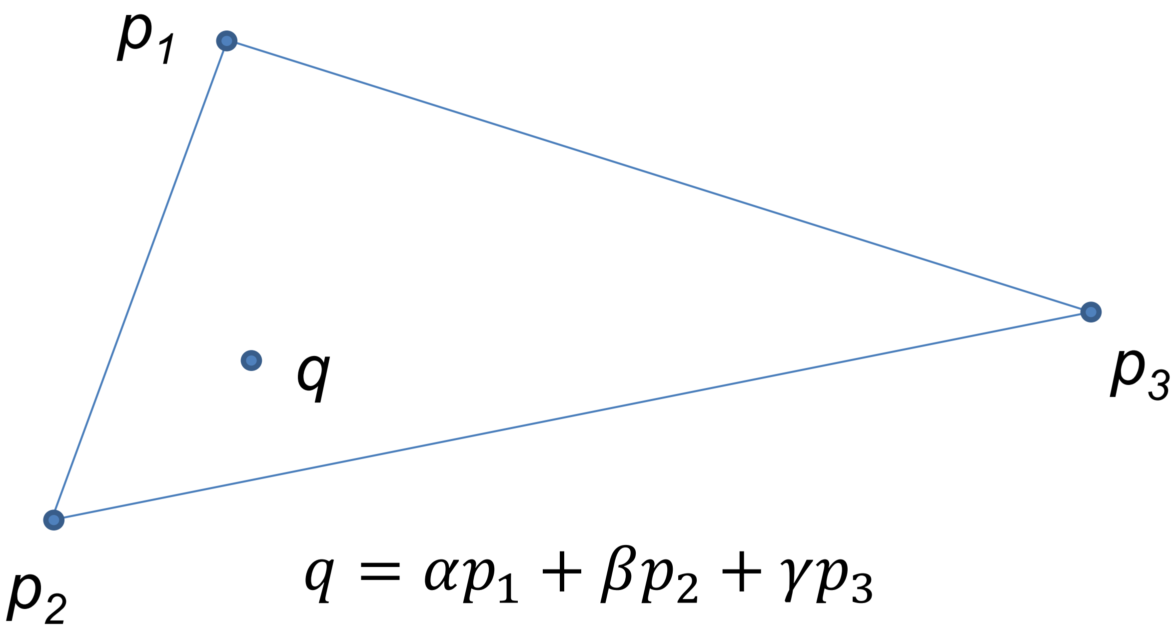


- Barycentric coords amongst many other ways..

Barycentric coordinates



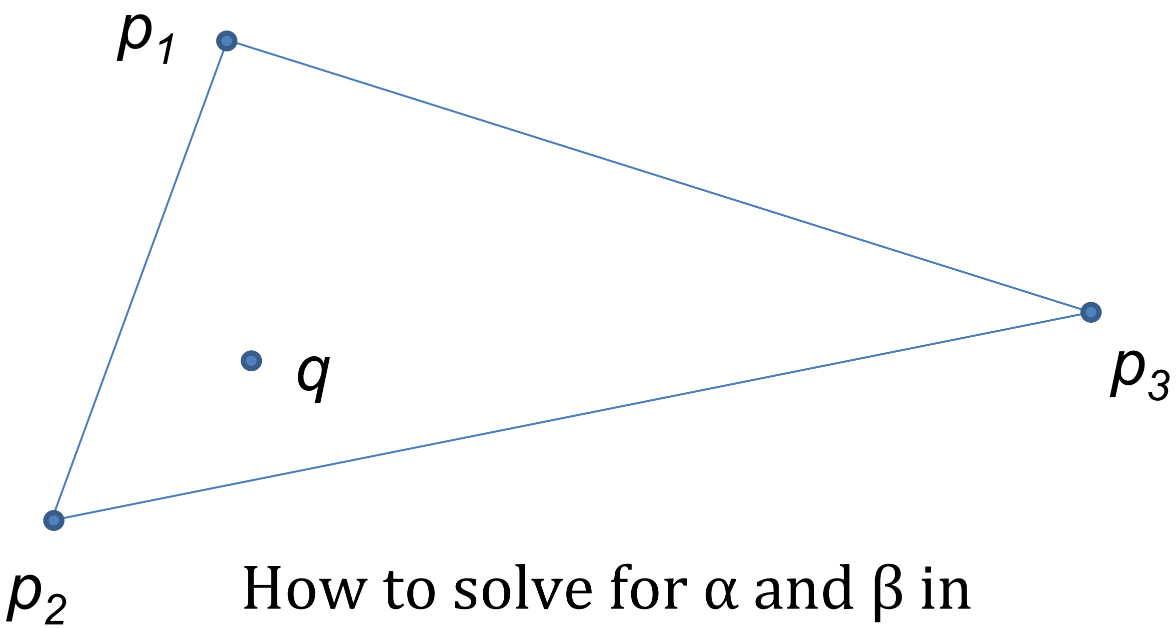
$$q = \alpha p_1 + \beta p_2 + \gamma p_3$$

If $[\alpha + \beta + \gamma = 1 \;\; and \;\; \{\alpha, \beta, \gamma\} \geq 0]$,
then q inside triangle (p_1, p_2, p_3)

Can also write:

$$q = \alpha p_1 + \beta p_2 + (1 - \alpha - \beta) p_3$$

Barycentric coordinates



How to solve for α and β in

$$q = \alpha p_1 + \beta p_2 + (1 - \alpha - \beta) p_3?$$

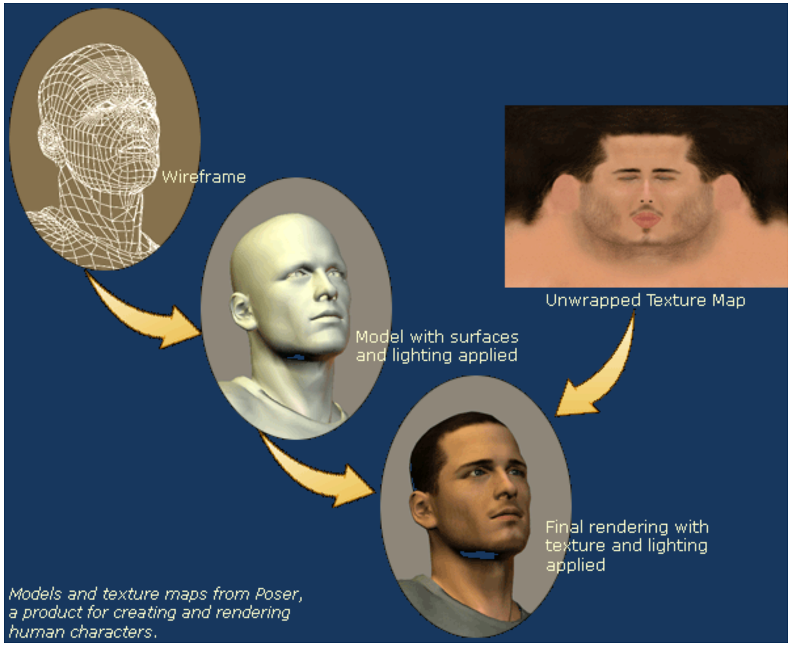
Two equations, two unknowns:
use 2x2 matrix inversion…

Additional concept: Texture mapping

- Model surface-detail with images
  - wrap object with photograph(s)
  - graphics object itself is a simpler model but "looks" more complex

Texture mapping

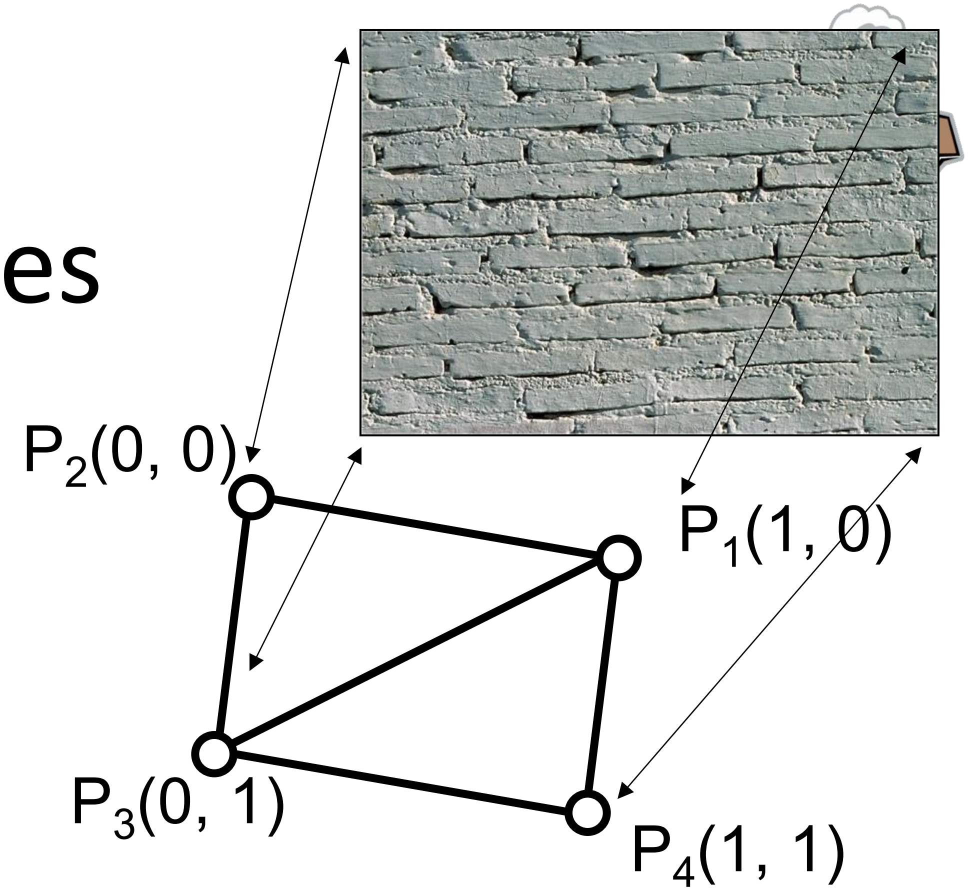
- Model surface-detail with images
  - wrap object with photograph(s)
  - graphics object itself is a simpler model but “looks” more complex

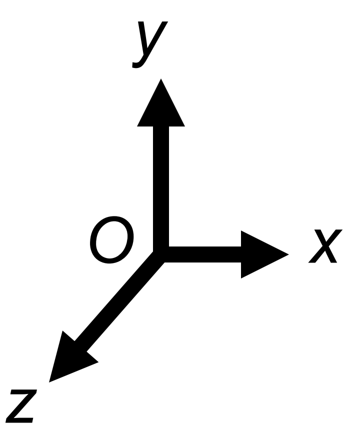


Texture coordinates

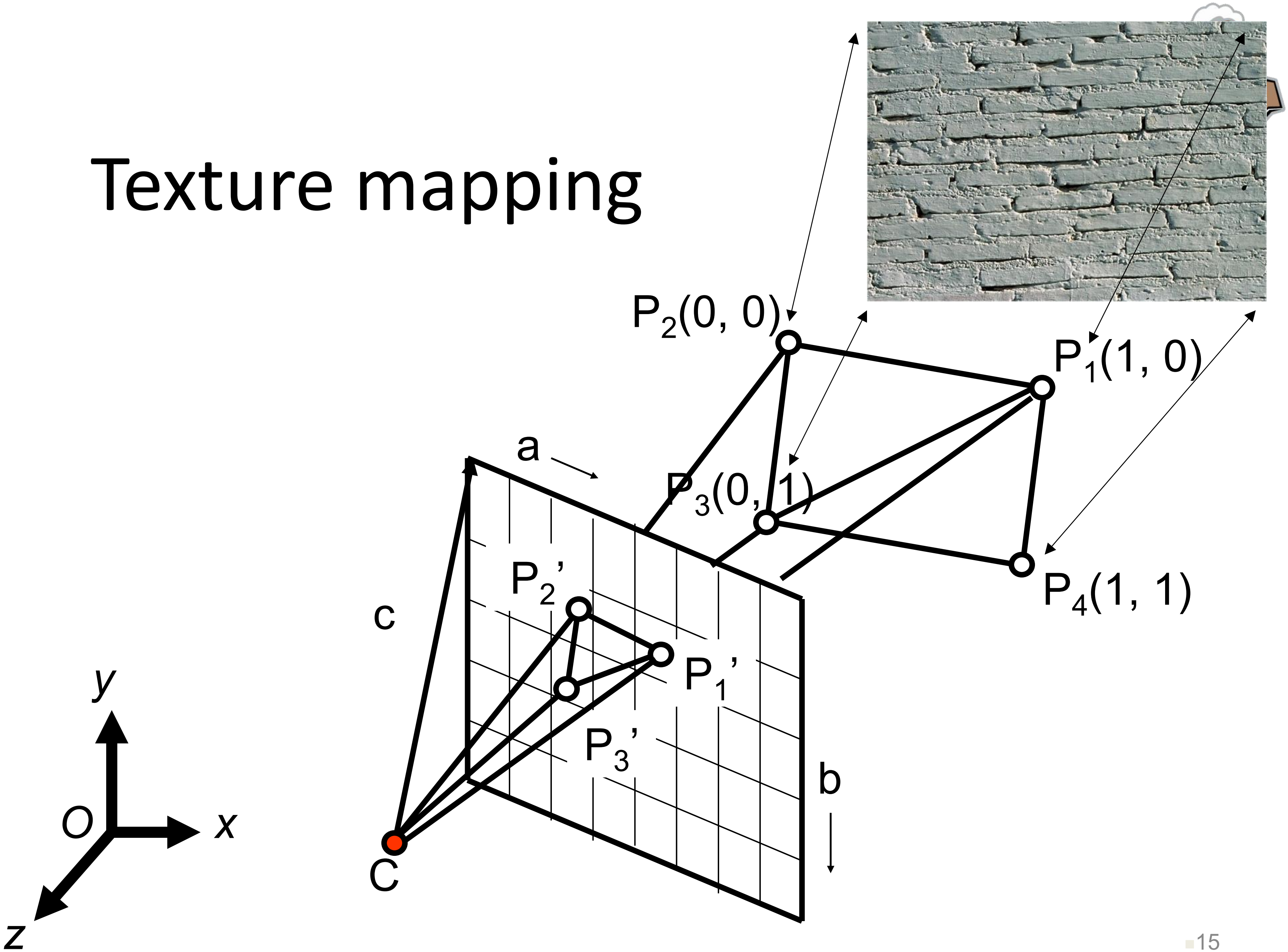
- Mechanism for attaching the texture map to the surface modeled
  - a pair of floats (s, t) for each triangle vertex
  - corners of the image are (0, 0), (0, 1), (1, 1), and (1, 0)
  - tiling indicated with tex. coords. > 1
  - *texels* – color samples in texture maps

Texture coordinates

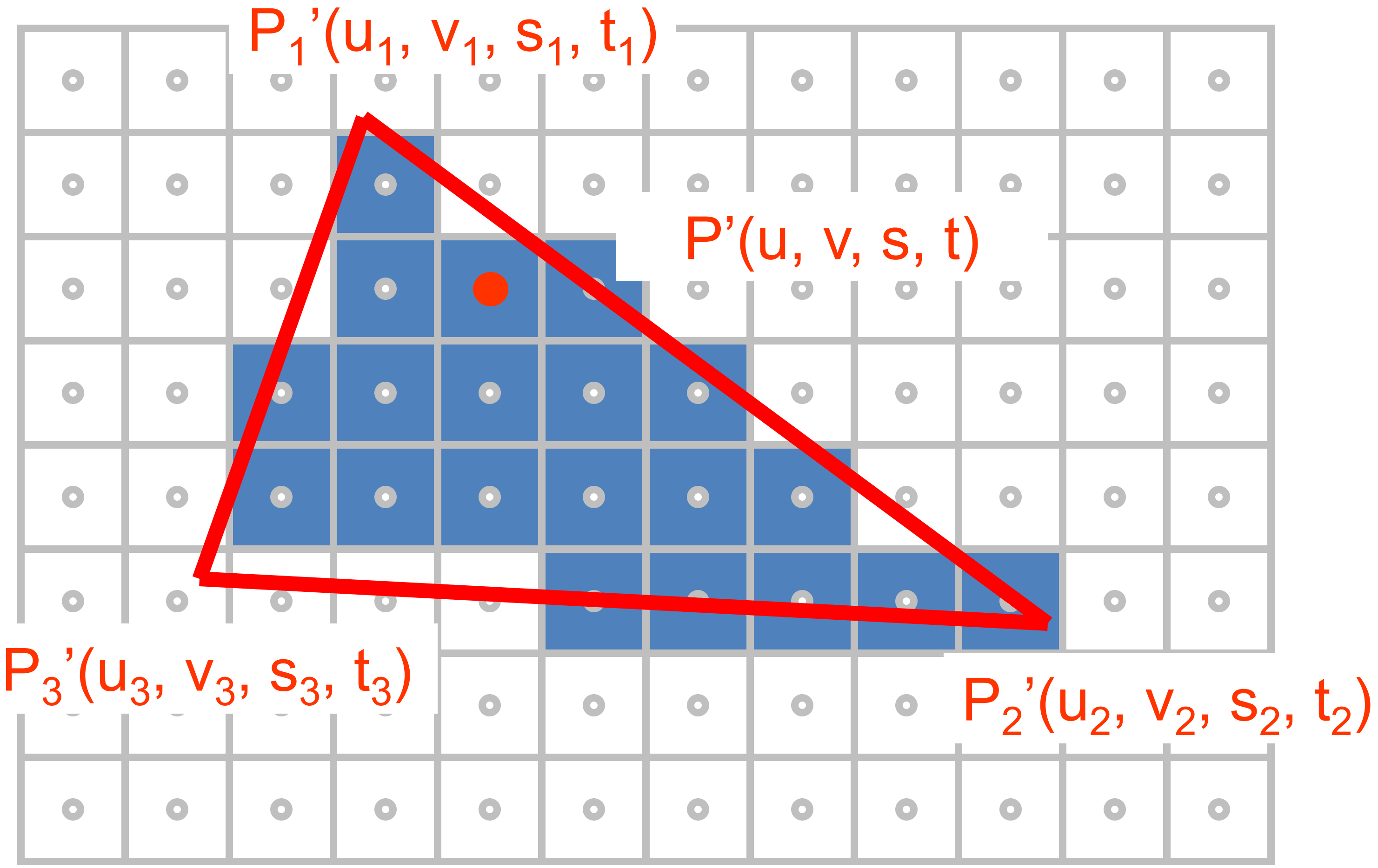




Texture mapping

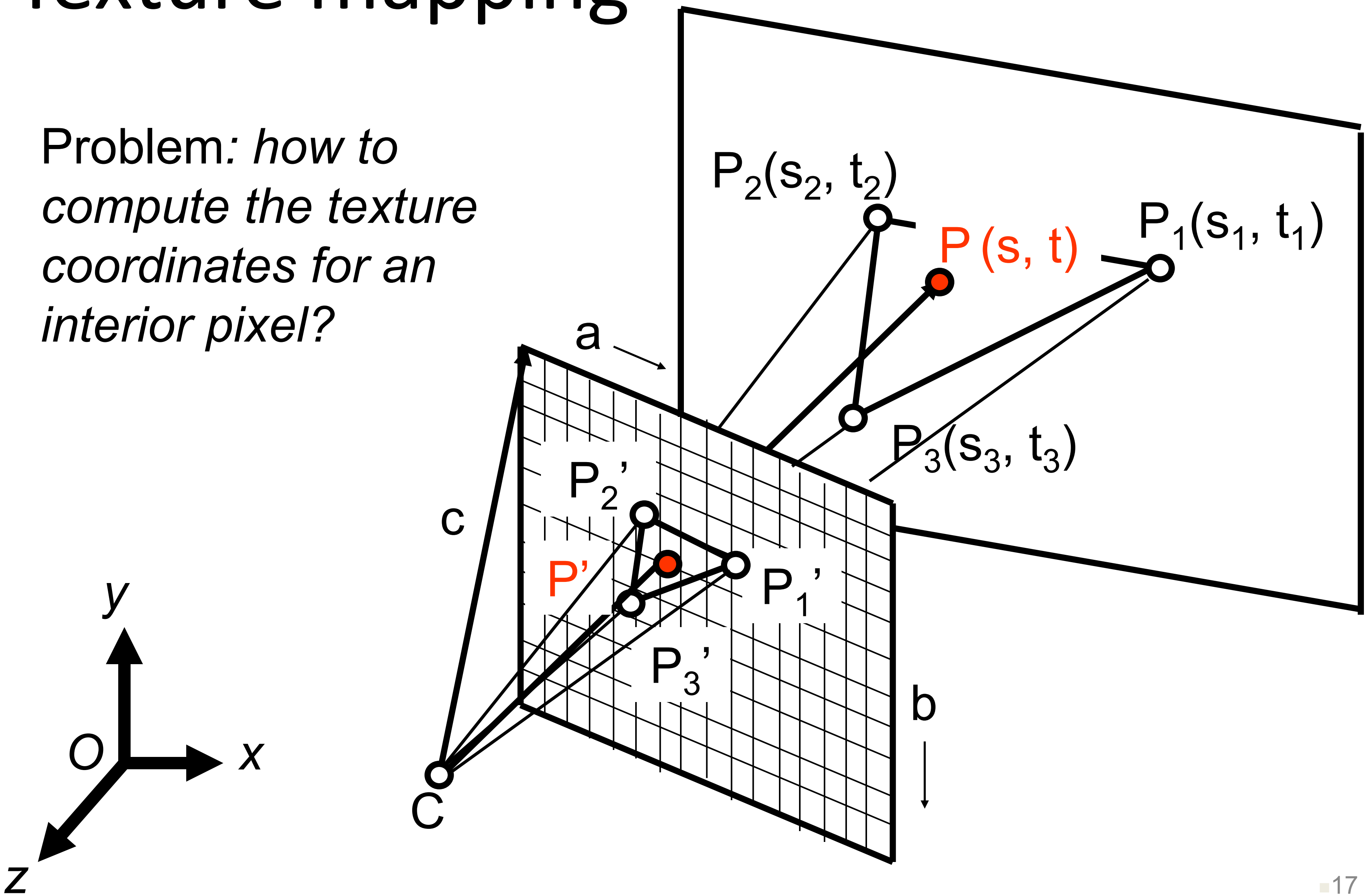


Texels: texture elements



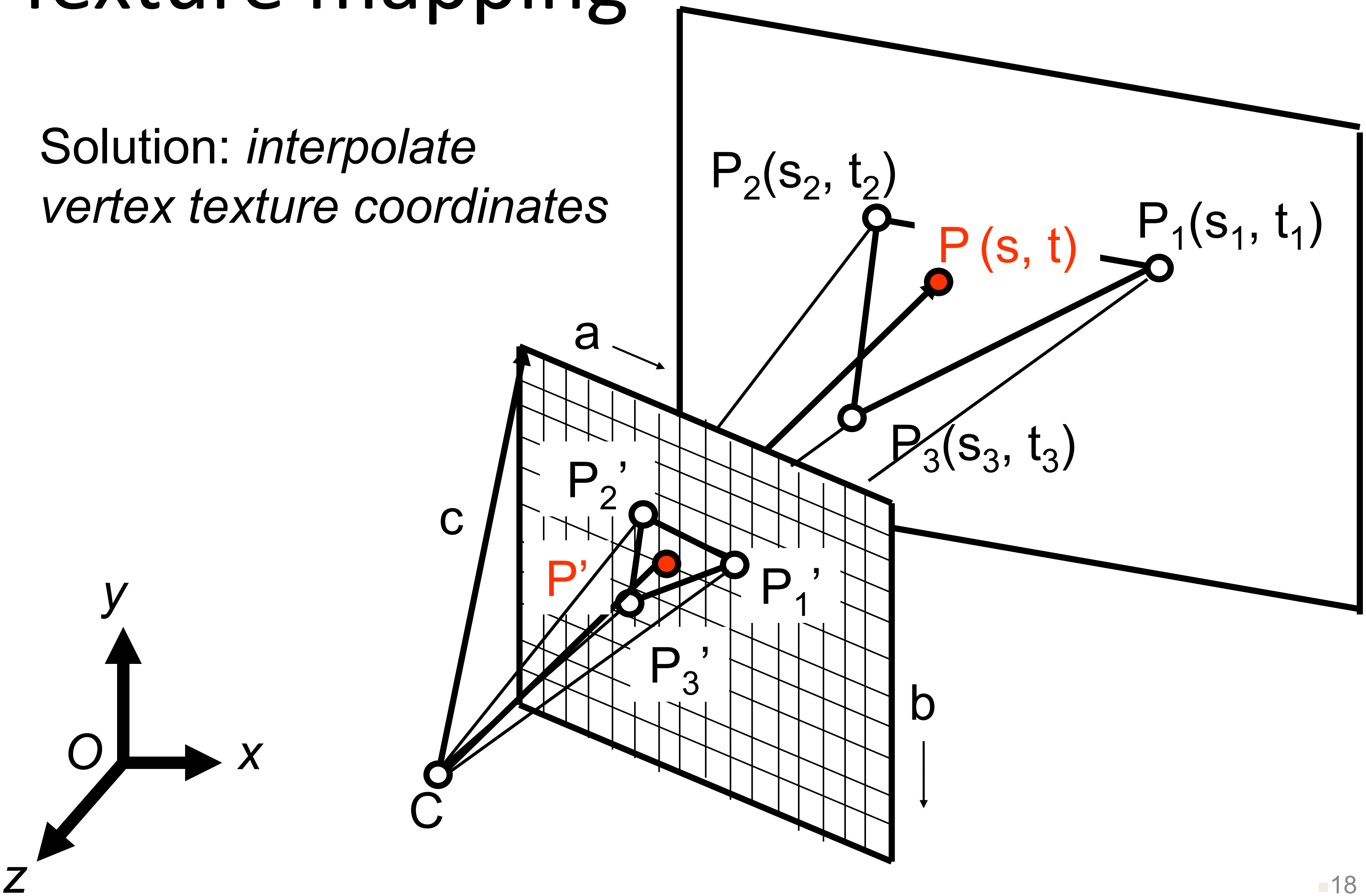
Texture mapping

Problem*: how to compute the texture coordinates for an interior pixel?*



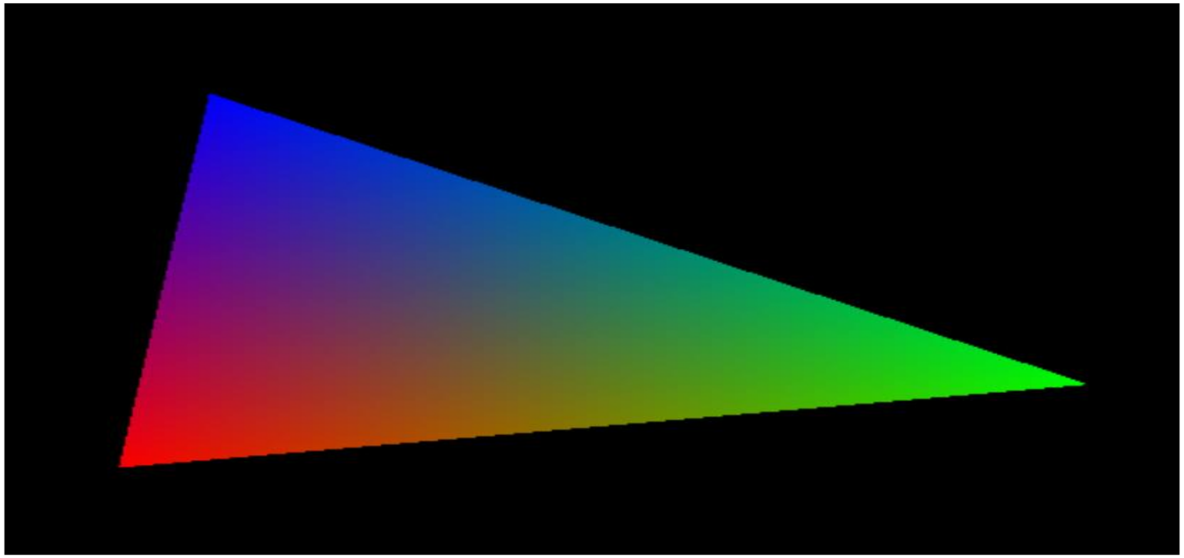
Texture mapping

Solution: *interpolate vertex texture coordinates*



Parameter Interpolation

- Texture coordinates, colors, normals, etc.



- How?
  - Again, use barycentric coordinates…

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Image

Modeling (or Content Generation)

- Manual Modeling (most prevalent approach):
  - Softimage (Microsoft+SoftImage3D, 1989-2002)
  - Maya (SGI+Alias+Wavefront, 1998-today)
  - Blender (open source, 1995-today)
  - Houdini (SideFX, 1996-today)
  - Rhino/Grasshopper (TLM, 1978? – today)
  - And more…
- Problem: TIME CONSUMING!
  - E.g., 3-20 days for a photorealistic character

Modeling (or Content Generation)

- Texture/Image Synthesis
- **Procedural Modeling**
- Generative Modeling
- Neural Modeling (and Rendering)

Illumination

- Local Illumination
  - **Diffuse, Specular**
- Global Illumination
  - **Ray Tracing**
  - Path Tracing (Single and Bidirectional)
  - Photon Mapping
  - **Radiosity**

Rendering

- Point rendering
- **Ray tracing**
- **Polygon/triangle rendering**
- Differentiable Rendering

Lighting and Illumination

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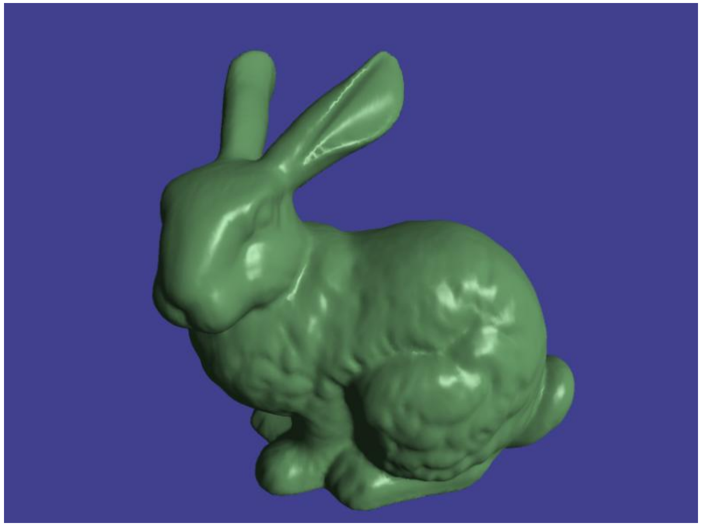
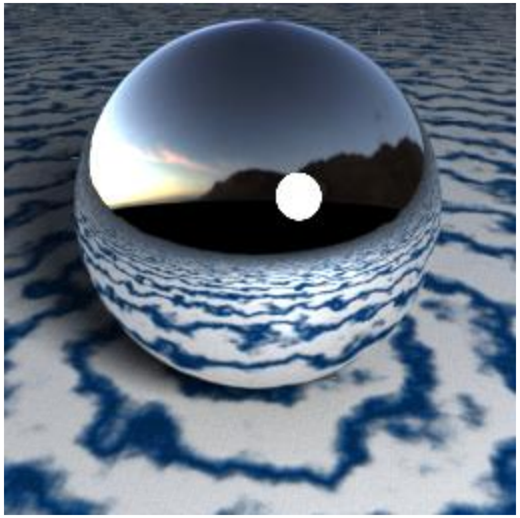
Diffuse Reflection

(mostly)

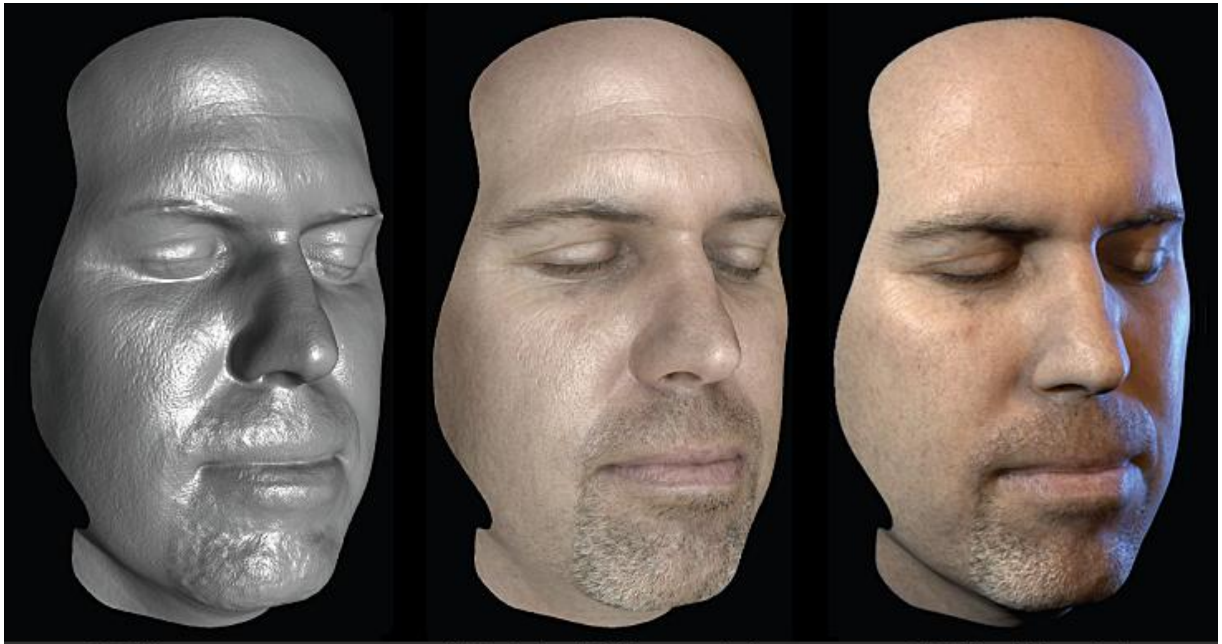
Specular Reflection





also Environment Mapping

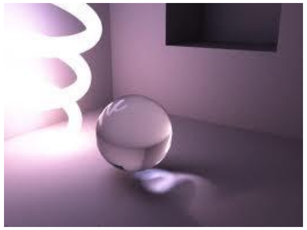
also Subsurface Scatterring

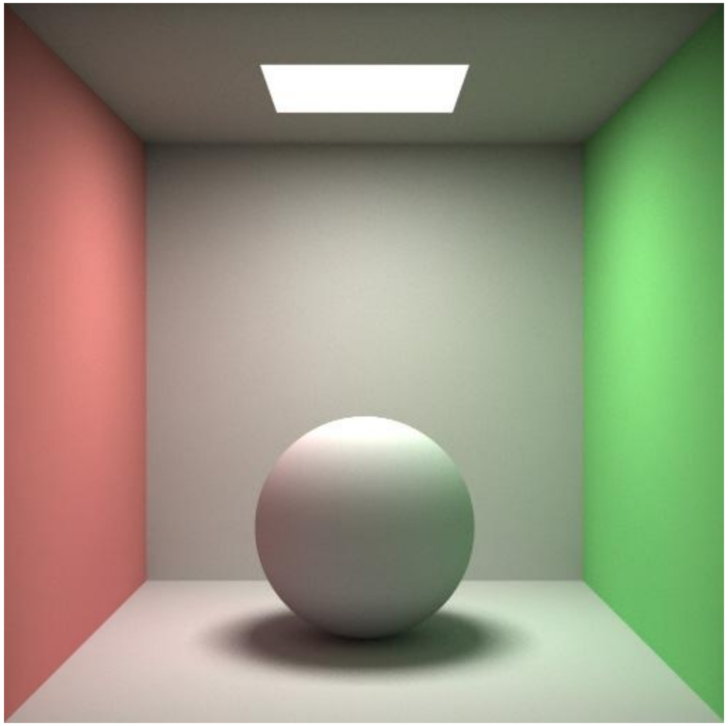


(a) High-res geometry (b) Real-time hybrid map rendering (c) Offline SSS rendering

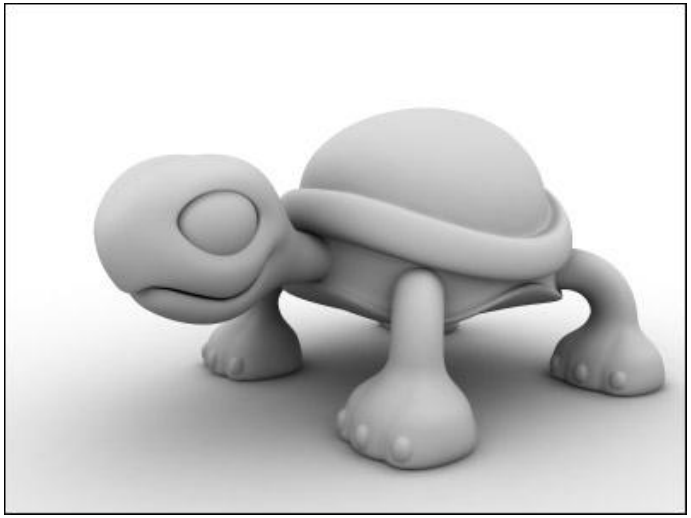
Towards Global Illumination…

Transparency





Radiosity



Ambient occlusion

Lighting and Illumination

- Light sources
  - Point light
    - Models an omnidirectional light source (e.g., a bulb)
  - Directional light
    - Models an omnidirectional light source at infinity
  - Spot light
    - Models a point light with direction
- Basic Direct Lighting Models
  - Ambient light
  - Diffuse reflection
  - Specular reflection

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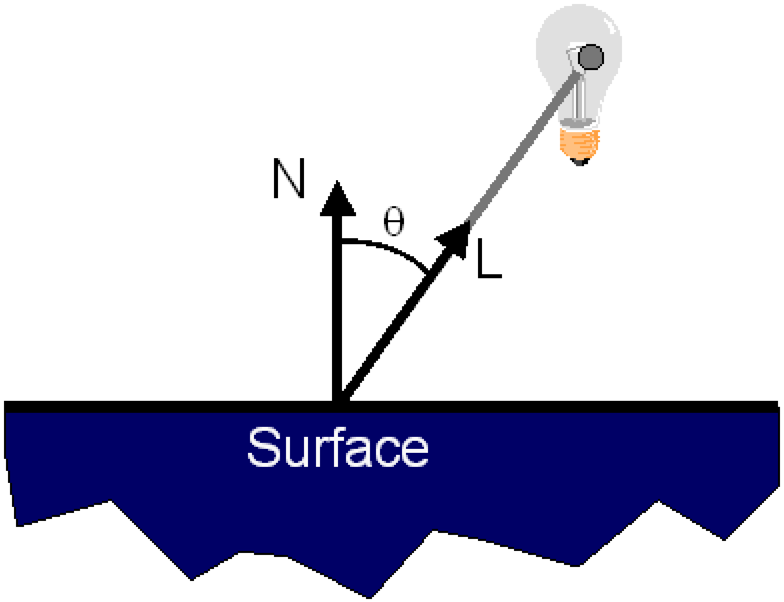
- Basic Direct Lighting Models
  - **Ambient light: just a “constant”, e.g., 0.1 of background color**
  - Diffuse reflection
  - Specular reflection

Lighting and Illumination

- Diffuse reflection
  - Lambertian model

Lighting and Shading

- Diffuse reflection
  - Lambertian model



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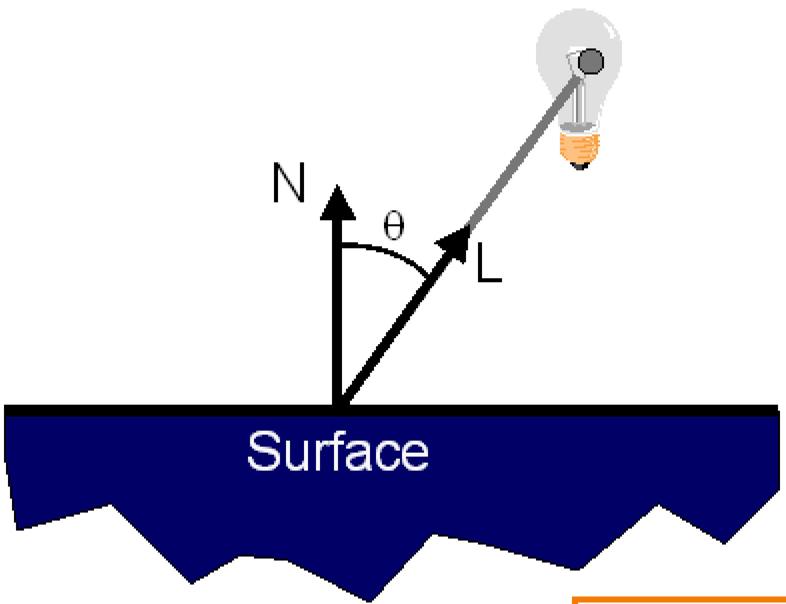

$$I_D = K_D(N \bullet L)I_L$$

Lighting and Illumination

- Specular reflection
  - Phong model

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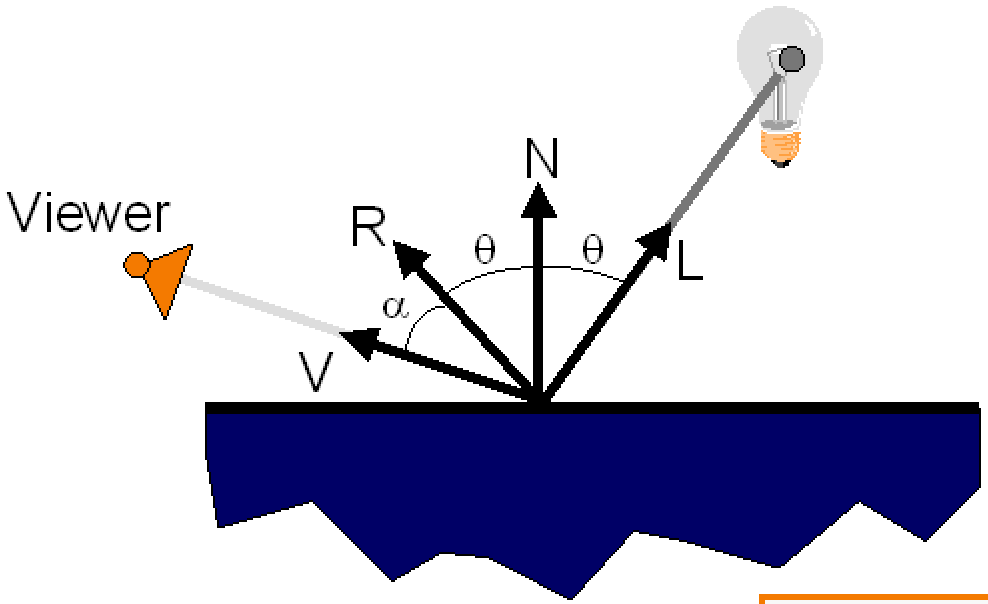

$$I_S = K_S (V \bullet R)^n I_L$$

Lighting and Illumination

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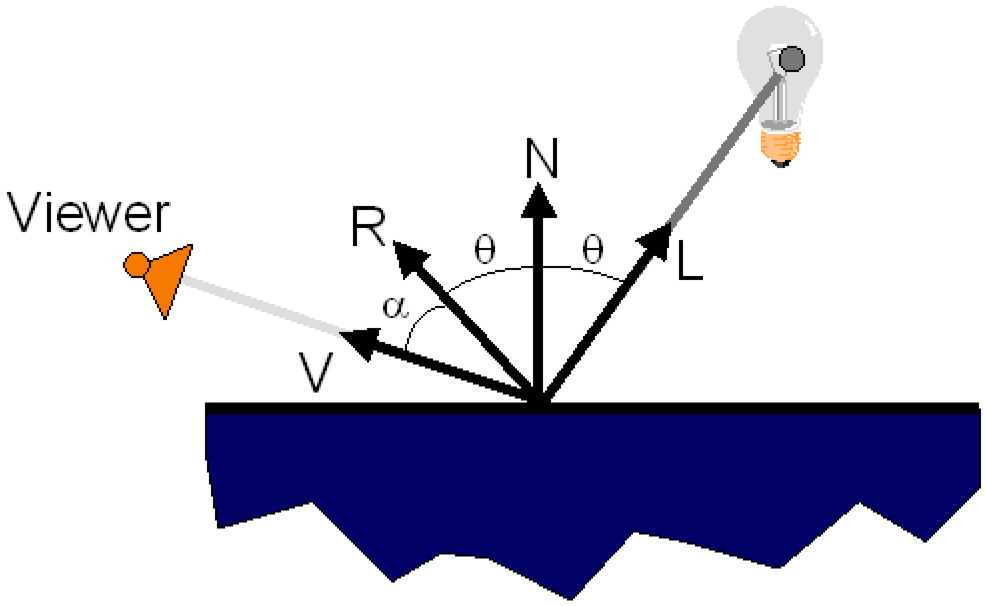
  - Phong model



But there is more…

Specular Reflection

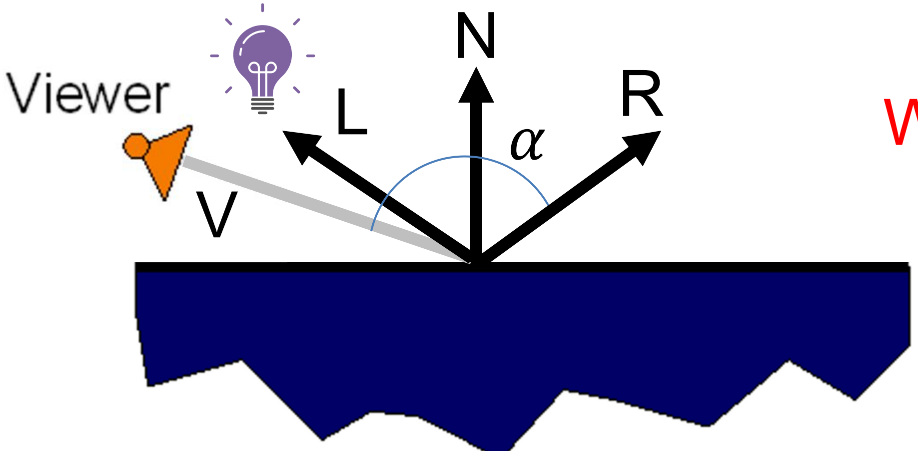
- Problem:

α is ?

$\alpha > 90$

$V \cdot R < 0$

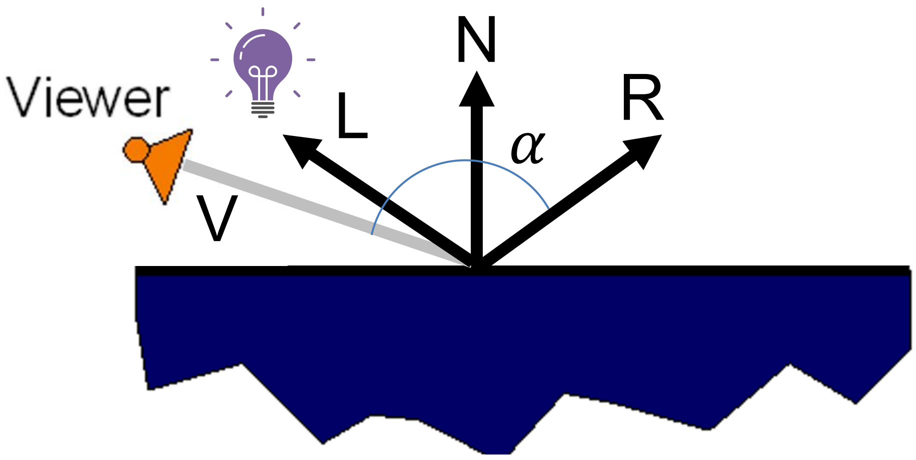
$K_S(V \cdot R)^n I_L < 0$



What can be done?

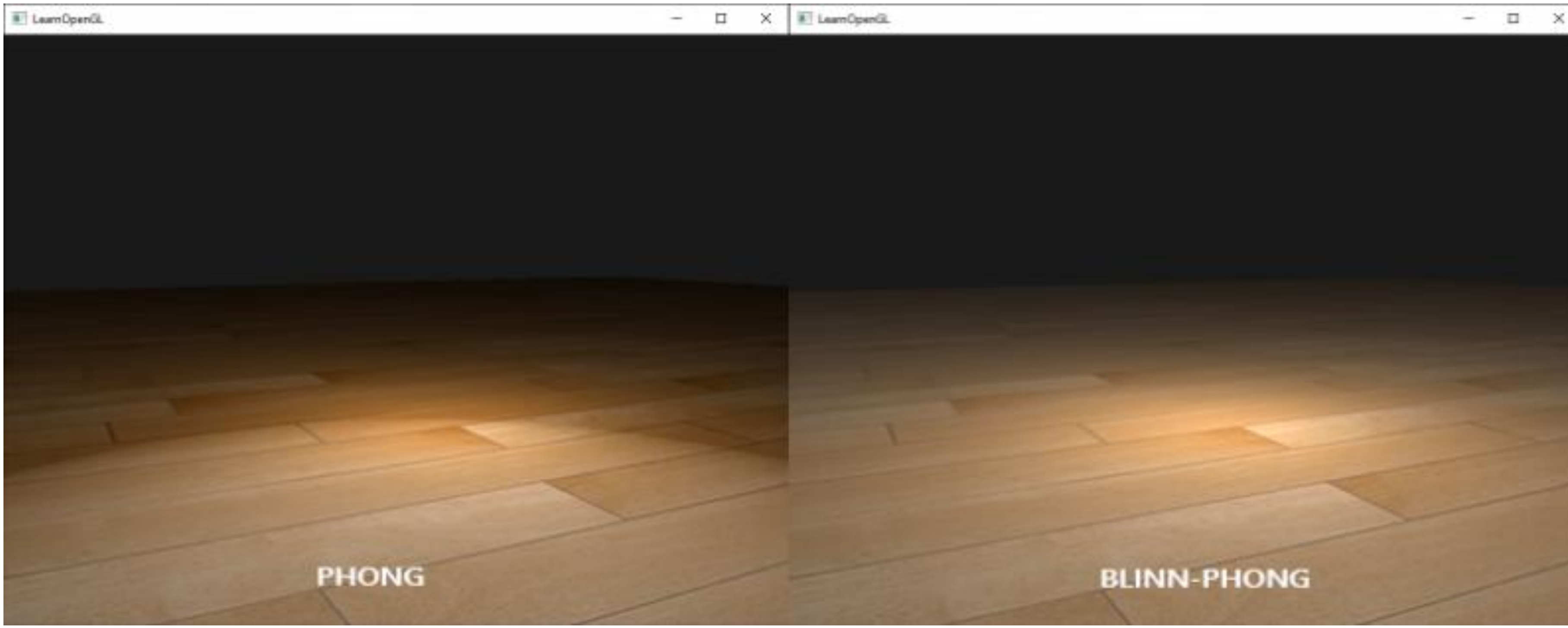
Specular Reflection

- Blinn-Phong Model (or Blinn Model)



$$H = (L + V)/\|L + V\|$$

$$I_{bp} = K_s(H \cdot N)^n I_L$$

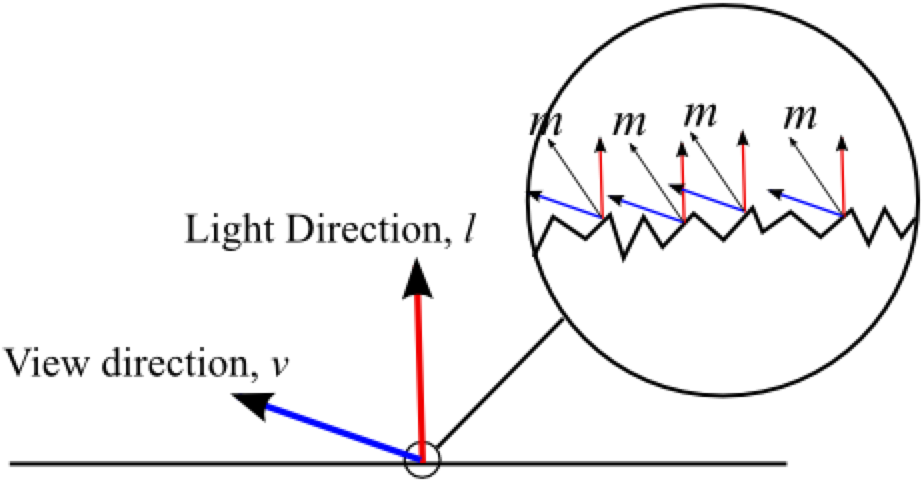

LearnOpenGL
LearnOpenGL
PHONG
BLINN-PHONG

Physically-Based Illumination Models

- Consider
  - Radiance
  - Irradiance
  - Microfacets
  - And more…

Microfacets

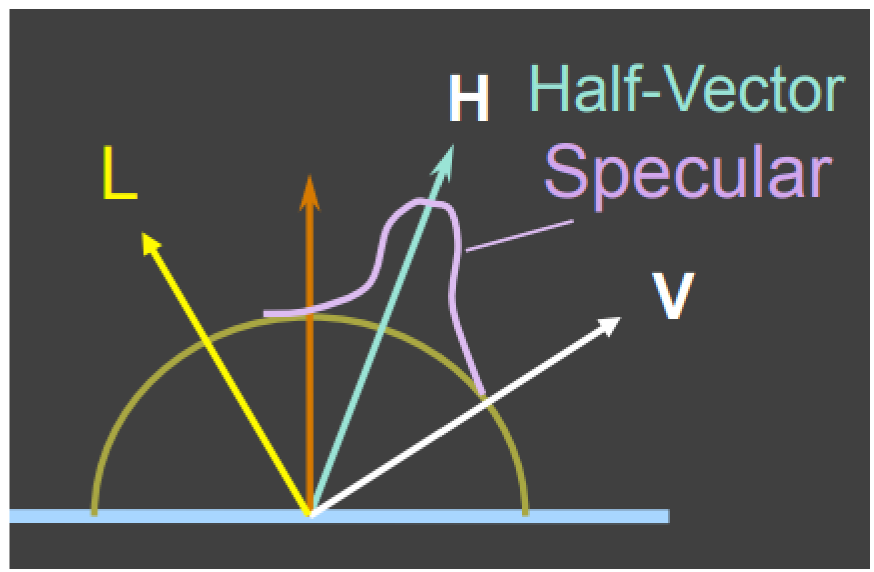
- Key question:
  - What proportion of the microfacets of the surface are oriented in such a way as to specularly reflect light towards the viewer?



Microfacets

- Key question:
  - What proportion of the microfacets of the surface are oriented in such a way as to specularly reflect light towards the viewer?

- If “very specular”, how does shape change?
- If “not very specular”, how does shape change?

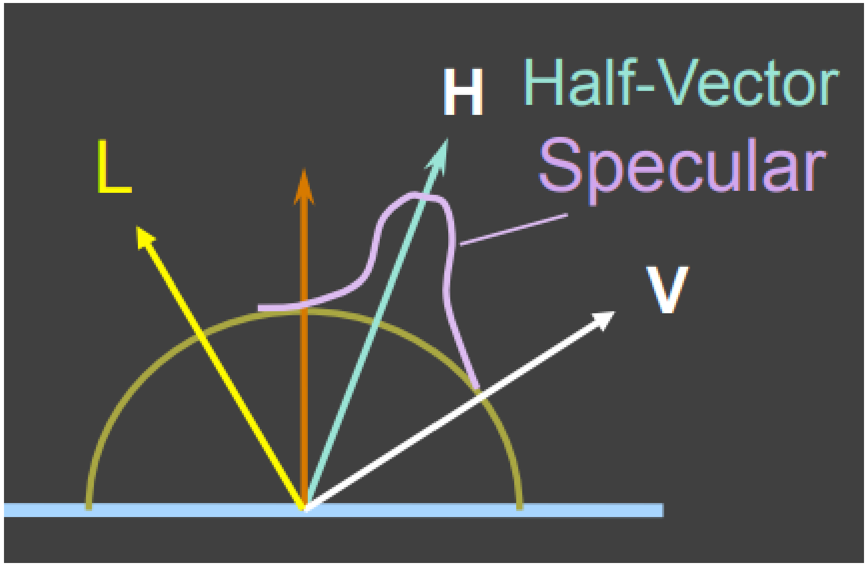


[courtesy Kavita Bala]

Microfacets

- Key question:
  - What proportion of the microfacets of the surface are oriented in such a way as to specularly reflect light towards the viewer?

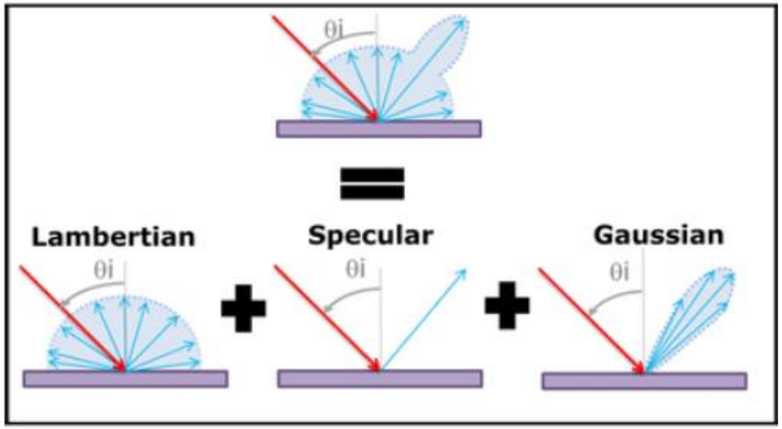
- How else can change shape?
  - Hint: a varying amount of microfacets reflect light in different directions…



[courtesy Kavita Bala]

Microfacets

- Key question:
  - What proportion of the microfacets of the surface are oriented in such a way as to specularly reflect light towards the viewer?
- An approximation:
  - Instead of $H \cdot N$ or $V \cdot R$, use Gaussian…



[courtesy L. Marot]

Microfacets

- Key question:
  – What proportion of the microfacets of the surface are oriented in such a way as to specularly reflect light towards the viewer?
- An approximation:
  – Instead of $H \cdot N$ or $V \cdot R$, use $e^{-\left(\frac{\alpha}{m}\right)^2}$ where $m \in [0,1]$ where larger m means rougher surface (=less specular)

$$I_{bp} = K_s e^{-\left(\frac{\mathrm{acos}(H \cdot N)}{m}\right)^2} I_L$$

Microfacets

- Torrance-Sparrow Model
  - Surface reflection model developed by physicists, assumes isotropic collection of planar microfacets

$$I_s = \frac{F_k}{\pi} \frac{DG}{(N \cdot V)(N \cdot L)}$$

where

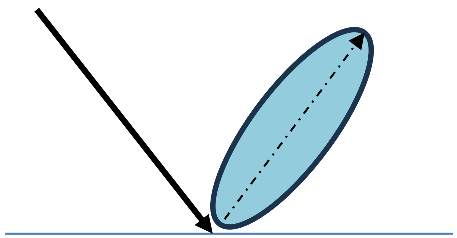
- D is distribution function of microfacet orientations
- G is geometrical attenuation factor (e.g., occlusion, shadowing)
- F_k is the Fresnel term describing the reflection and transmission of light incident on different media

Microfacets

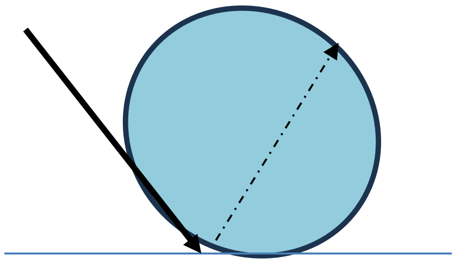
- Torrance-Sparrow Model

$$D = \frac{1}{4m^2 \cos^4 \beta} e^{-\left(\frac{\tan \beta}{m}\right)^2}$$

  - β is angle between N and H
  - m is root-mean-square slope of the microfacets
    - small m = facets similar to normal; highly directional
    - large m = facets slopes are steep; rough surface



m=0.2



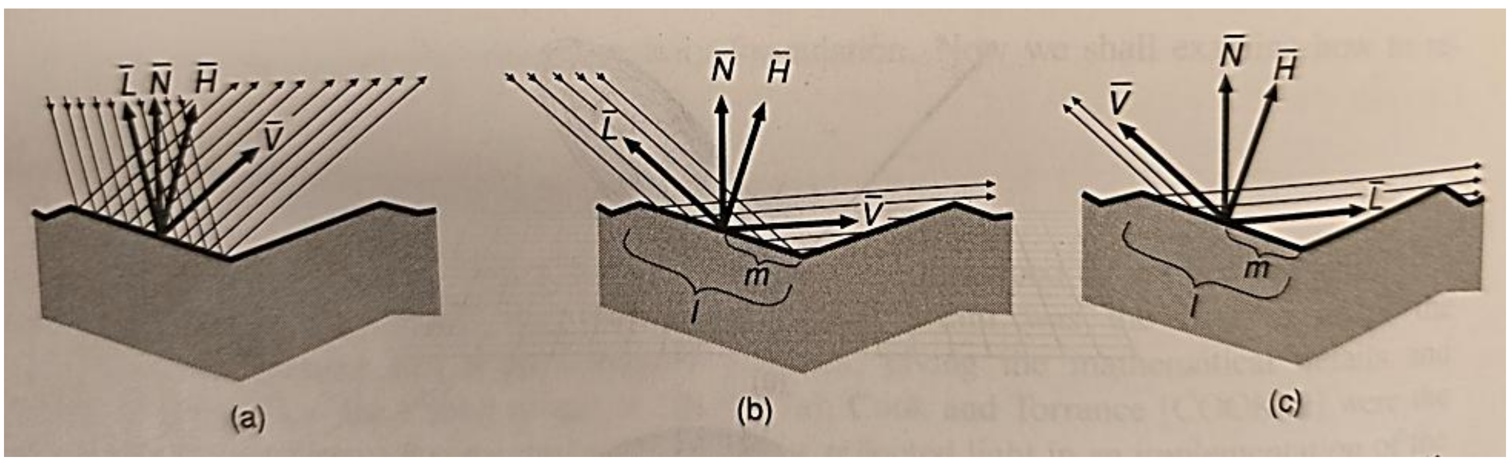
m=0.6

Microfacets

- Torrance-Sparrow Model

$$G = \min\{1, \frac{2(N\cdot H)(N\cdot V)}{V\cdot H}, \frac{2(N\cdot H)(N\cdot L)}{V\cdot H}\}$$

…quantifies how much microfacets shadow each other



Microfacets

- Torrance-Sparrow Model

$$F_k = \frac{1}{2}\frac{(g-c)^2}{(g+c)^2}\left(1+\frac{[c(g+c)-1]^2}{[c(g-c)+1]^2}\right)$$

where $c = \mathrm{L} \cdot H$, $g^2 = \eta_\lambda^2 + c^2 - 1$, $\eta_\lambda = \frac{\eta_{t\lambda}}{\eta_{i\lambda}}$

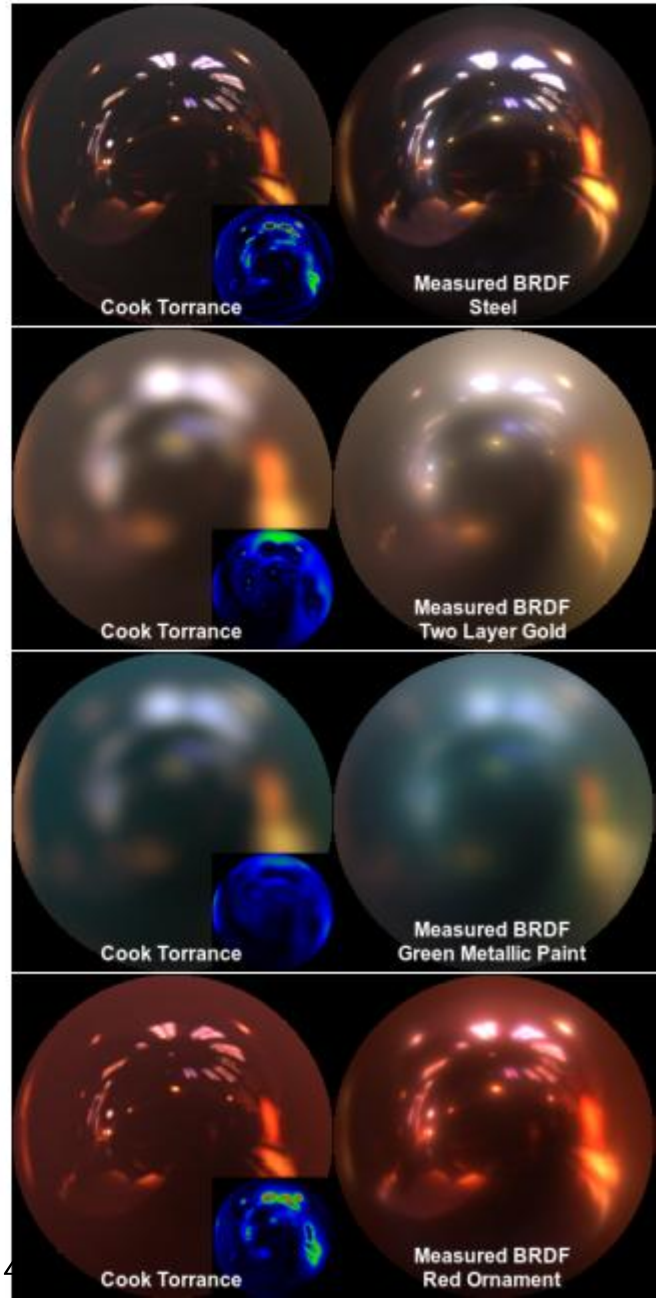
or, simplified to

$F_k \approx F_0 + (1 - F_0)(1 - (V \cdot H))^5$ [Schlick's approximation]

where F_0 is reflectance at normal incidence (e.g., RGB color)

Microfacets

- Cook-Torrance Model
  - Similar, with some improvements
  - D is a weighted sum of distribution functions to model surfaces of multiple scales of roughness



[Brady et al., TOG 2014

What about rough (diffuse) surfaces?

- Must consider occlusion, shadowing, and diffuse interreflection – thoughts?
- $I_d = N \cdot L$
- Is that it?

What about rough (diffuse) surfaces?

- Must consider occlusion, shadowing, and diffuse interreflection – thoughts?
- Derived from microfacets model
  - Oren-Nayar Model:

$$L_r = L_1 + L_2$$

$$L_1 = \frac{\rho}{\pi} E_0 \cos\theta_i \left(C_1 + C_2 \cos(\phi_i - \phi_r) \tan\beta + C_3 (1 - |\cos(\phi_i - \phi_r)|) \tan\frac{\alpha+\beta}{2} \right),$$

$$L_2 = 0.17 \frac{\rho^2}{\pi} E_0 \cos\theta_i \frac{\sigma^2}{\sigma^2 + 0.13} \left[1 - \cos(\phi_i - \phi_r) \left(\frac{2\beta}{\pi} \right)^2 \right],$$

where

$$C_1 = 1 - 0.5 \frac{\sigma^2}{\sigma^2 + 0.33},$$

$$C_2 = \begin{cases} 0.45 \frac{\sigma^2}{\sigma^2 + 0.09} \sin\alpha & \text{if } \cos(\phi_i - \phi_r) \geq 0, \\ 0.45 \frac{\sigma^2}{\sigma^2 + 0.09} \left(\sin\alpha - \left(\frac{2\beta}{\pi} \right)^3 \right) & \text{otherwise,} \end{cases}$$

$$C_3 = 0.125 \frac{\sigma^2}{\sigma^2 + 0.09} \left(\frac{4\alpha\beta}{\pi^2} \right)^2,$$

$$\alpha = \max(\theta_i, \theta_r),$$

$$\beta = \min(\theta_i, \theta_r),$$

What about rough (diffuse) surfaces?

- Oren-Nayar Model (simplified)

$$L_r = \cos\theta_i\,[A + B\max(0, \cos(\theta_r - \theta_i))\sin\alpha\tan\beta]$$

where using numerical experiments:

$$A \approx \rho(\frac{1}{\pi} - 0.09\,\frac{\sigma^2}{\sigma^2 + 0.4})$$

$$B \approx \rho(0.125\,\frac{\sigma^2}{\sigma^2 + 0.18})$$

Real Image Lambertian Model Oren-Nayar Model