



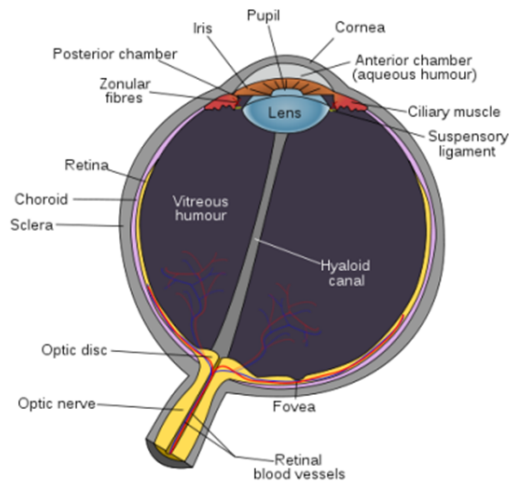
Camera Models

Spring 2025

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Department of Computer Science
Purdue University

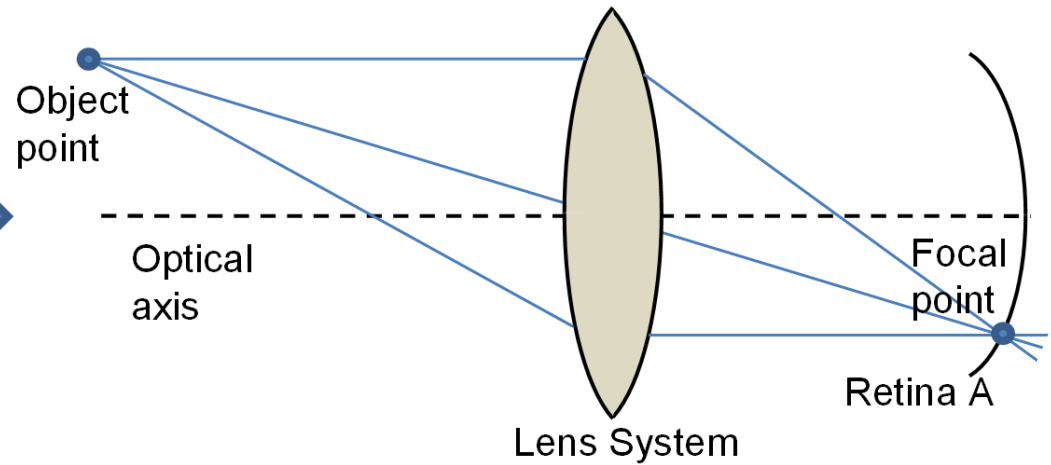
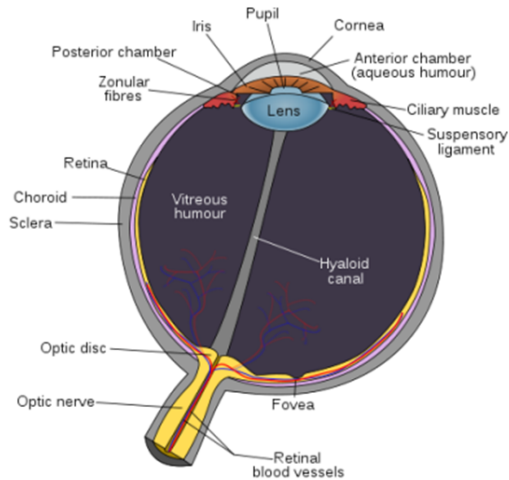


Human Eye



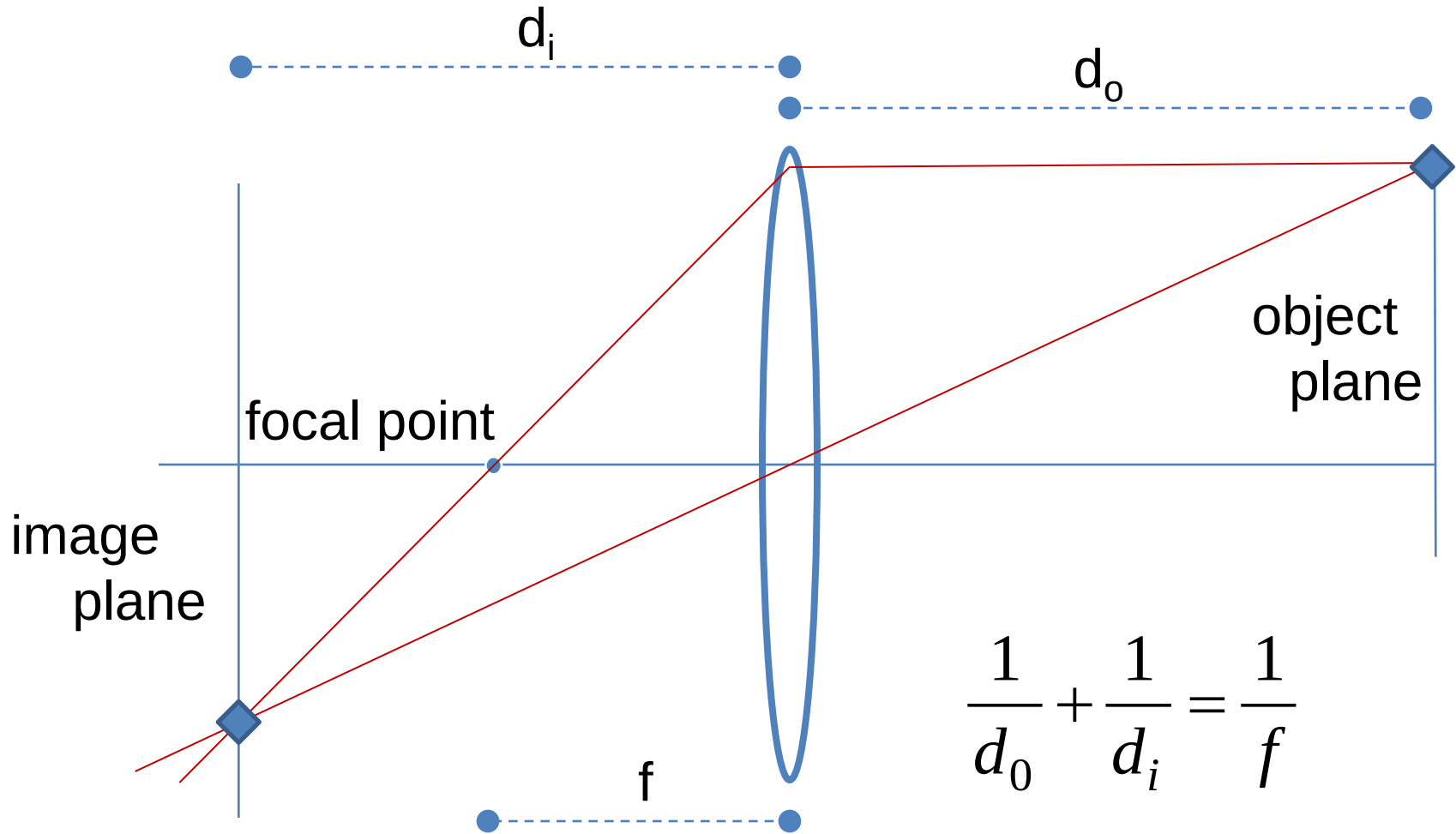


Thin Lens System





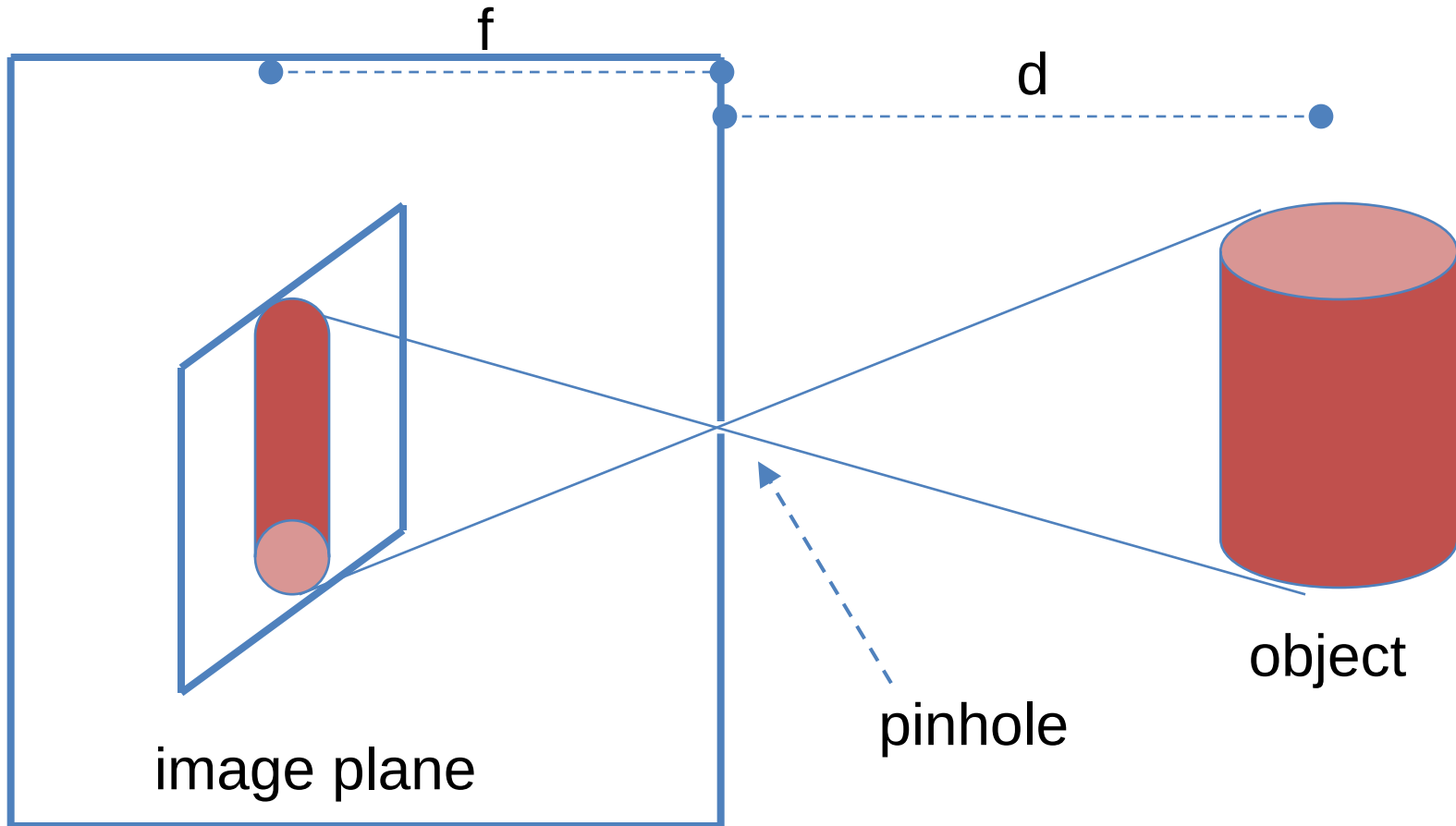
Thin Lens Equation



$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

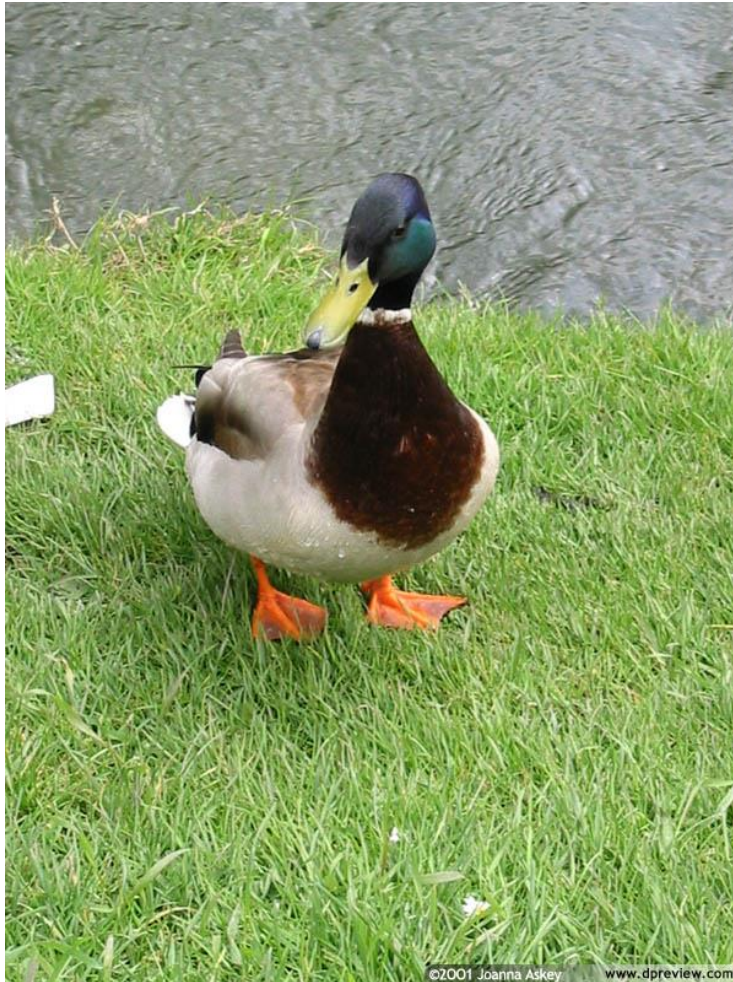


Pinhole Camera Model

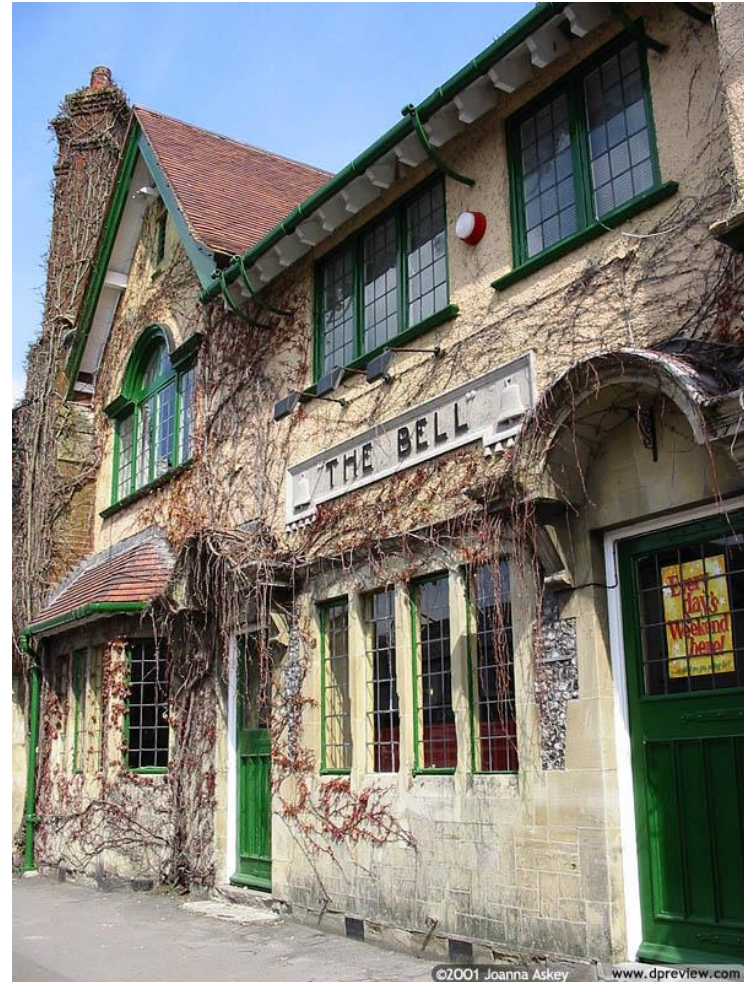




“Pinhole” Camera Image



©2001 Joanna Askey www.dpreview.com



©2001 Joanna Askey www.dpreview.com



Digression: Non-Pinhole Camera Models...

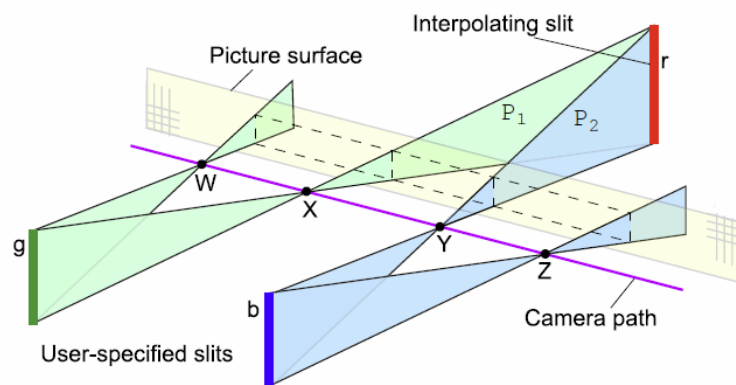
- Why restrict the camera to a pinhole camera model?



Multiperspective Imaging



Hand-crafted



semi-automated...

to produce this...



[Roman-Vis04]



Multiperspective Imaging



(a)



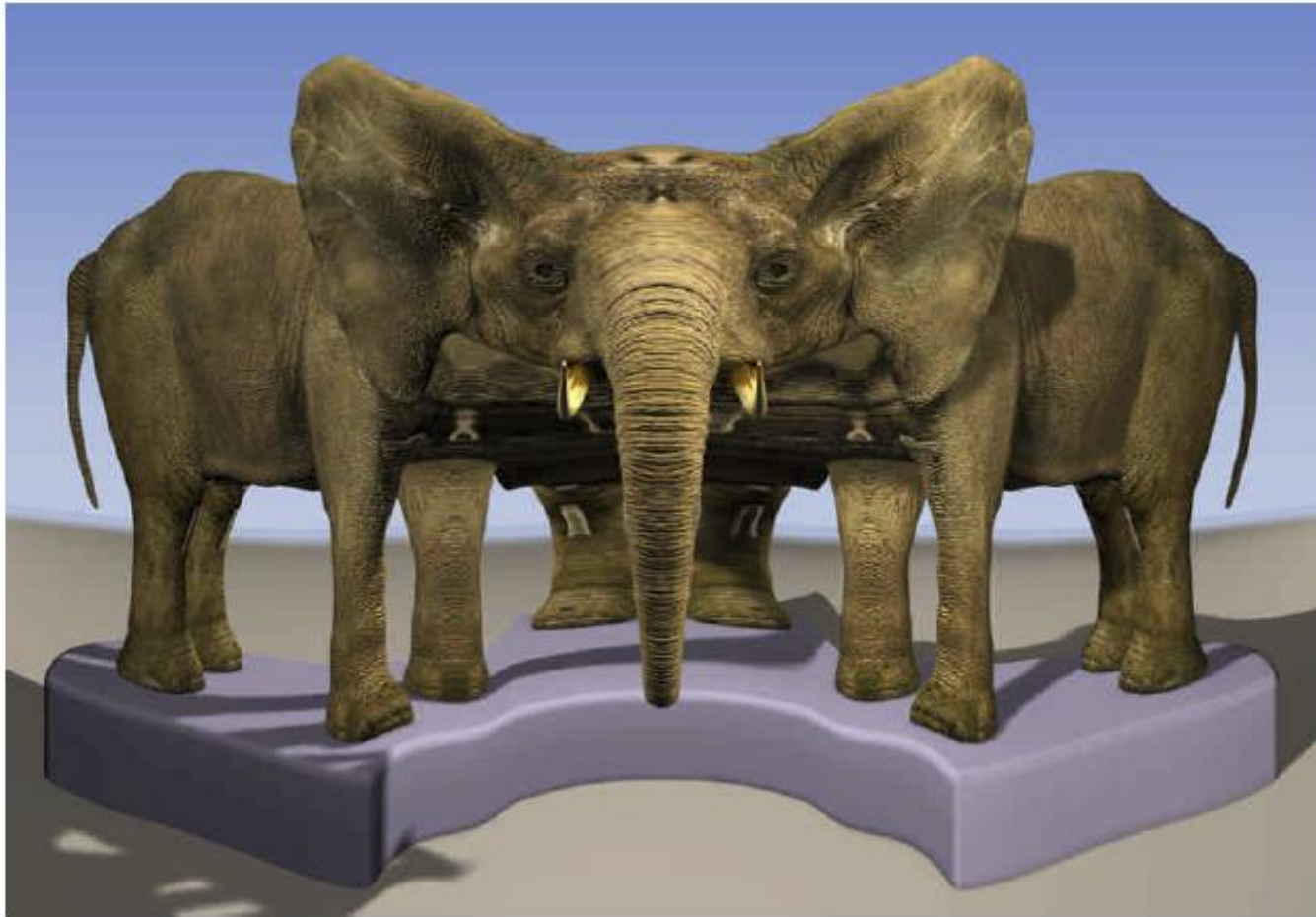
(b)



(c)

[Seitz-CGA03]

Multiple COP Images



[Rademacher-SIG98]₁₀

Multiple COP Images

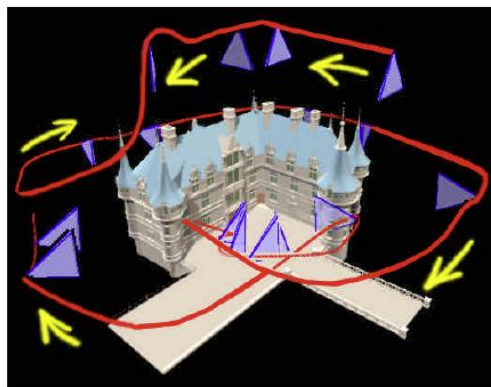


Figure 5 Castle model. The red curve is the path the camera was swept on, and the arrows indicate the direction of motion. The blue triangles are the thin frusta of each camera. Every 64th camera is shown.

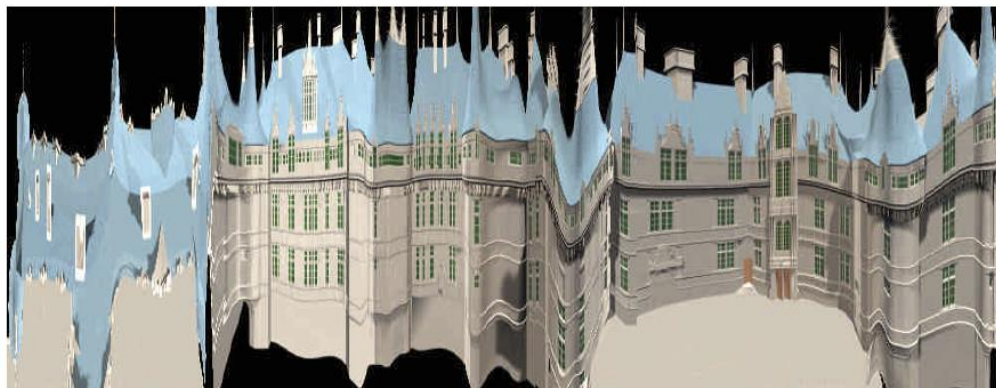


Figure 6 The resulting 1000×500 MCOP image. The first fourth of the image, on the left side, is from the camera sweeping over the roof. Note how the courtyard was sampled more finely, for added resolution.

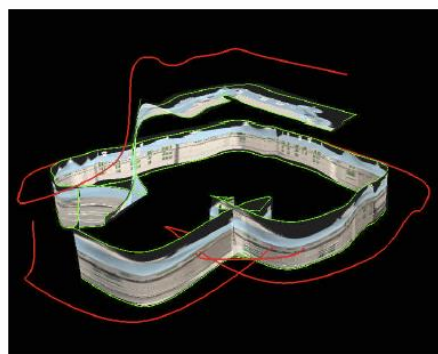


Figure 7 The projection surface (image plane) of the camera curve.



Figure 8 Three views of the castle, reconstructed solely from the single MCOP image above. This dataset captures the complete exterior of the castle.

Multiperspective Imaging for Cel Animation

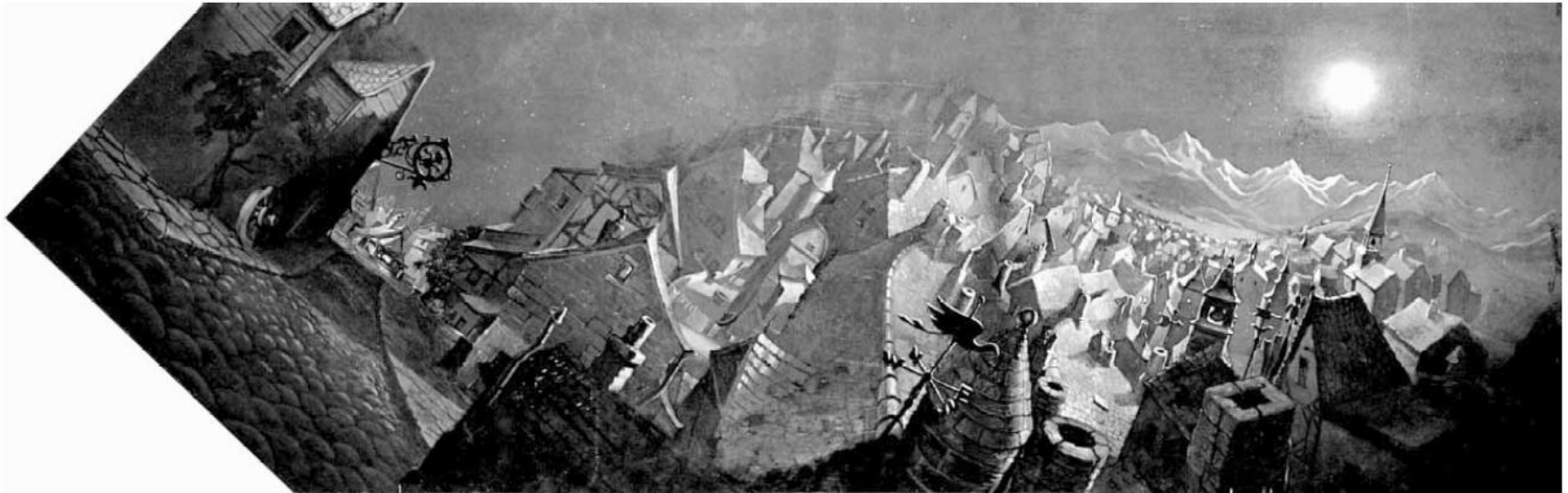
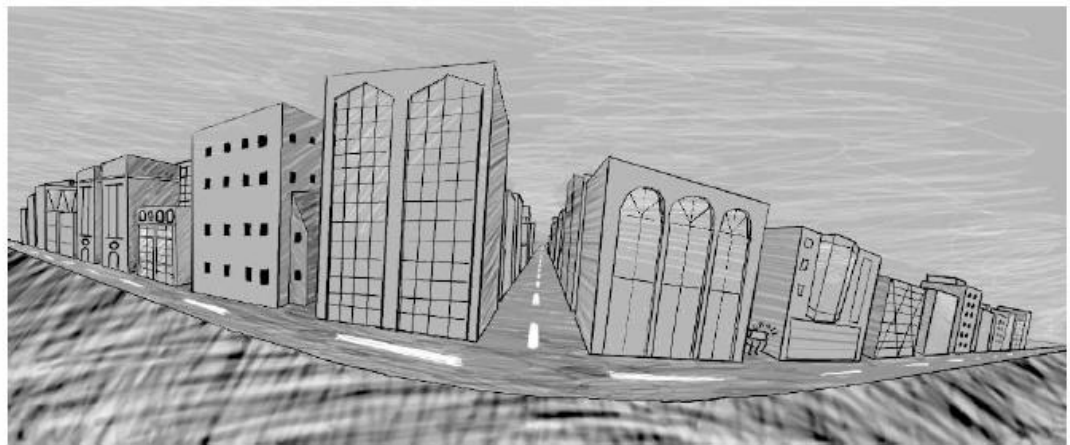
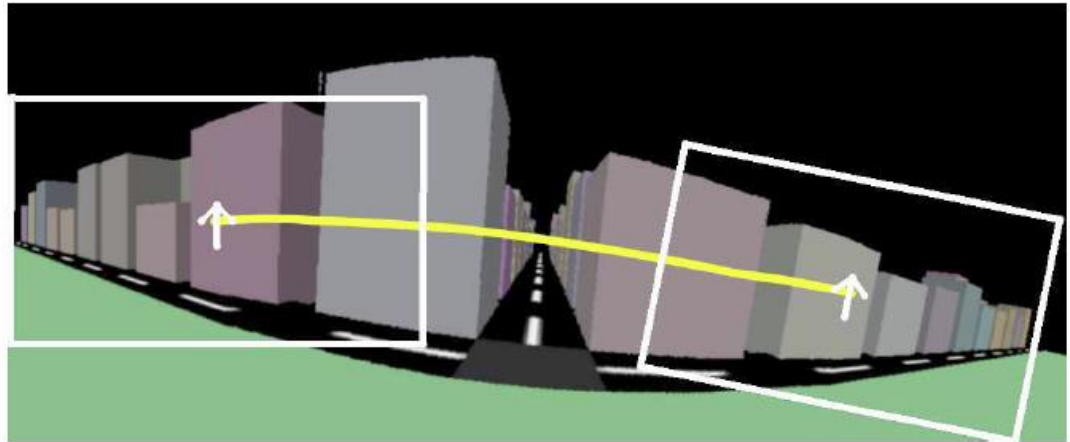
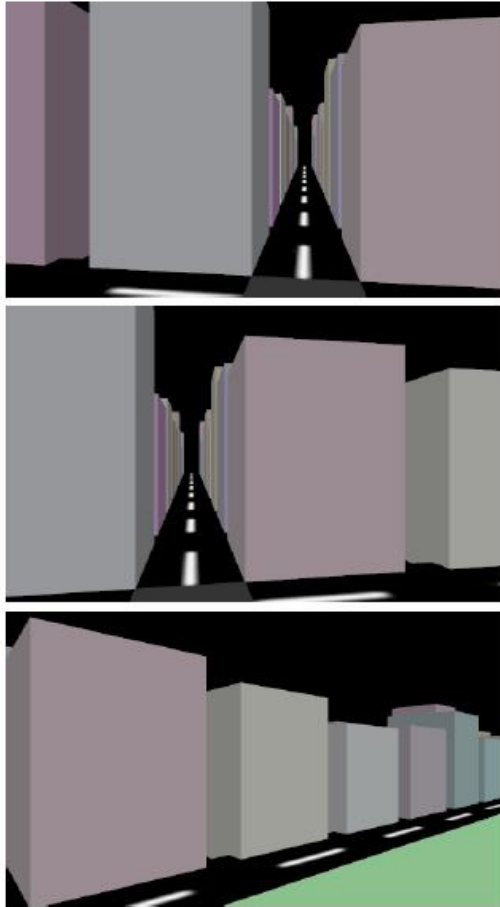


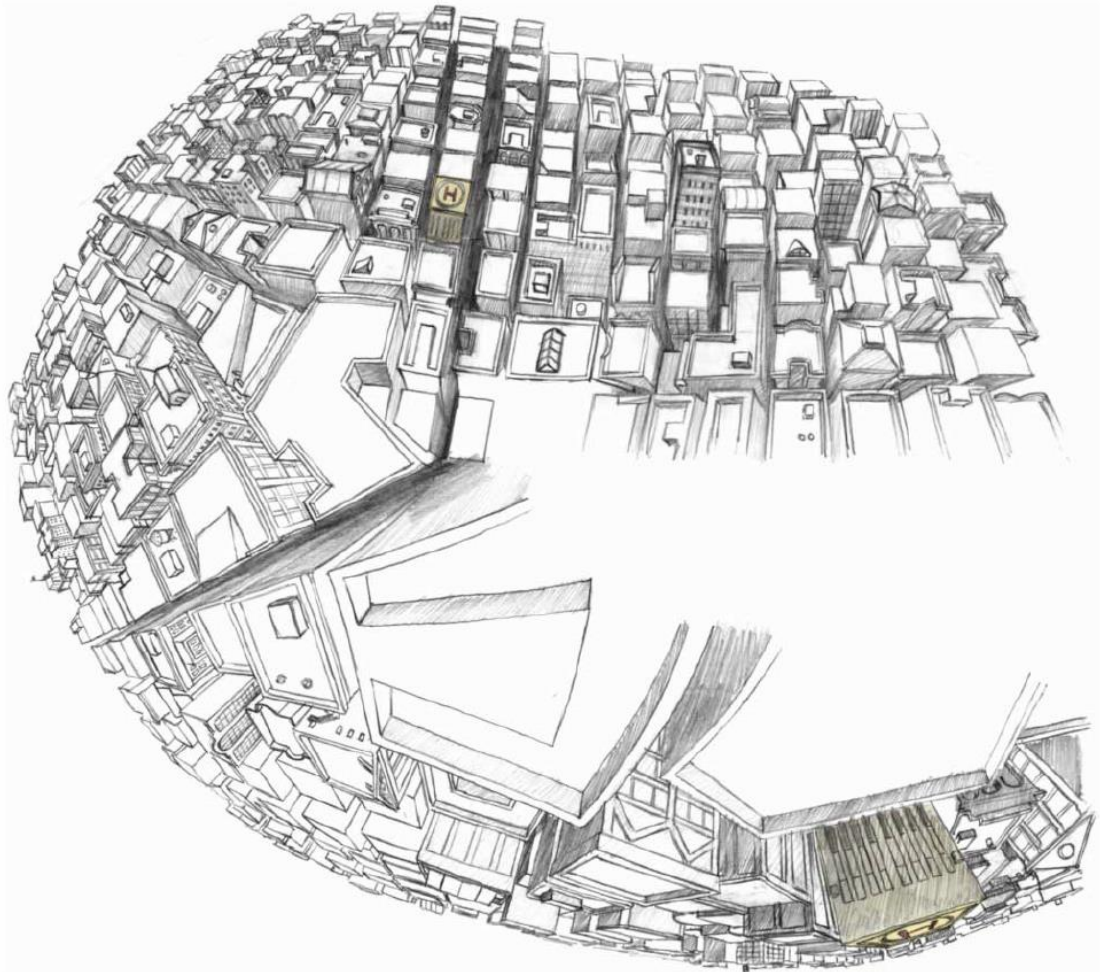
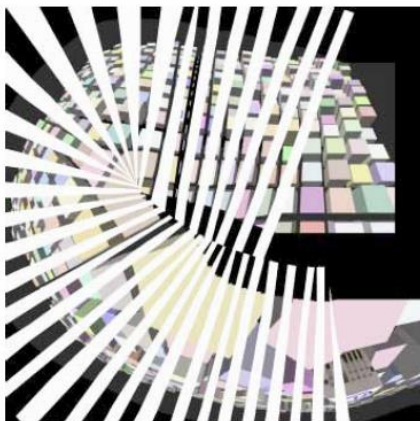
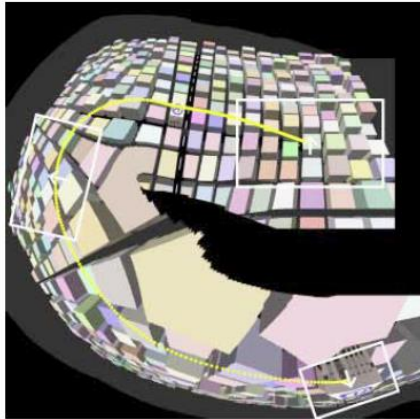
Figure 1 A multiperspective panorama from Disney's 1940 film *Pinocchio*. (Used with permission.)

Multiperspective Imaging for Cel Animation



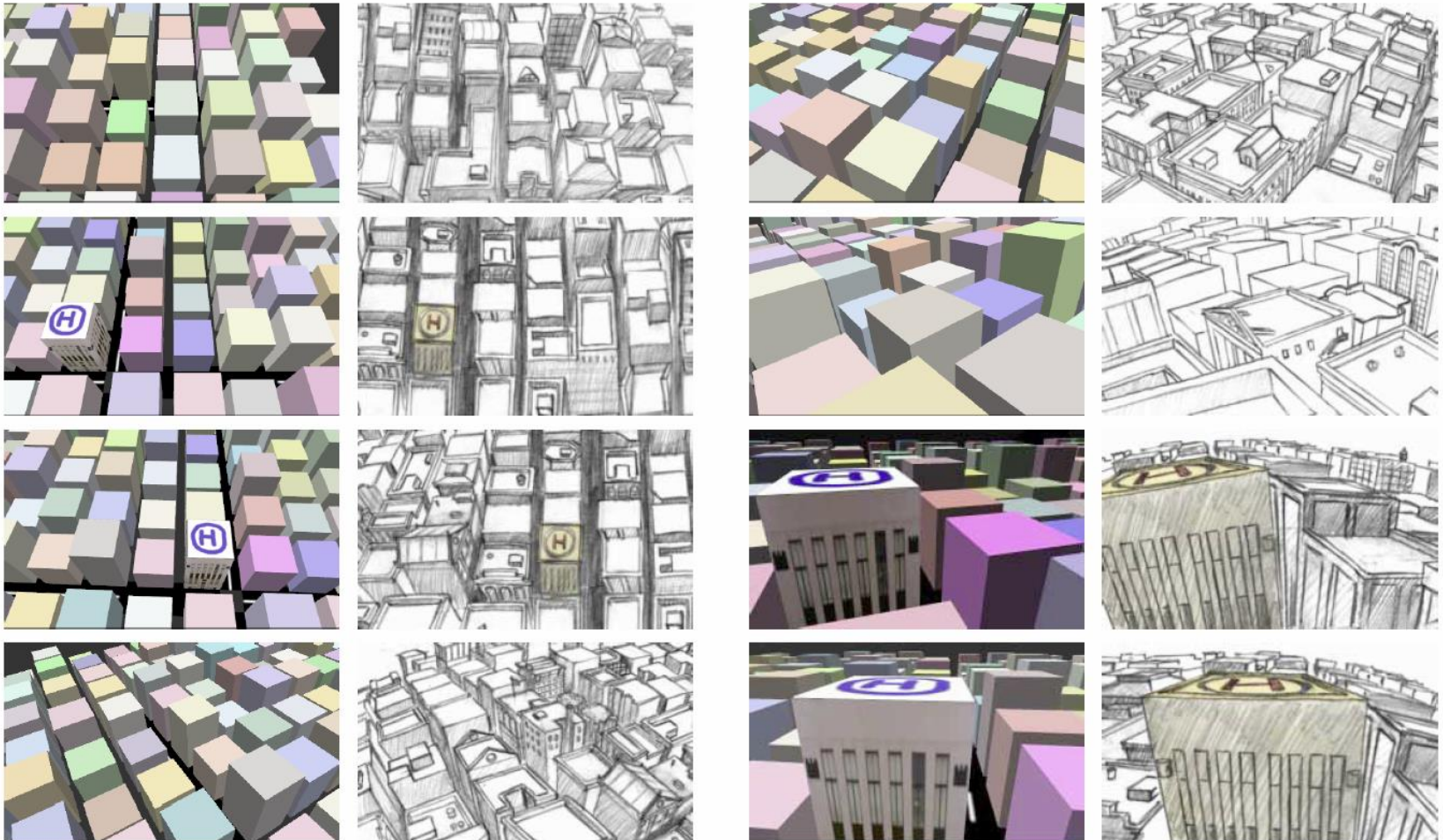
[Wood-SIG97]

Multiperspective Imaging for Cel Animation



[Wood-SIG97]

Multiperspective Imaging for Cel Animation

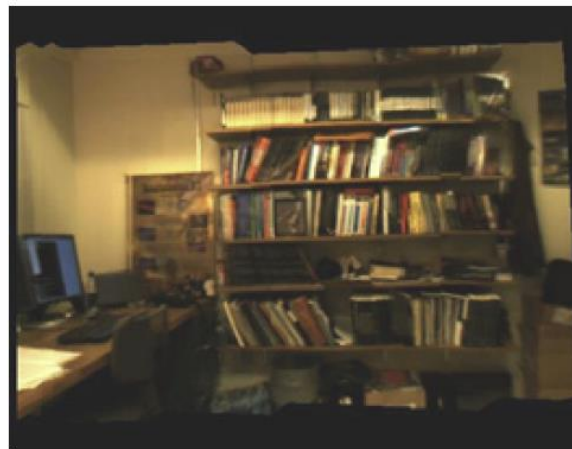


[Wood-SIG97]

Occlusion-Resistant Cameras



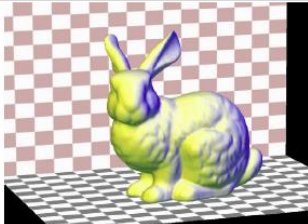
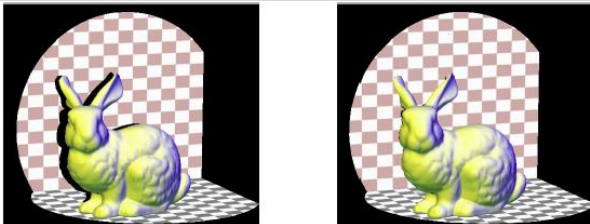
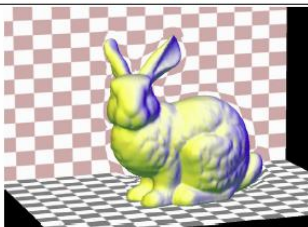
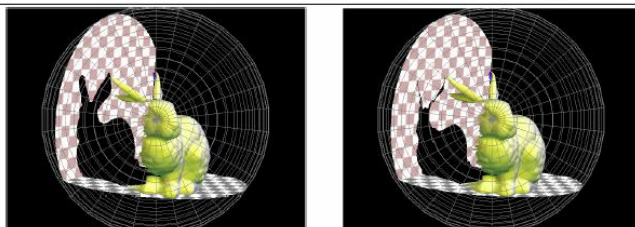
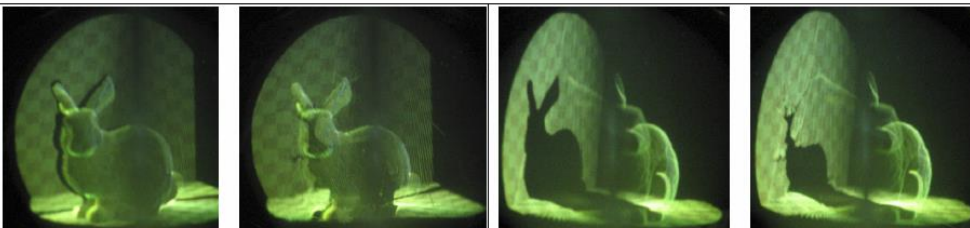
Input images



Output images

[Aliaga-CGA07]

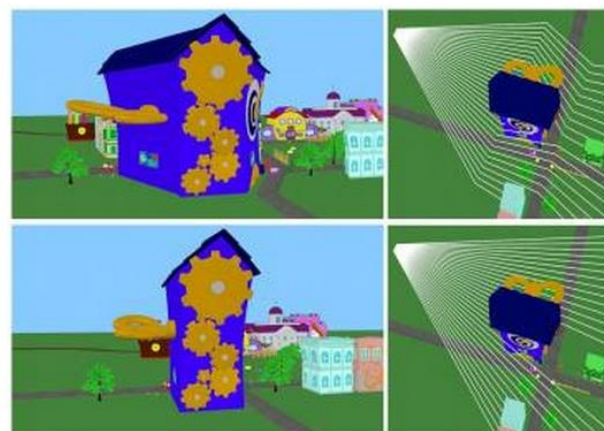
Occlusion Cameras

SIMULATOR IMAGES			
	Figure 1 Depth image (DI).	Figure 2 Images rendered from DI and OCRI, viewpoint 4° left of reference viewpoint.	
			
	Figure 3 OCRI.	Figure 4 Images rendered from DI and OCRI. Wireframe shows spherical display volume.	
PHOTOGRAPHS OF 3D DISPLAY			
	Figure 5 3D images rendered from DI (left), OCRI (middle), and original geometric model (right), all photographed from reference view.		
			
	Figure 6 DI and OCRI 3D images from viewpoint translated 4° left.		Figure 7 DI and OCRI 3D images from side view.

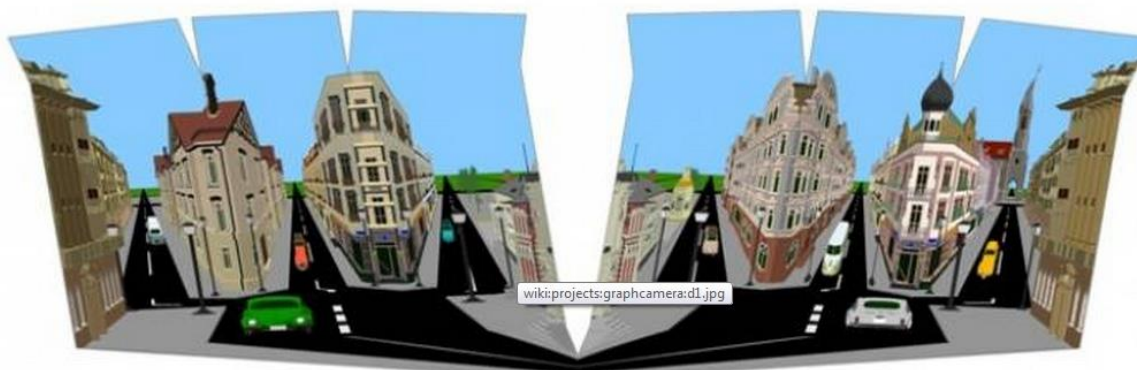
Graph Cameras



Portal-based graph camera image (top left and fragment right) and PPC image for comparison (bottom left)



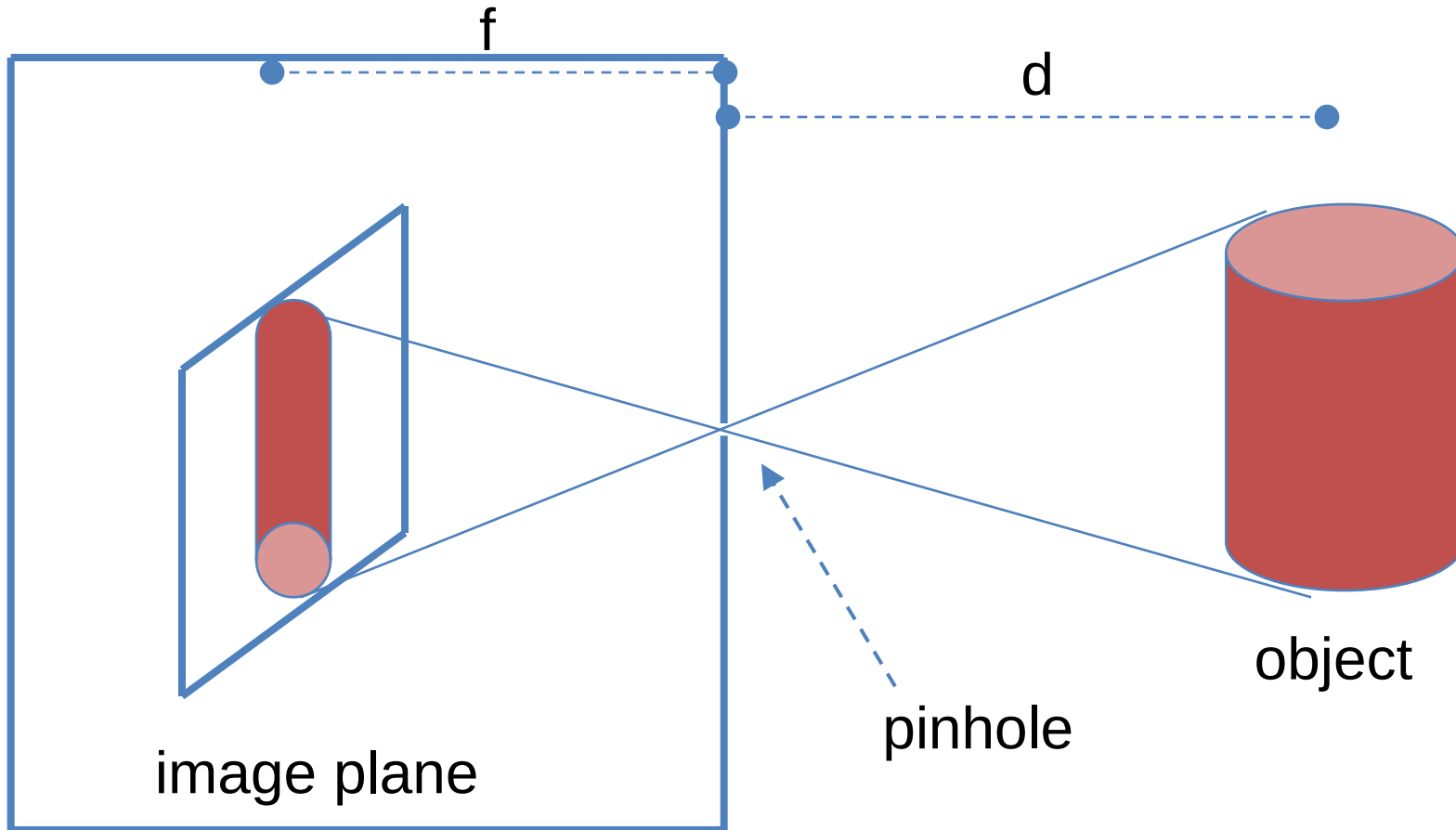
Occluder-based graph camera image (top left), PPC image for comparison (bottom left), and ray visualizations (right)



Enhanced street-level navigation



Let's get back on track: Pinhole Camera Model...





Photographic Camera History 101

- First photos:
 - 1826:
 - https://www.youtube.com/watch?v=sOkd8ObhN_M
- First videos:
 - 1874
 - <https://www.youtube.com/watch?v=VC-yTYyE2w0>
- Femto camera (10^{12} frames per second)
 - 2013
 - <https://www.youtube.com/watch?v=EtsXgODHMWk>

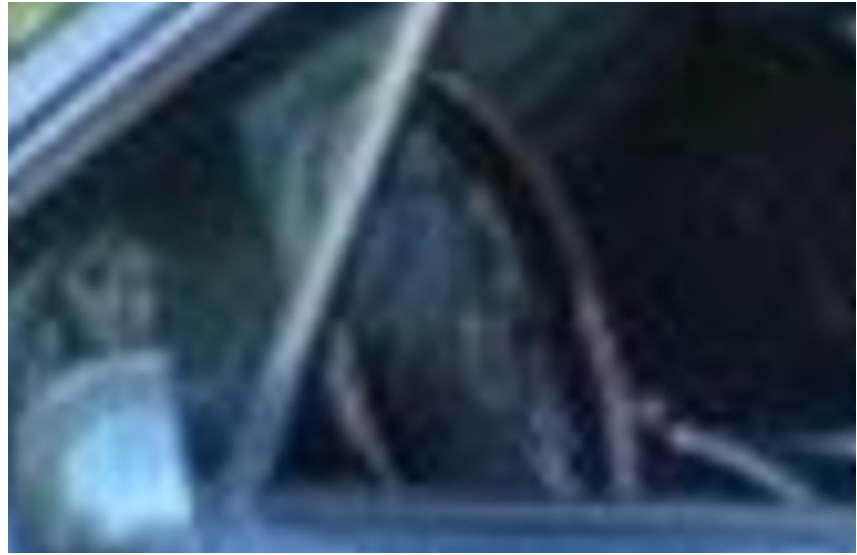


Camera Resolution

- First digital camera (~1985): 0.01 MP (100x100)
- Consumer ware: up to 100 MP
- Gigapixel camera (assembled pics): ~300 GP



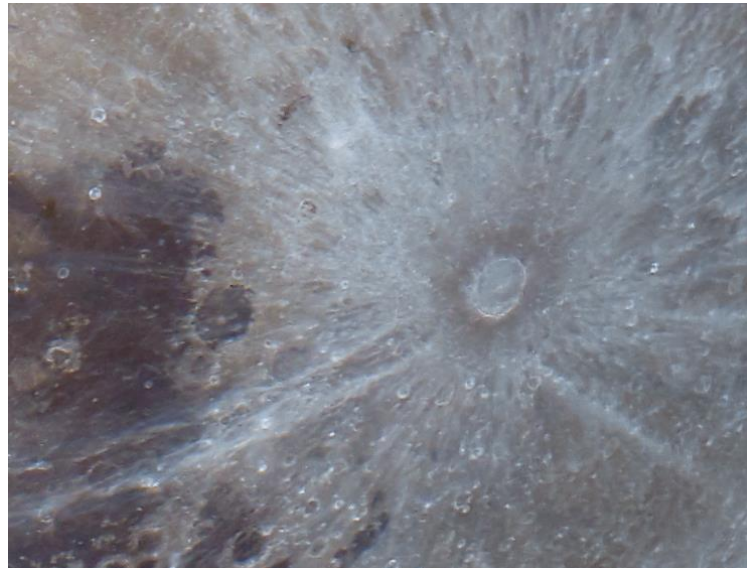
Camera Resolution



0.25 MP



Camera Resolution



100 MP
(astrophotography)



Camera Resolution



100 MP



Camera Resolution



272 GP



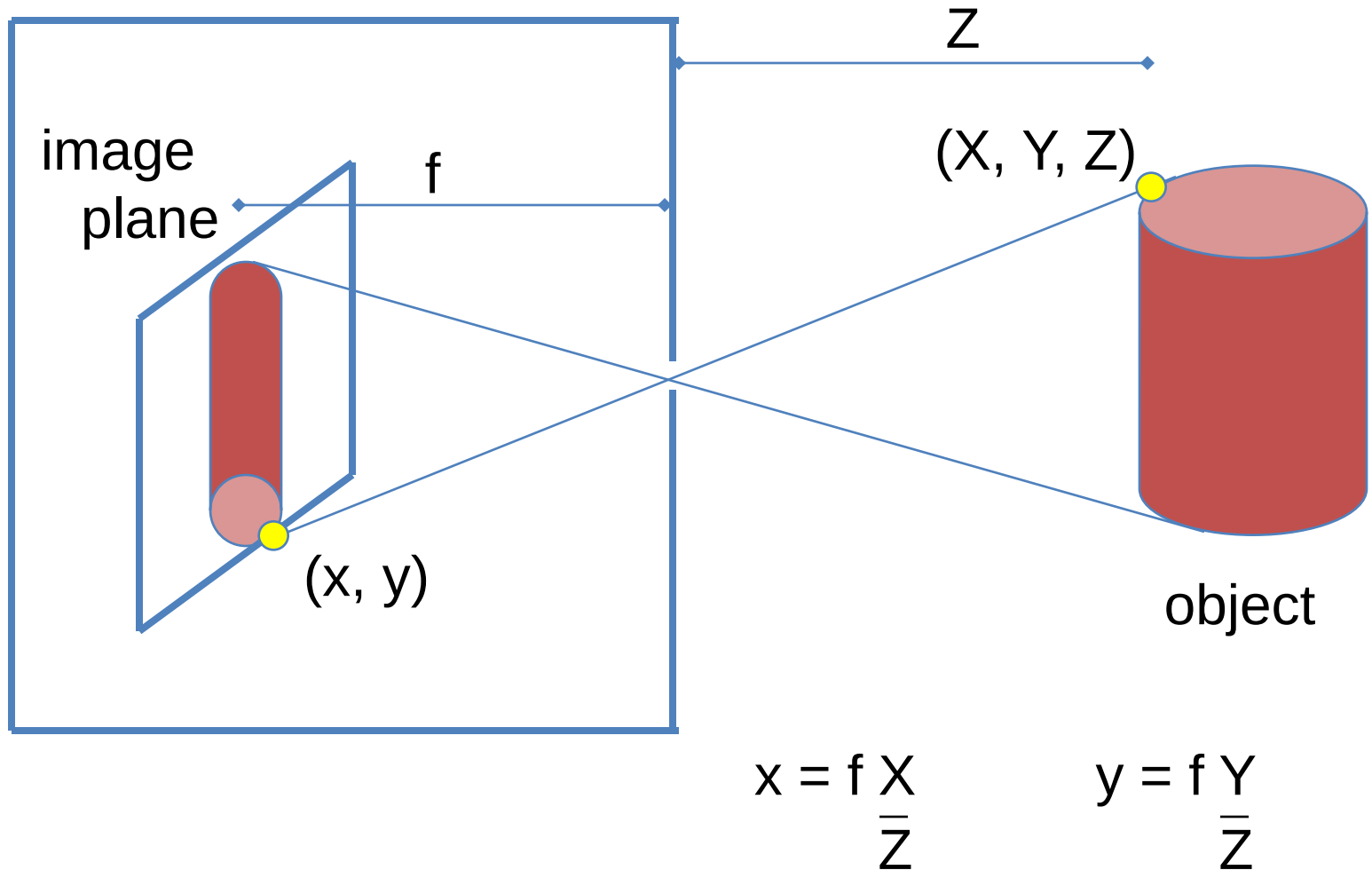
Used 12,000 images....Calibration is crucial!!!!



Camera Resolution

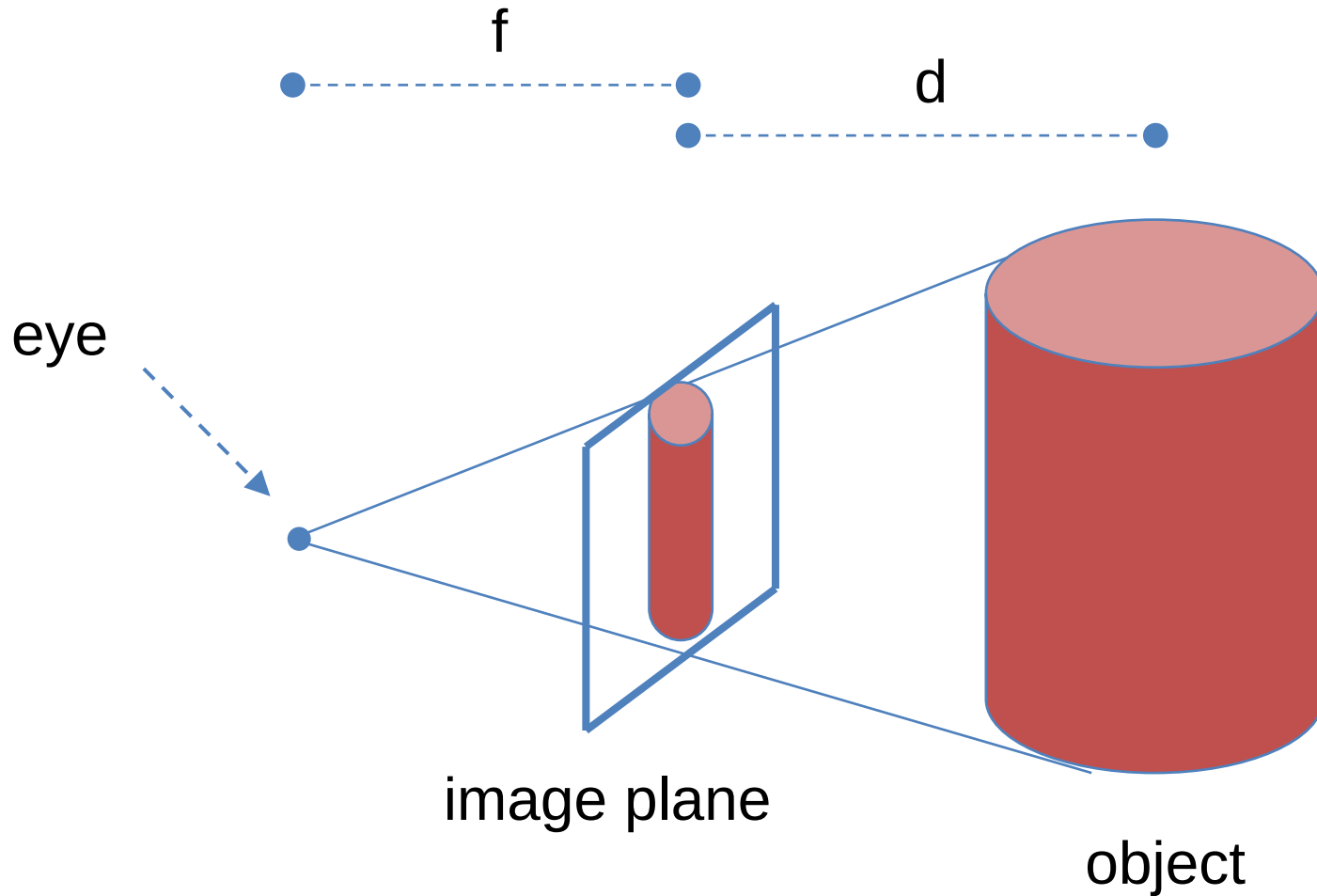
- London (565,000 x 565,000 pixels)
 - <https://www.dpreview.com/articles/7683341128/320-gigapixel-photo-of-london-is-the-worlds-largest-panoramic-photo>

What is perspective projection?

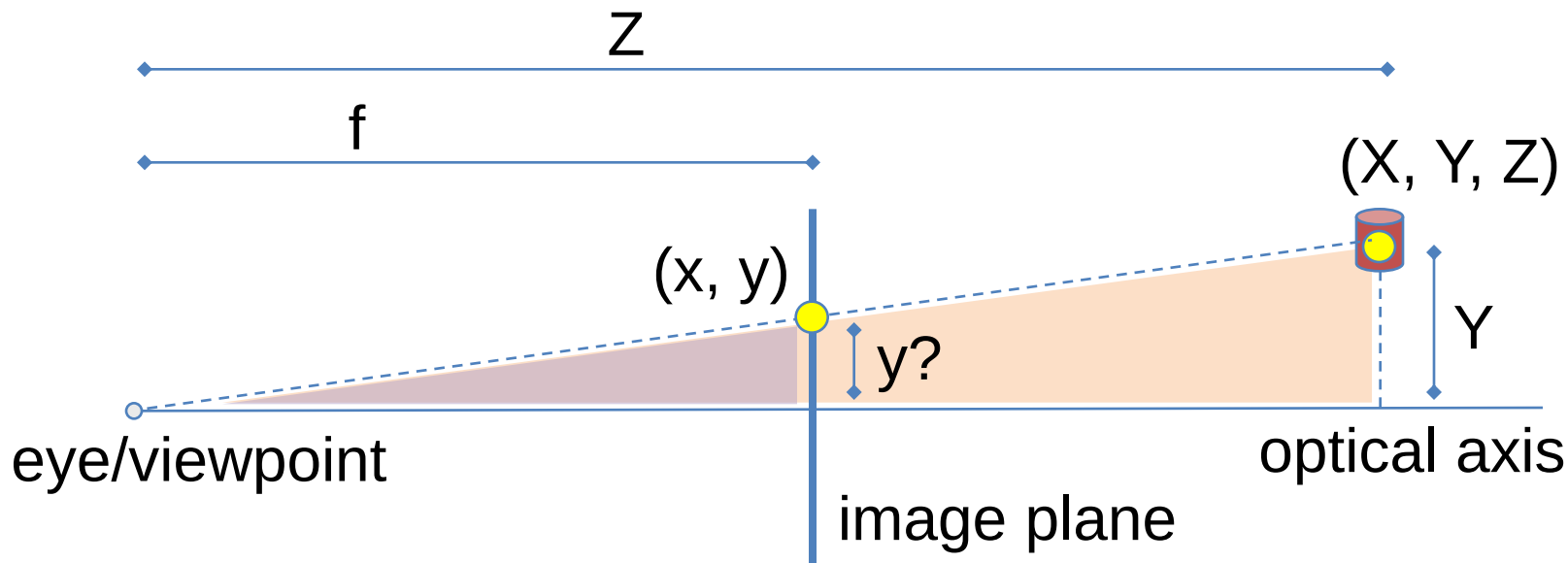




“Computer Graphics” Pinhole Camera Model



What is perspective projection?



$$\frac{y}{f} = \frac{Y}{Z}$$



$$y = f \frac{Y}{Z} \quad \& \quad x = f \frac{X}{Z}$$

Perspective Camera Parameters



- Intrinsic/Internal
 - Focal length f
 - Principal point (center) p_x, p_y
 - Pixel size s_x, s_y
 - (Distortion coefficients) $k_1, \dots$
- Extrinsic/External
 - Rotation ϕ, φ, ψ
 - Translation t_x, t_y, t_z

Perspective Camera Parameters



- Intrinsic/Internal

- Focal length

f

- Principal point (center)

(=middle of image)

- Pixel size

(=1, irrelevant)

- (Distortion coefficients)

(=0, assuming no bugs ☺)

- Extrinsic/External

- Rotation

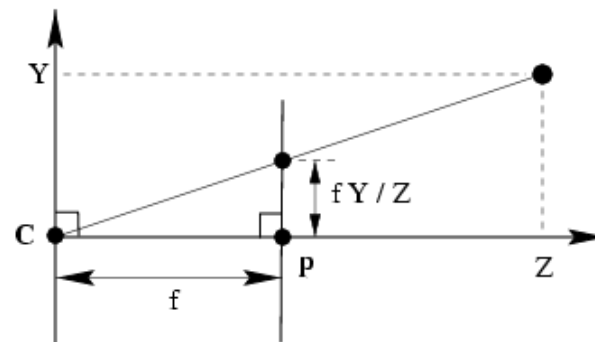
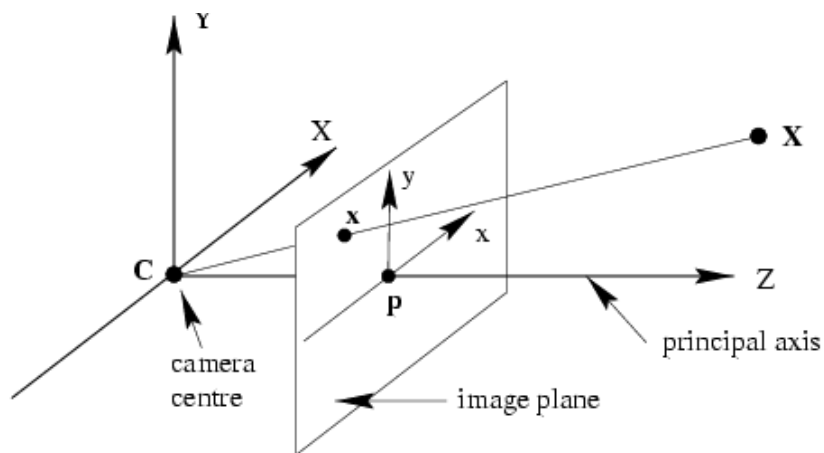
ϕ, φ, ψ

- Translation

t_x, t_y, t_z



Focal Length

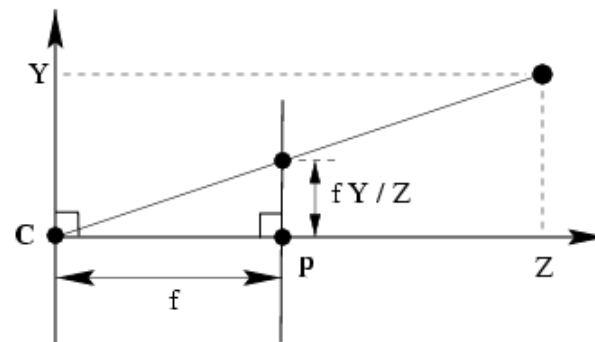
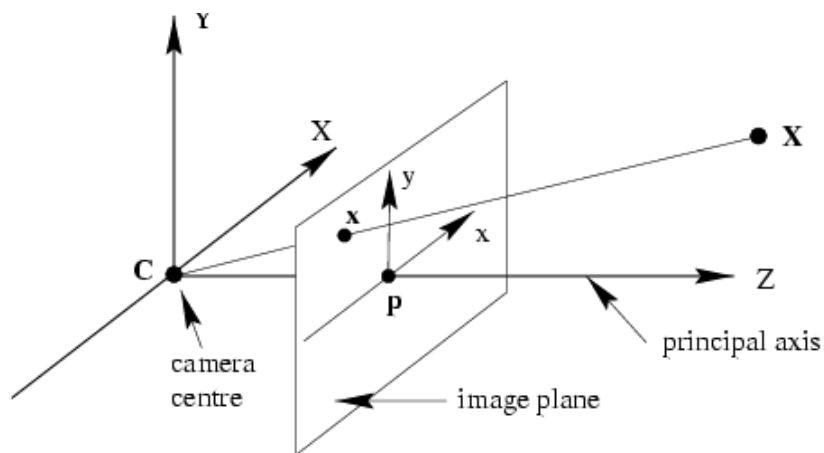


Assume $c = 0$:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} fX/Z \\ fY/Z \end{pmatrix} \quad \leftarrow \quad \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \end{bmatrix}$$



Focal Length

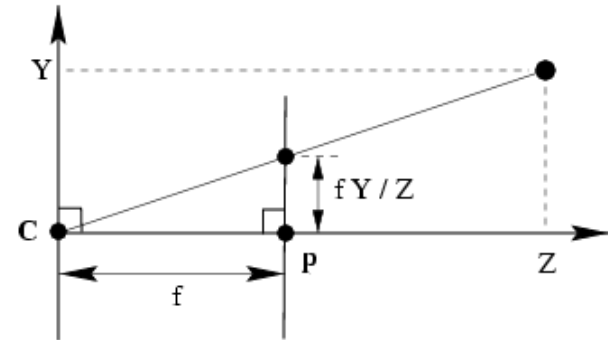
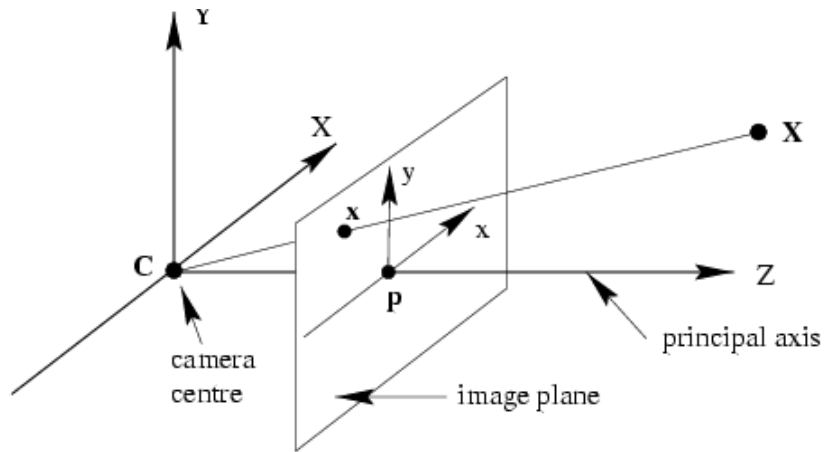


Assume $c \neq 0$:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} fX'/Z' \\ fY'/Z' \end{pmatrix} \leftarrow \begin{pmatrix} fX' \\ fY' \\ Z' \end{pmatrix} = \begin{bmatrix} - \\ - \\ - \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$



Focal Length

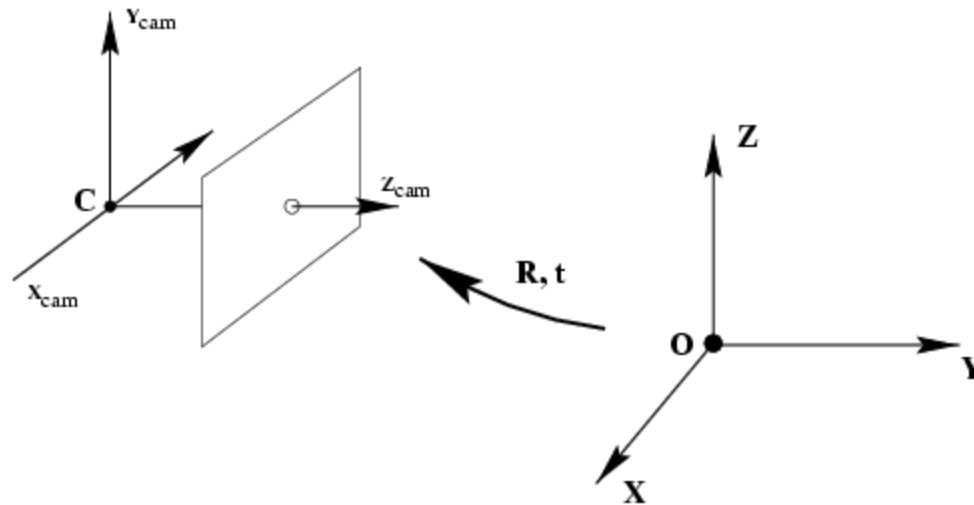


In general, where M is some 4x4 matrix:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} fX'/Z' \\ fY'/Z' \end{pmatrix} \quad \leftarrow \quad \begin{pmatrix} fX' \\ fY' \\ Z' \end{pmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} M_{4 \times 4} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$



World to Camera Matrix M



$$\left. \begin{aligned} \tilde{x}_c &= R(\tilde{X} - C) \\ \tilde{x}_c &= R\tilde{X} - RC \end{aligned} \right\}$$

$\downarrow$
 $-t$

$$\tilde{x}_c = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

M

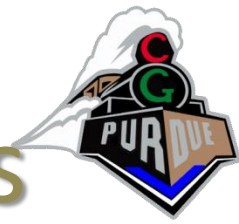
$$R = R_x R_y R_z$$

3x3 rotation matrices

$$t = \begin{bmatrix} t_x & t_y & t_z \end{bmatrix}^T$$

translation vector

Perspective Projection Process



- Thus, given $\tilde{X} = \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$

...the perspective projection is

$$\tilde{x}_p = PM \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \longrightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \tilde{x}_{p_x} / \tilde{x}_{p_z} \\ \tilde{x}_{p_y} / \tilde{x}_{p_z} \end{bmatrix}$$

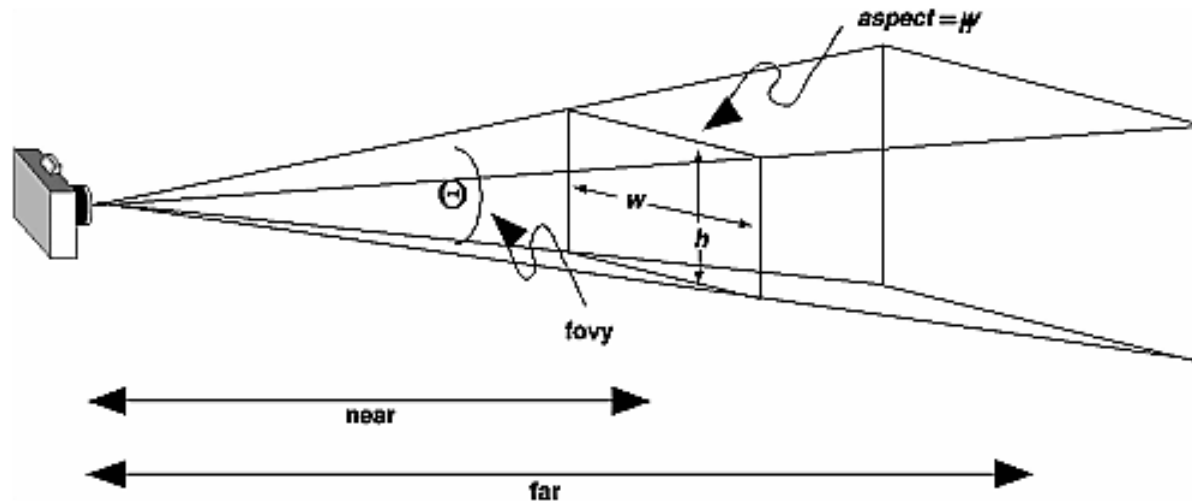


OpenGL Equivalent

```
...  
glMatrixMode(GL_PROJECTION);  
...  
gluPerspective(60, 1.0, 0.1, 1000.0);  
...  
glMatrixMode(GL_MODELVIEW);  
...  
glTranslatef(tx,ty,tz);  
glRotatef(rx,1,0,0);  
glRotatef(ry,0,1,0);  
glRotatef(rz,0,0,1);  
  
/* or glLoadMatrixf(mat); */  
...
```



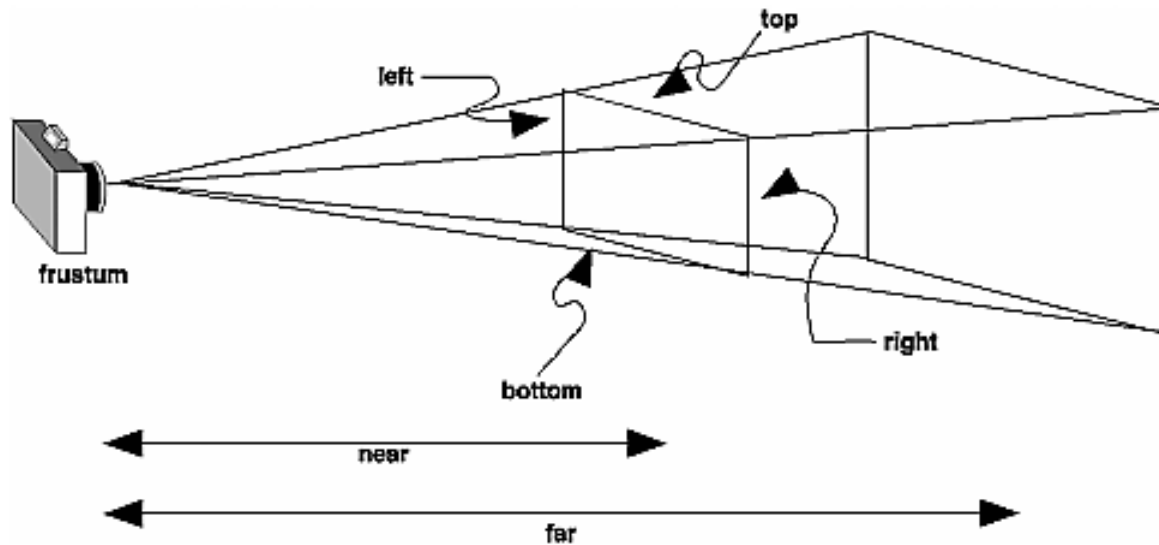
Projection Transformations



```
void gluPerspective(GLdouble fovy, GLdouble aspect, GLdouble  
near, GLdouble far);
```



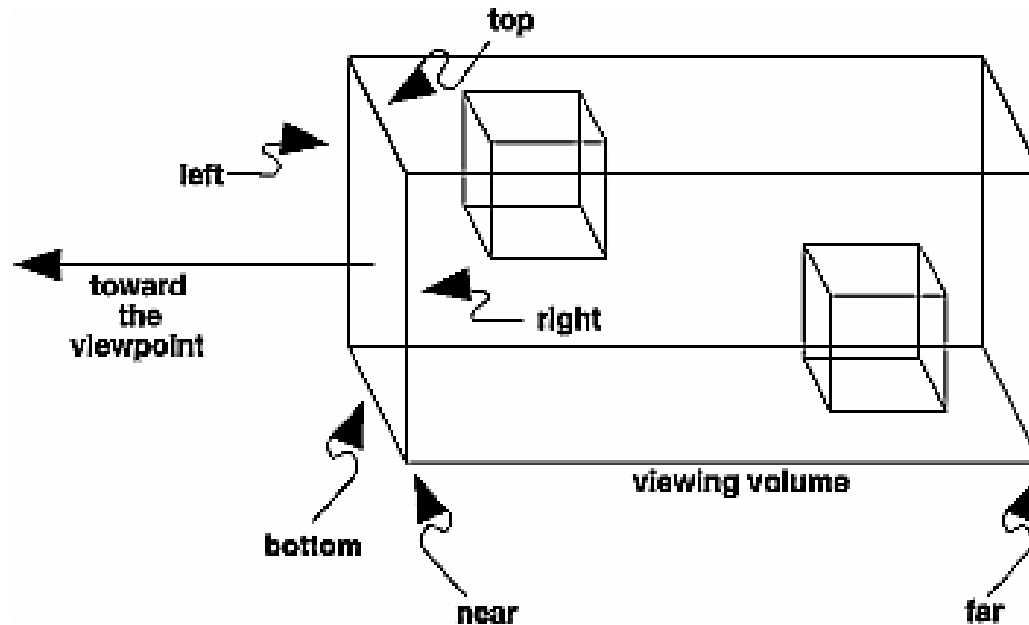
Projection Transformations



```
void glFrustum(GLdouble left, GLdouble right, GLdouble  
    bottom, GLdouble top, GLdouble near, GLdouble far);
```



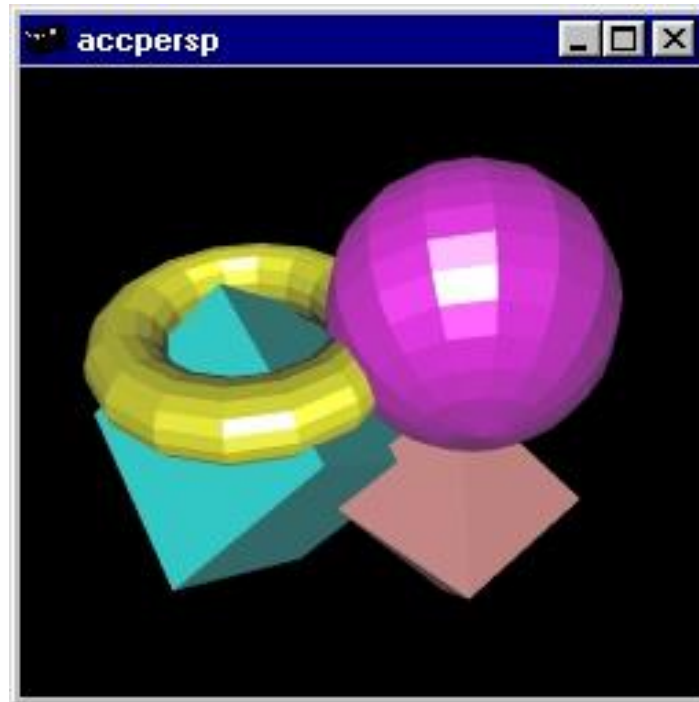

Projection Transformations



```
void glOrtho(GLdouble left, GLdouble right, GLdouble  
    bottom,  
    GLdouble top, GLdouble near, GLdouble far);
```

```
void gluOrtho2D(GLdouble left, GLdouble right,  
    GLdouble bottom, GLdouble top);
```

Example OpenGL Rendering



- Perspective is “mathematically” accurate...



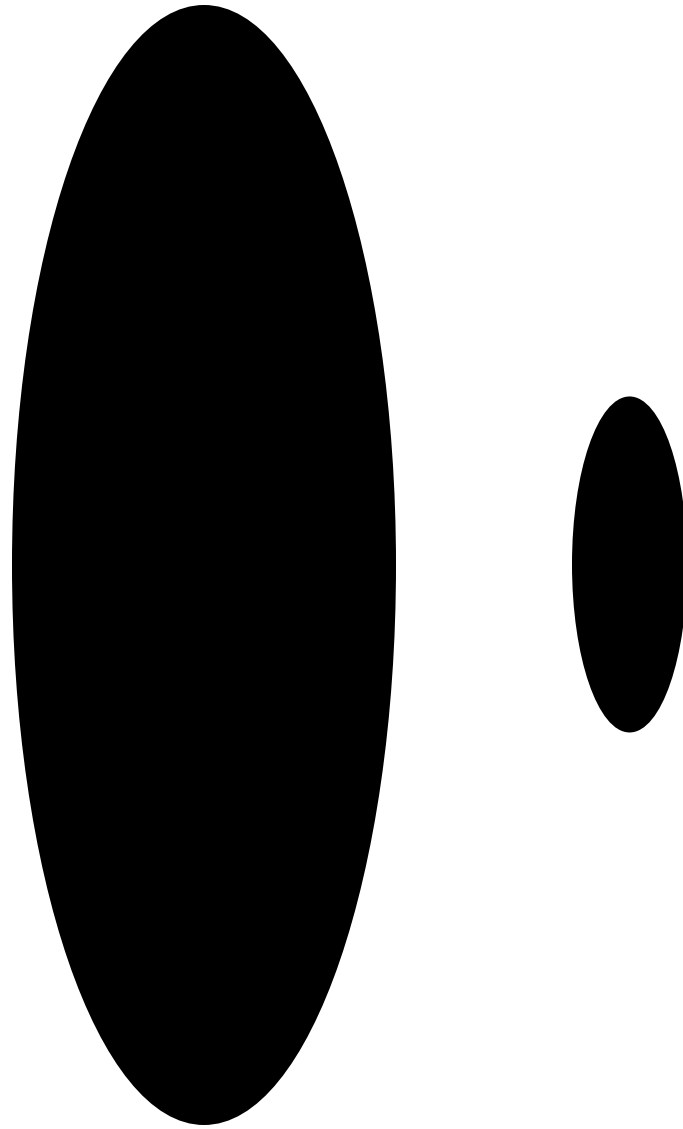
(3D) Perception

- There is lot more to it...



Perception

- Size-distance relationship
 - Same size object at two distances...

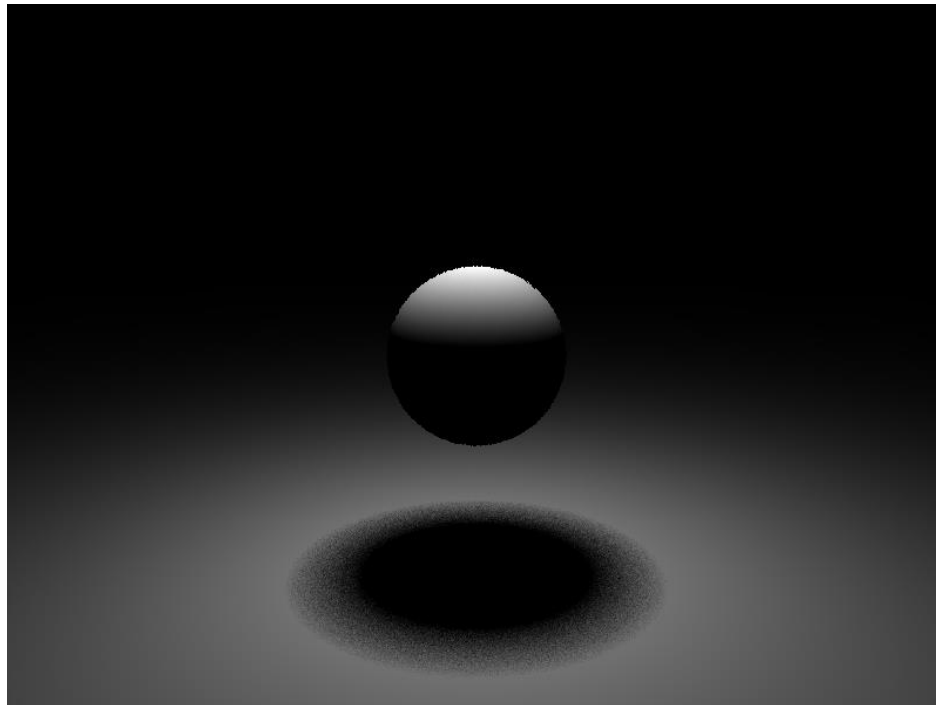






Perception

- Importance of ground plane, shadows...



Perception

- Alto Relief

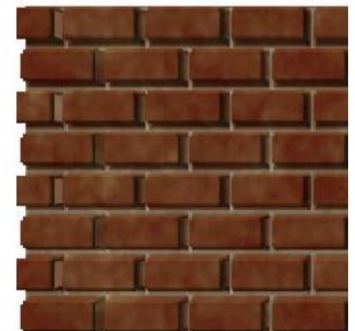
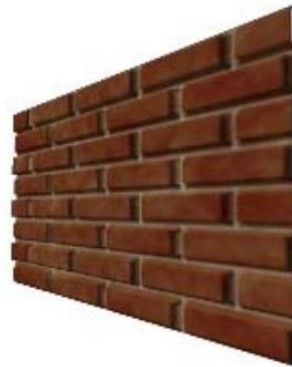
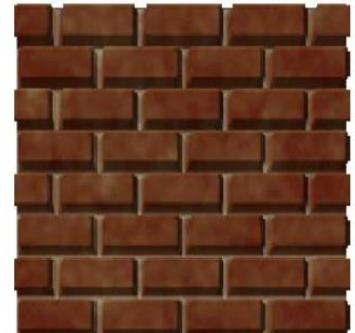
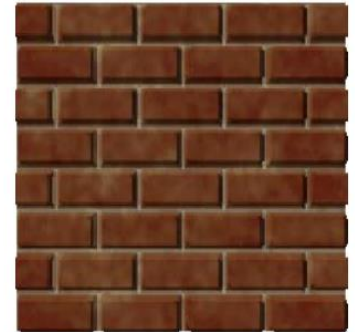
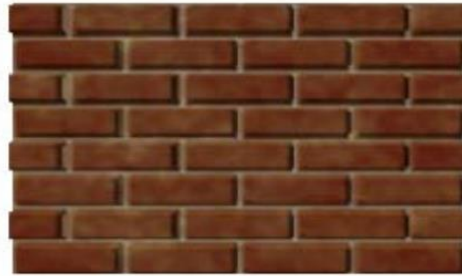


- Sunken Relief



(Relief Images)

- We will see this type of things later...



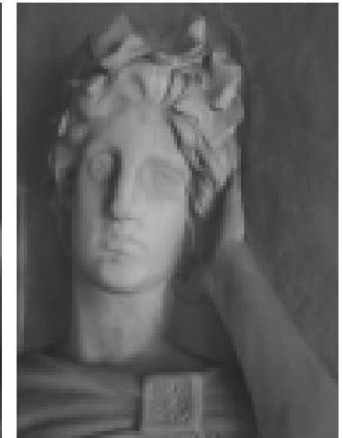


Perception: Bas-Relief





Perception: Bas-Relief



Perception: Bas Relief Ambiguity

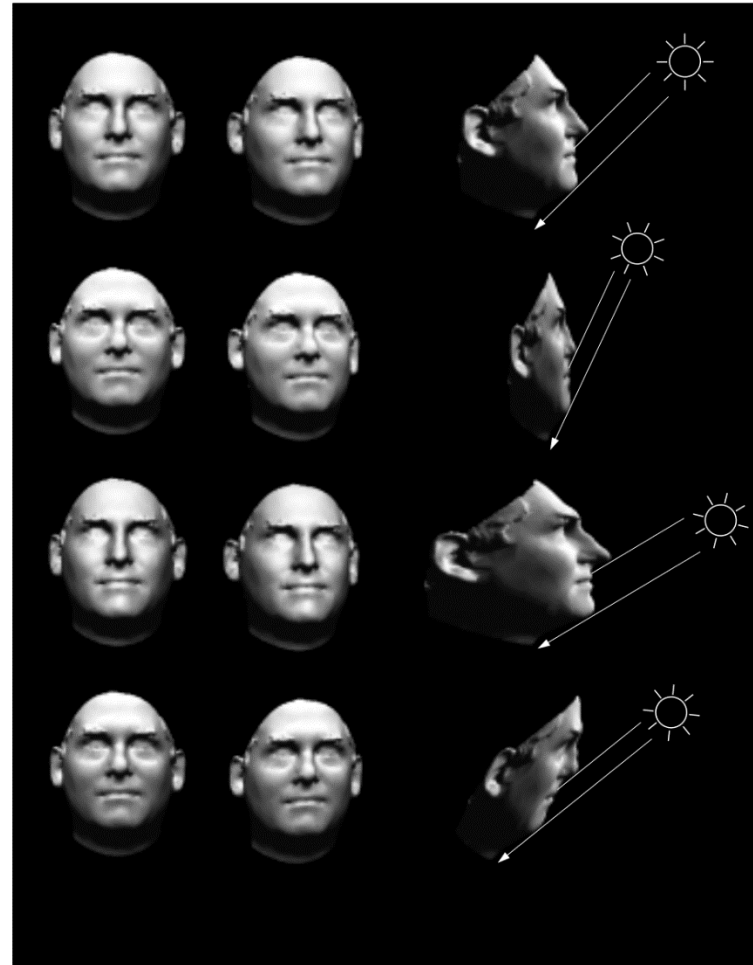
$$NL^T = C$$

or

$$NRR^T L^T = C$$

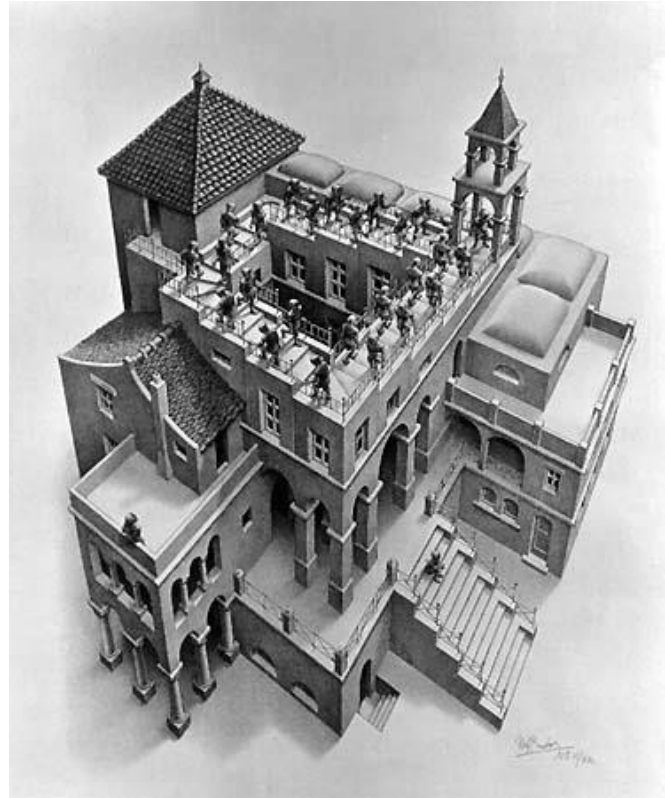
$$NGG^{-1} L^T = C$$

??



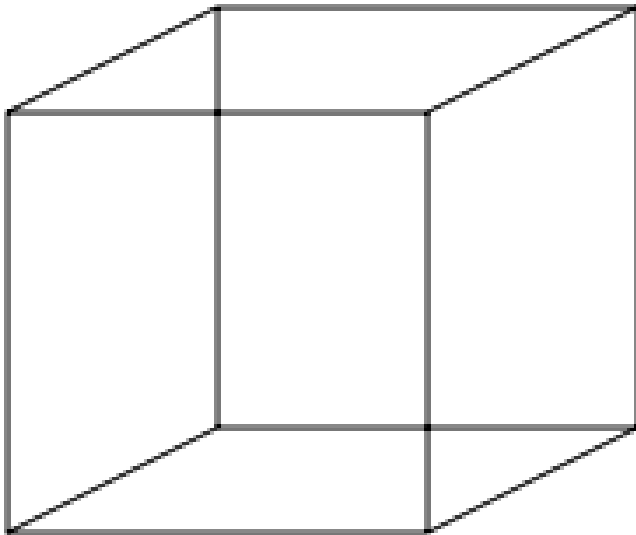
Perception: Inconsistencies

- Escher



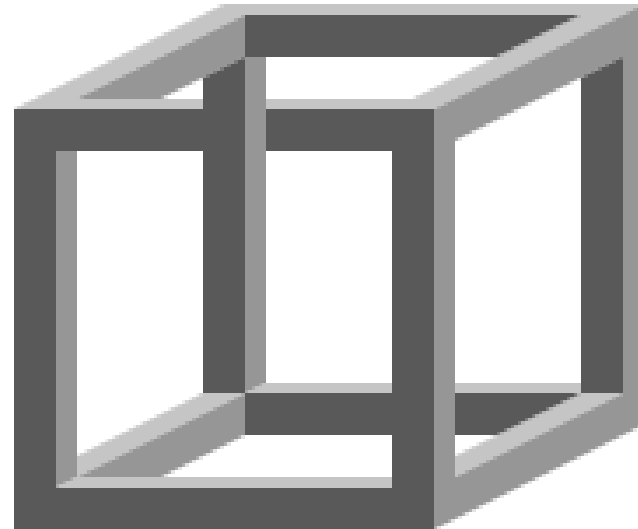
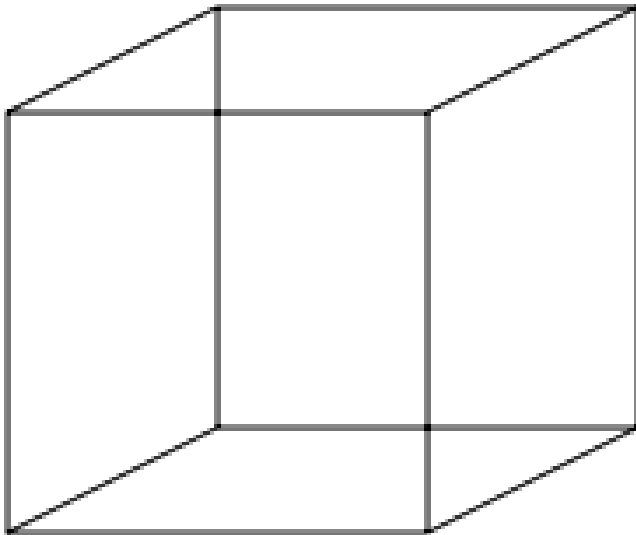


Perception: Nekker Cube

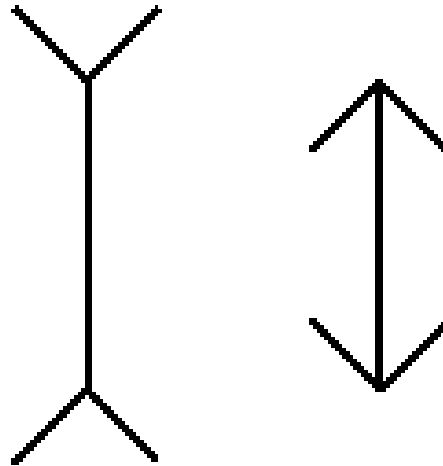




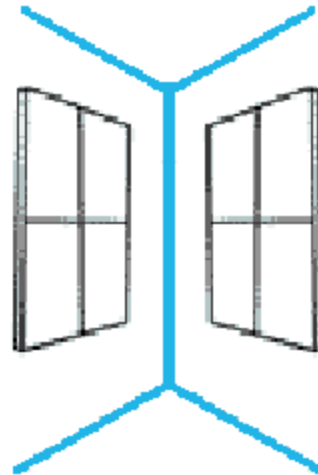
Perception: Nekker Cube



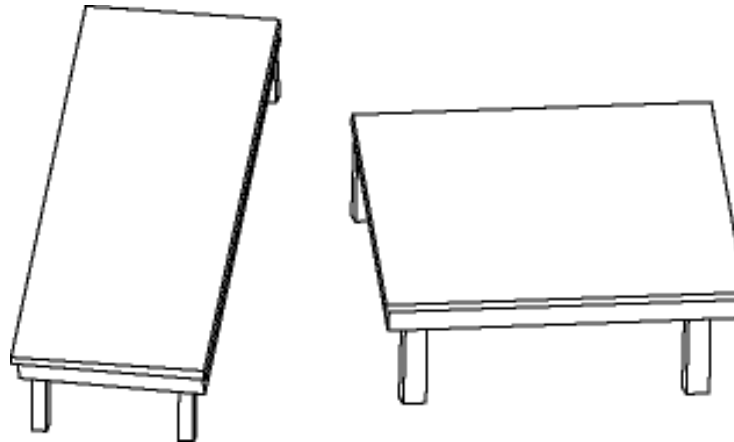
Perception: Muller-Lyer Illusion



Perception: Muller-Lyer Illusion



Perception: Muller-Lyer Illusion





Back to Projection Matrices...

- Basic Perspective Projection:

$$\begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Basic Orthographic:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Projection Matrix

- Problem (maybe?):
 - The formulation give does not pass z, or depth, though the transformation pipeline
 - What can we do?

IDEAS????



Options

- Trivial changes?
- Funky matrices...
- Re-mapping options...



Projection Matrices

- Passthrough Z Perspective Projection:

$$\begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n + f & -nf \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\text{If } (Z = n), \text{ then } z = \left(n + f - \frac{nf}{n} \right) = n$$

$$\text{If } (Z = f), \text{ then } z = \left(n + f - \frac{nf}{f} \right) = f$$



Projection Matrices

- To map $[-1,+1]$ to viewing frustum (l,r,b,t,n,f) :

$$\begin{bmatrix} 2n/(r-l) & 0 & 0 & -(r+l)/(r-l) \\ 0 & 2n/(t-b) & 0 & -(t+b)/(t-b) \\ 0 & 0 & 2nf/(n-f) & -(n+f)/(n-f) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$