Spatial Data Structures and Hierarchies

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Spatial Data Structures

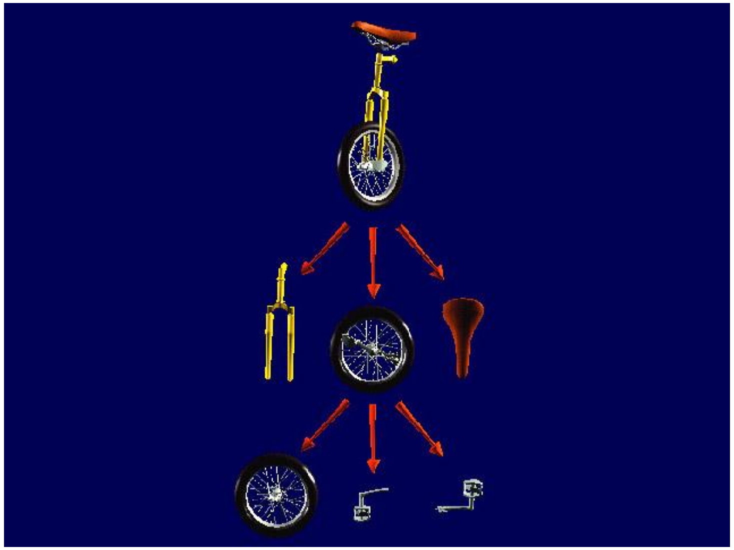
- Store geometric information
- Organize geometric information
- Permit fast access to/of geometric information
- Applications
  – Heightfields
  – Collision detection (core to *many* uses)
  – Simulations (e.g., surgery, games)
  – Rendering (e.g., need to render fast!)

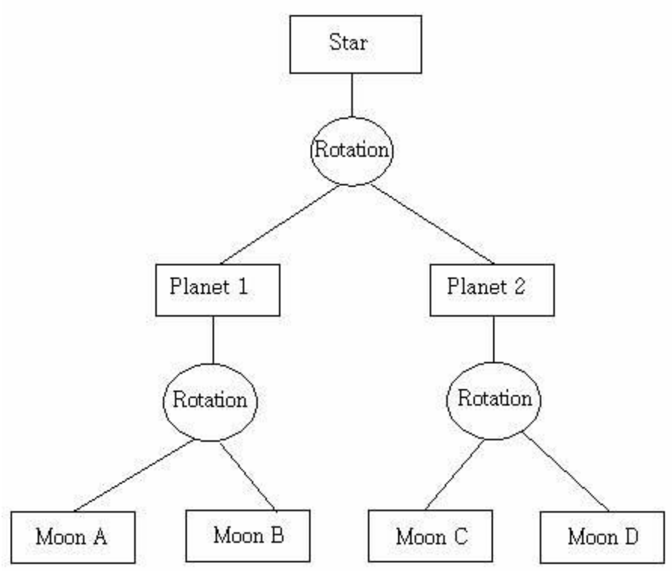
Hierarchical Modeling

- Concept is old but fundamental
  – "Hierarchical geometric models for visible surface algorithms", James Clark - 1976

Hierarchical Modeling

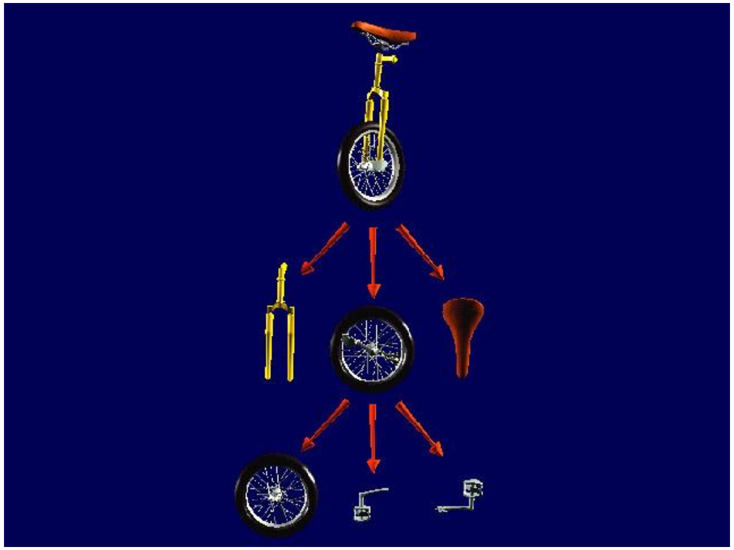
- Trees and Scene Graphs

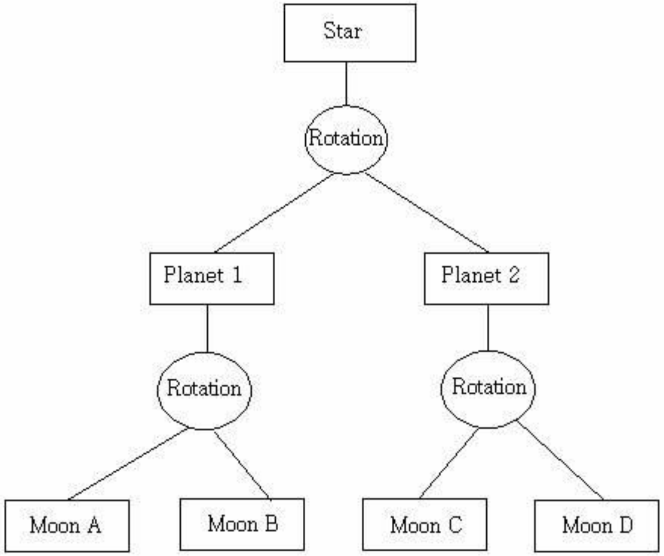




Hierarchical Modeling

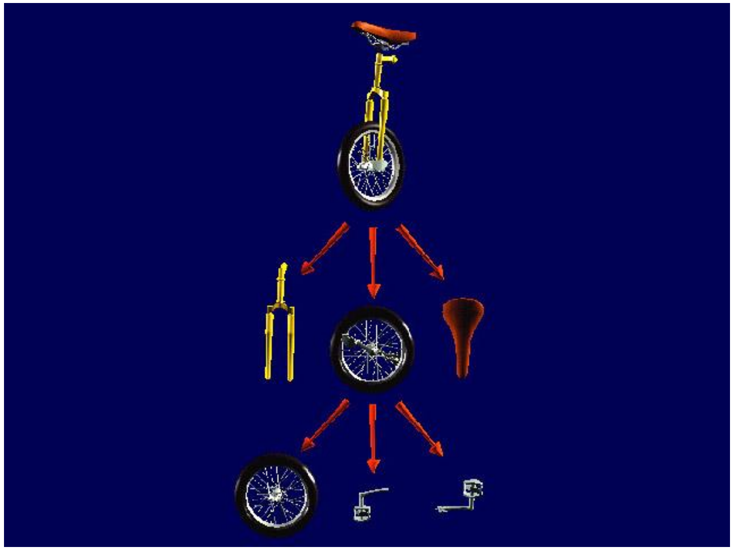
- Trees and Scene Graphs

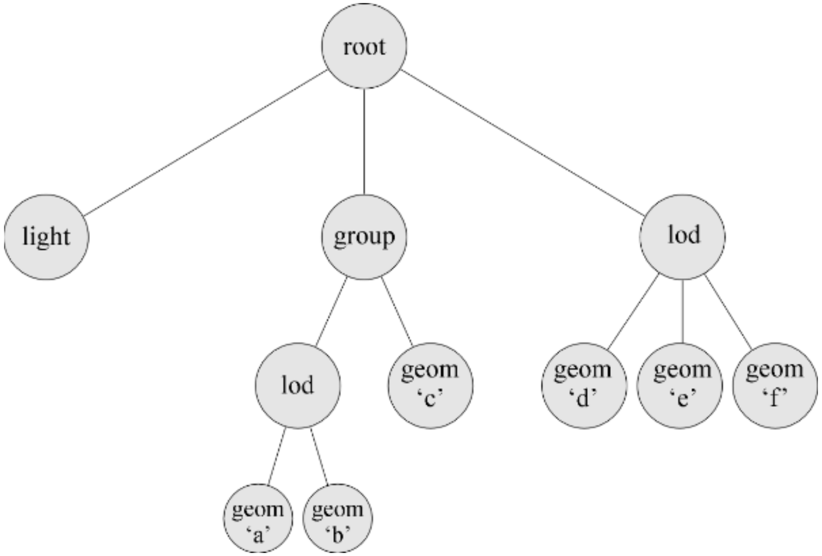




Hierarchical Modeling

- Trees and Scene Graphs



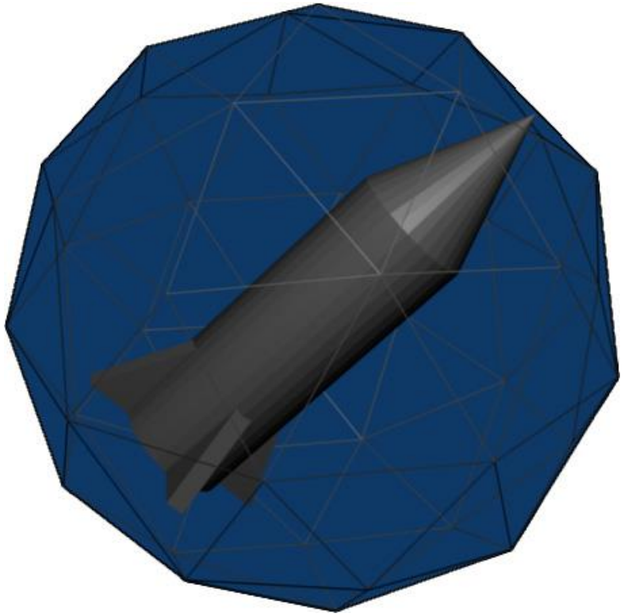


Bounding Volumes

- Problem:
  - Suppose you need to intersect rays with a scene…
  - Suppose you have a scene divided into objects…
- Solution: bottom-up
  - Wrap complex objects into simple ones
    - Boxes, spheres, other shapes…
  - Organize into a tree

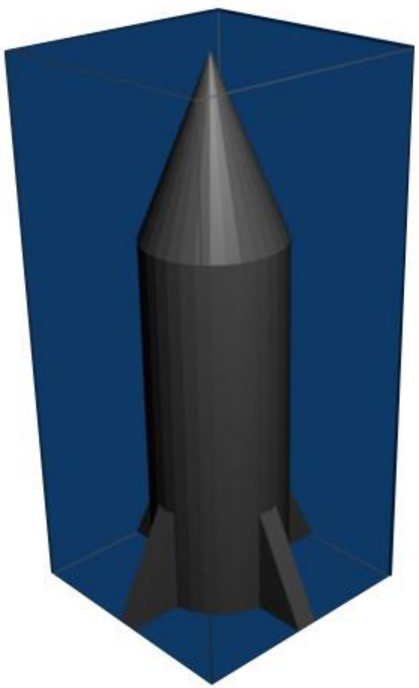
Bounding Sphere

- Simplest way to bound an object
- Good for small or round objects

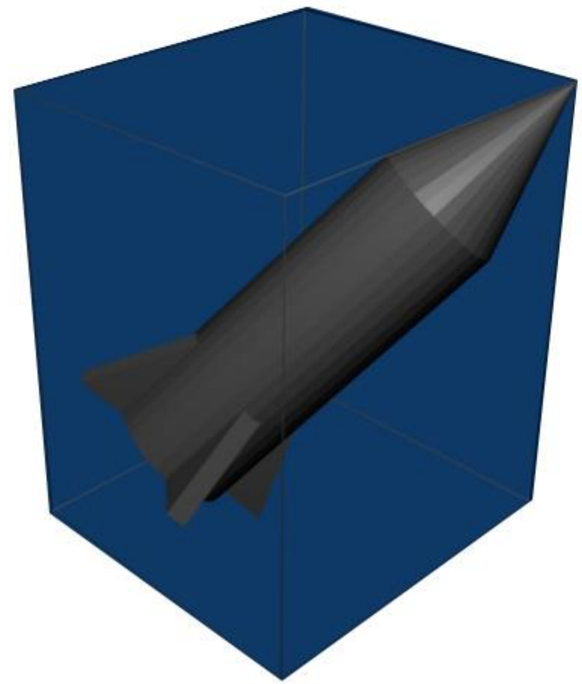


Bounding Boxes

- Axis Aligned Vs Orientated



Orientated
More Expensive



Axis Aligned
Cheaper

Bounding Volume Hierarchy (BVH)

- How to building an axis aligned bounding box (AABB) BVH?
- How to intersect?
- Complexity? Problem cases?

AABB BVH

- Example construction
  - Given M 2D points, use k-means clustering to determine clusters
  - Then group nearby clusters (e.g., use Voronoi diagram or Delaunay triangulation)
  - And iteratively form a tree from the bottom-up
  - In each node, approximate the contained points using an axis-aligned bounding box
    - e.g., box = [min of all contained pts, max of all contained pts]

Bounding Volume Hierarchy (BVH)

- How to building an oriented bounding box (OBB) BVH?
- How to intersect?
- Complexity? Problem cases? Advantages over axis-aligned?

OBB BVH

- Example construction
  - Similar to AABB BVH but “fit” an oriented box to the points within each cluster/node of the tree
  - Methods:
    - Sample possible rotations and sizes in order to pick the best box
    - Compute distance of points to a line and optimize the line equation parameters until finding the line that best approximates all points
    - Then compute a box width – consider the benefit/cost of the box size
      - e.g., totally containing all points might make the box very large; could also choose to mostly contain the points – however, what does this mean with regards to operations using the BVH?

An Application of BVH: Collision Detection

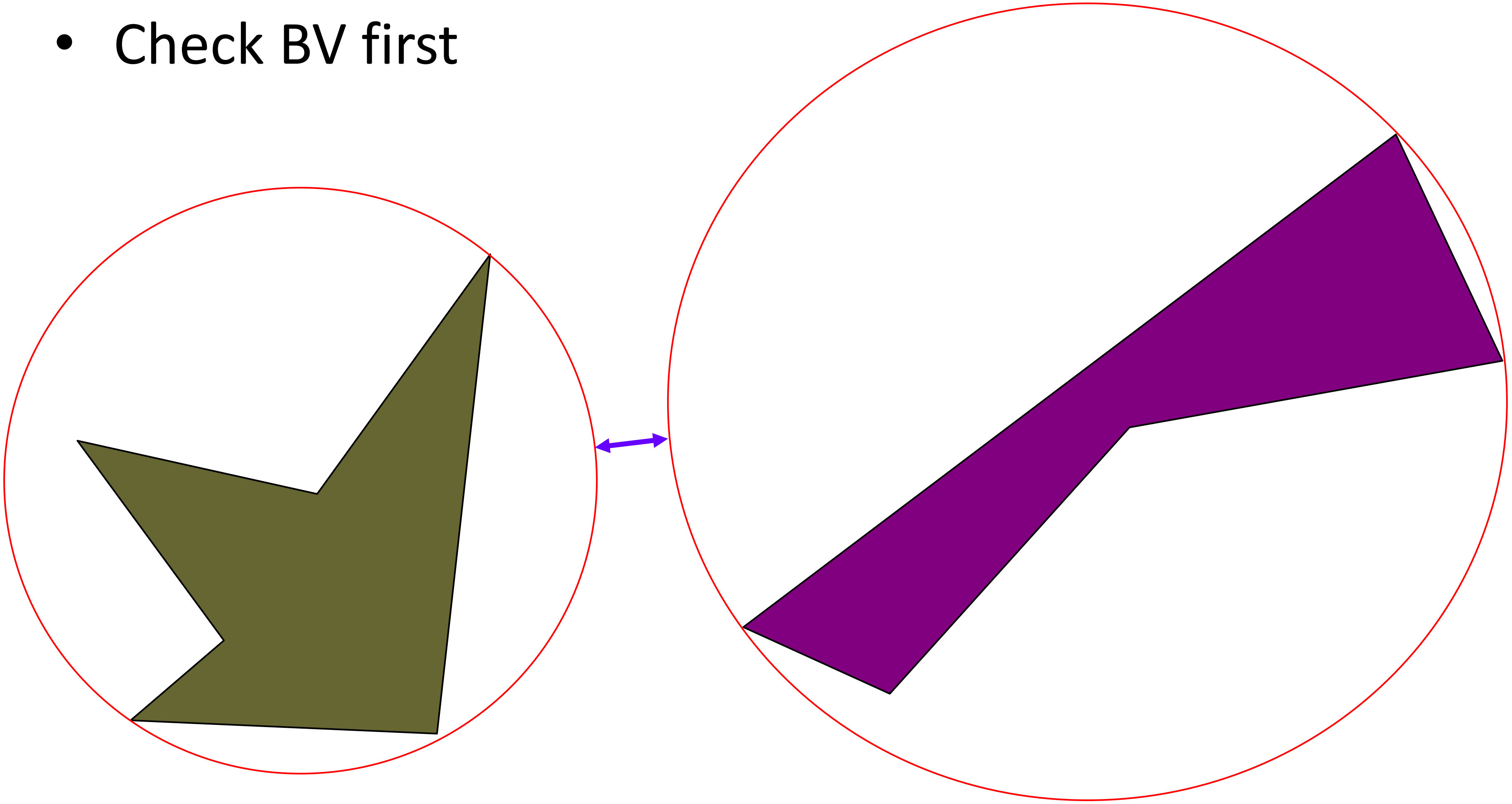
- Turn complex objects into bounded volumes for collision testing
- Fast, but not reliable
- Great initial test, but should be followed by another more precise test

An Application of BVH: Collision Detection

- Intersect these!

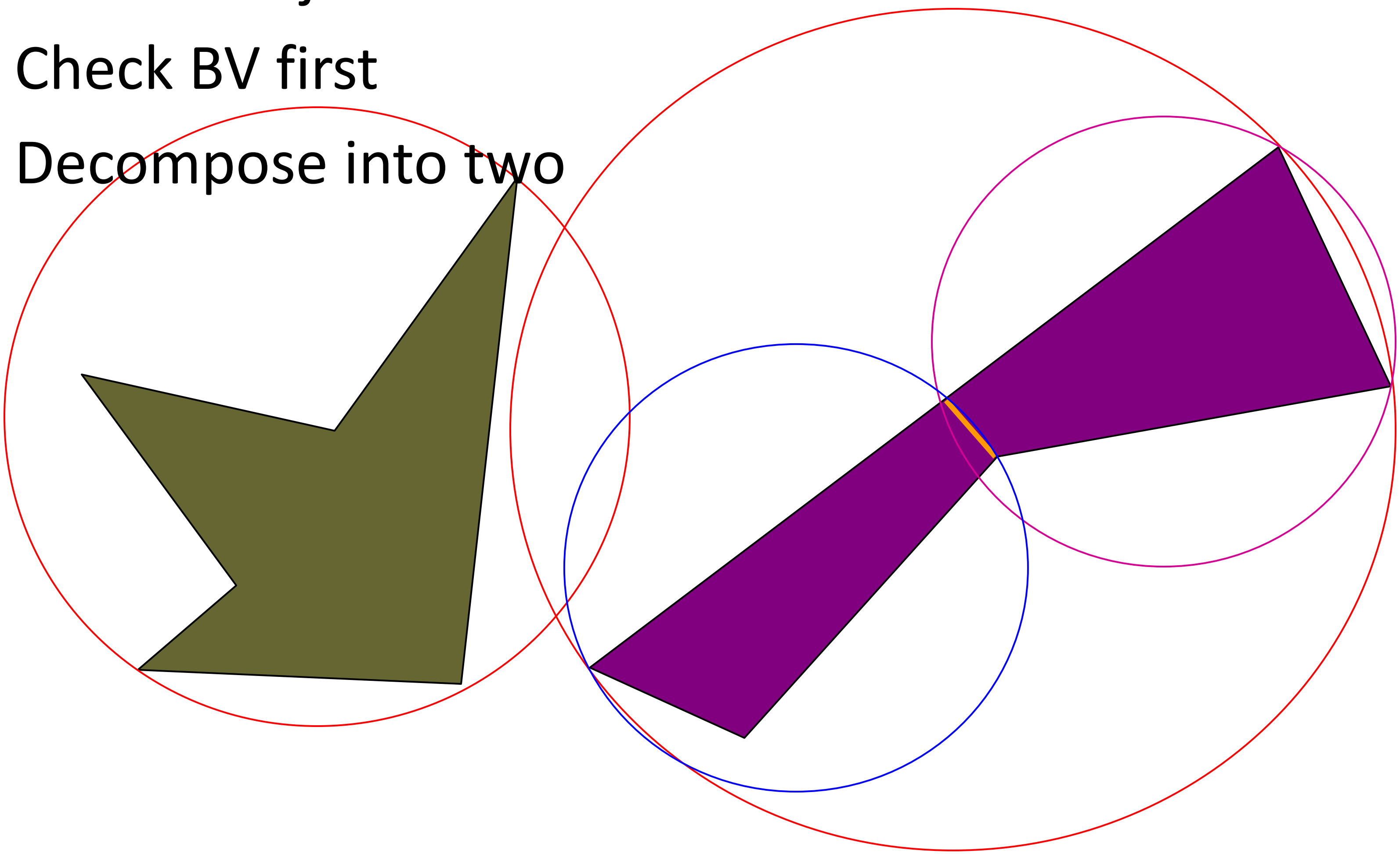
Bounding Volume Hierarchy

- Enclose objects into BVs
- Check BV first



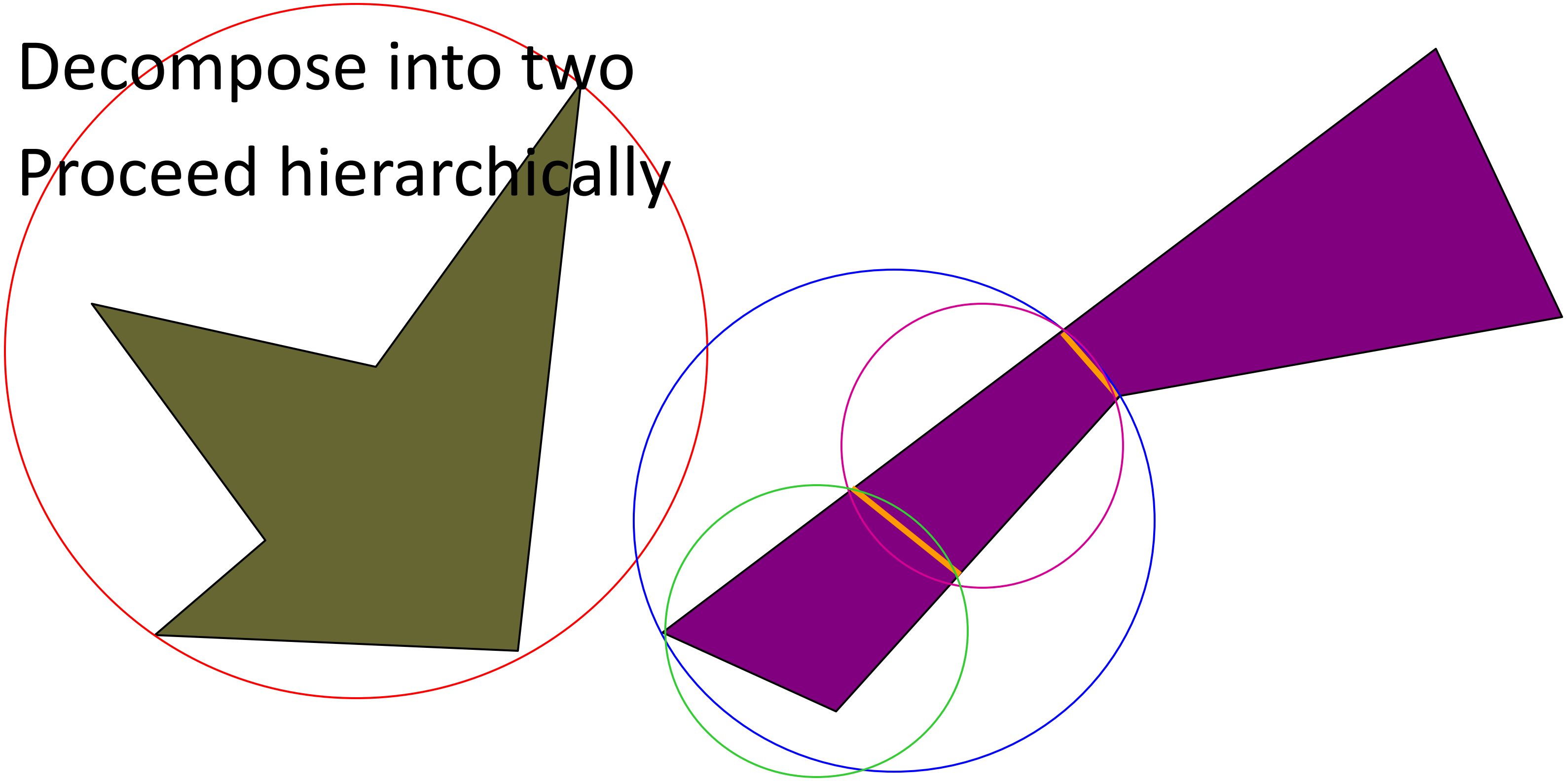
Bounding Volume Hierarchy

- Enclose objects into BVs
- Check BV first
- Decompose into two



Bounding Volume Hierarchy

- Enclose objects into BVs
- Check BV first
- Decompose into two
- Proceed hierarchically



Bounding Volume Hierarchy

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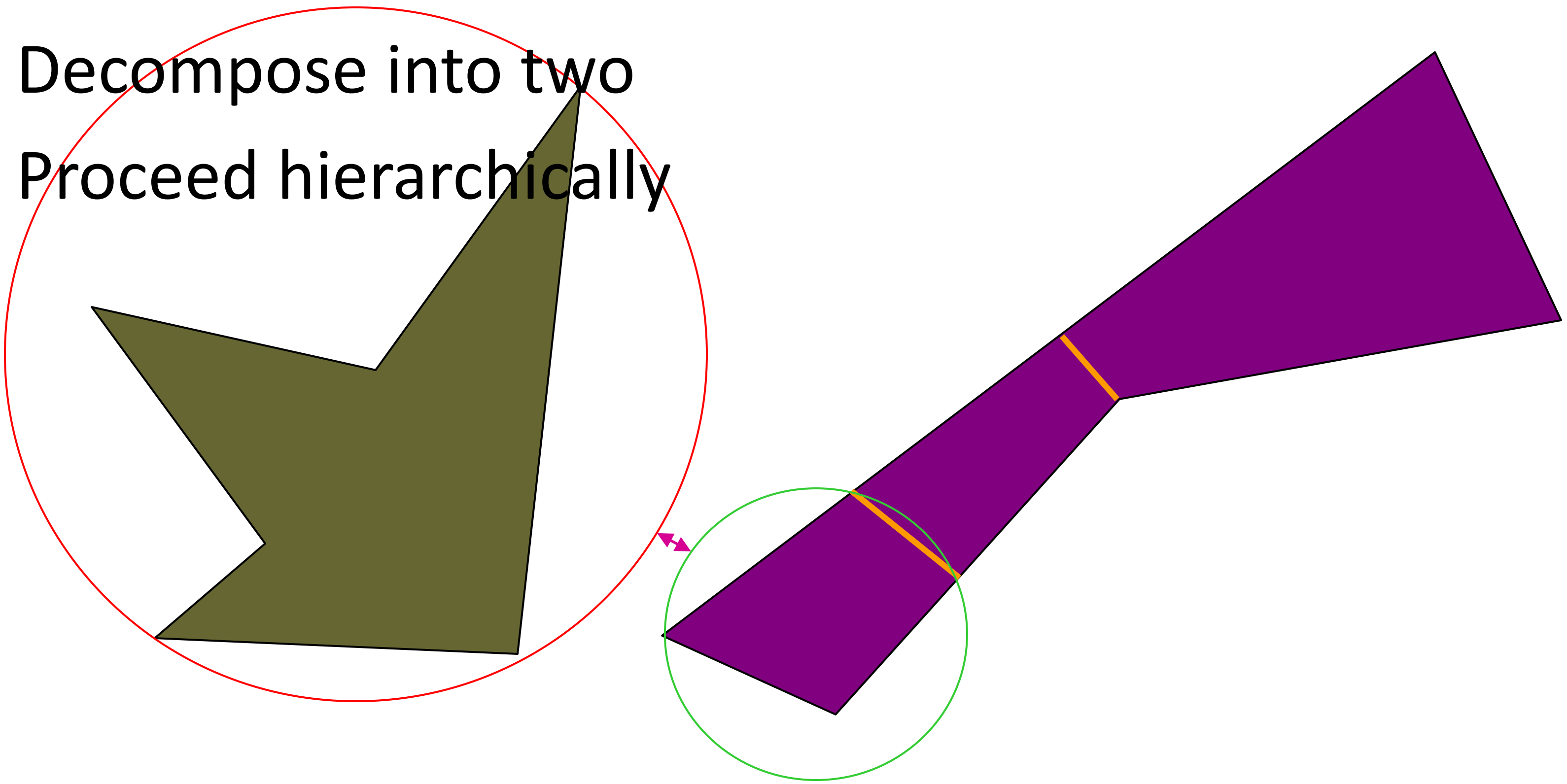
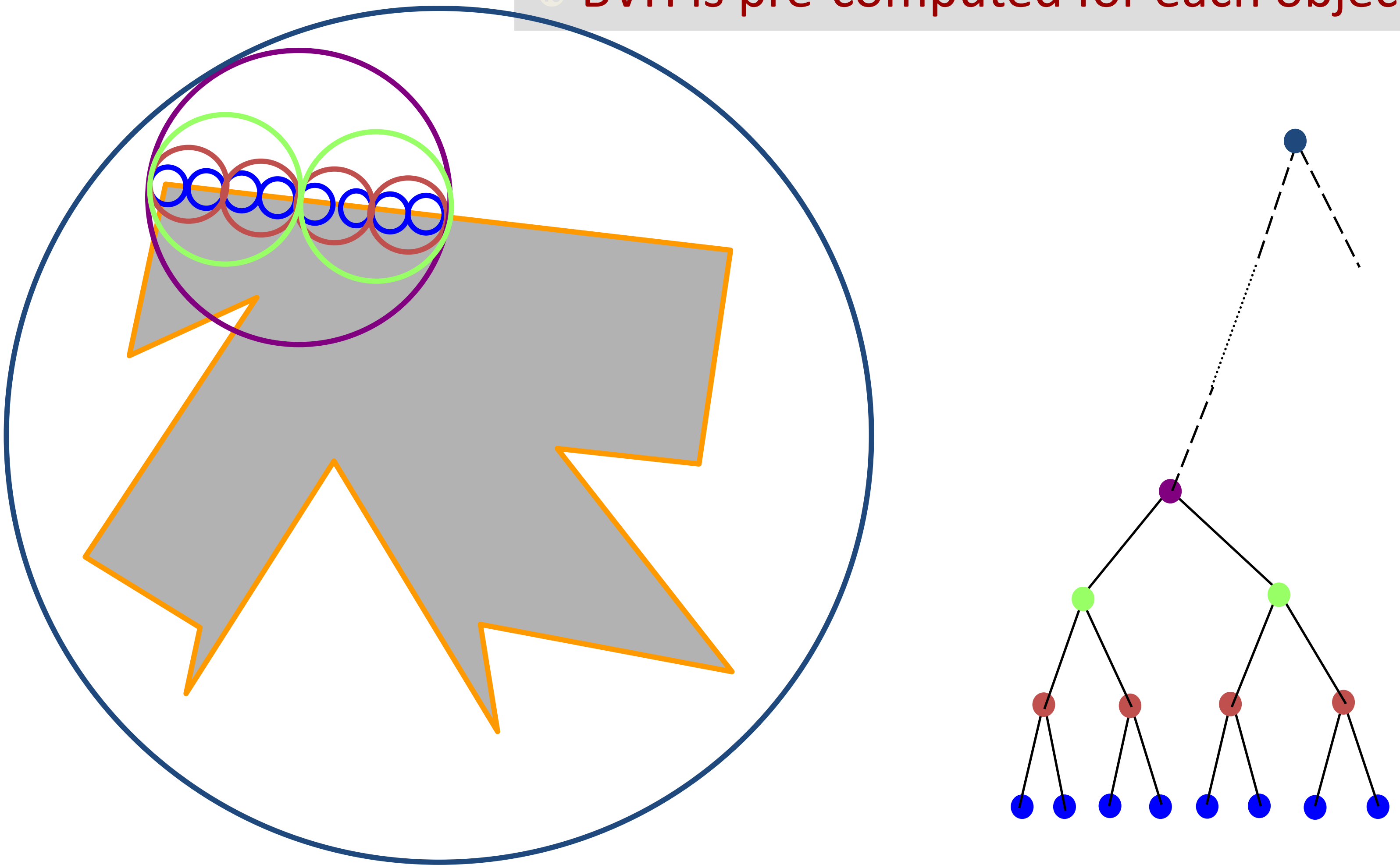


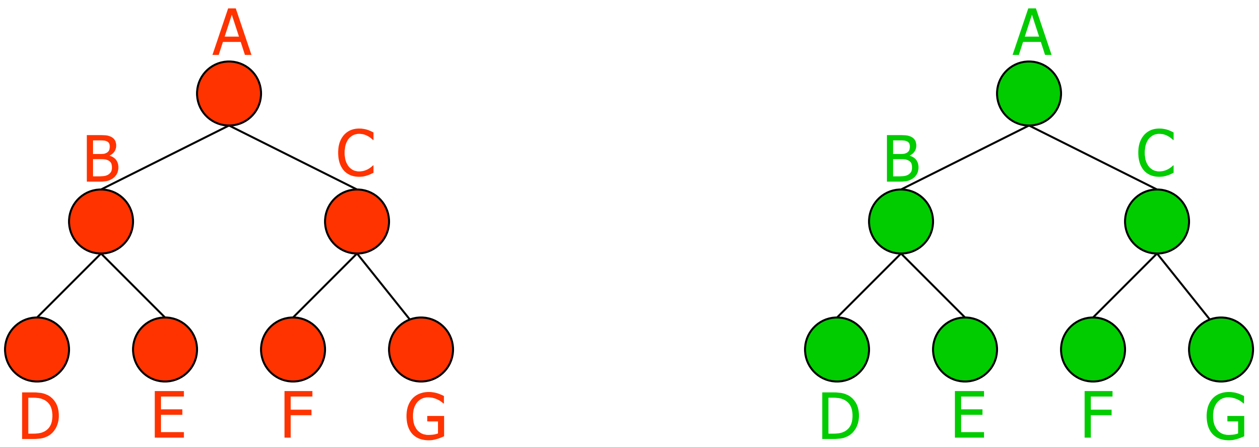
Bounding Volume Hierarchy

- BVH is pre-computed for each object



Bounding Volume Hierarchy in 3D

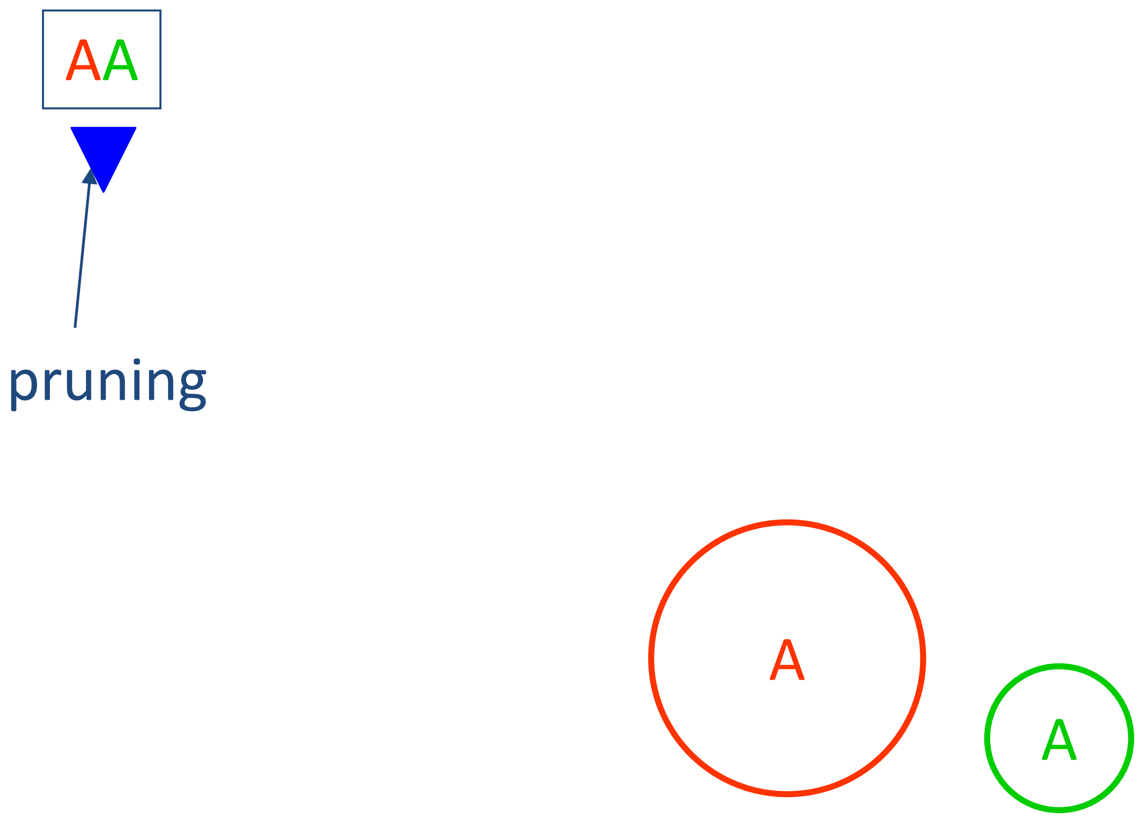
Collision Detection



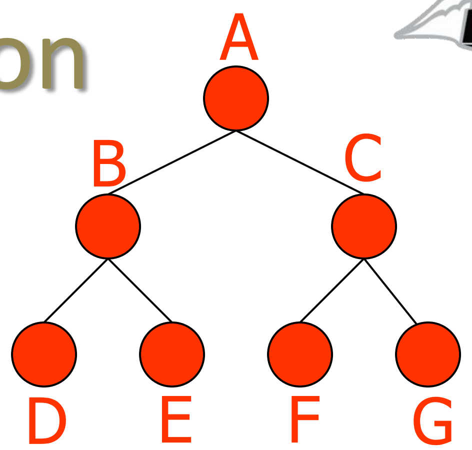
Two objects described by their precomputed BVHs

Collision Detection

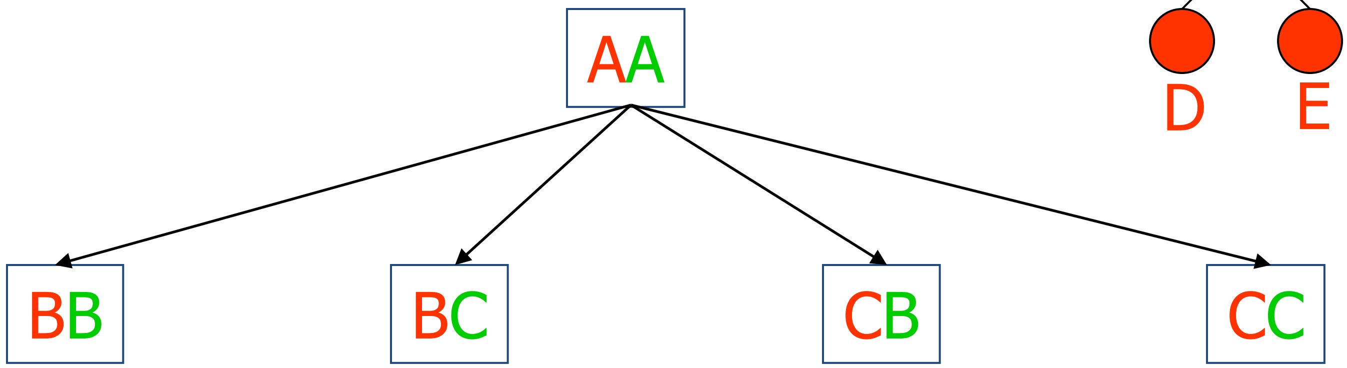
Search tree

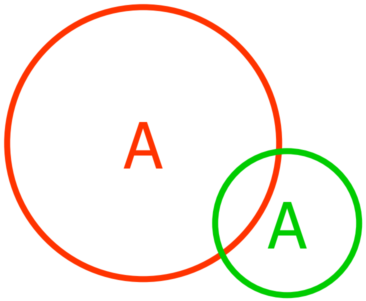


Collision Detection

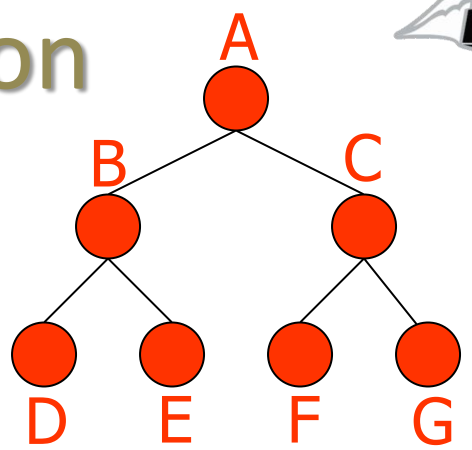


Search tree

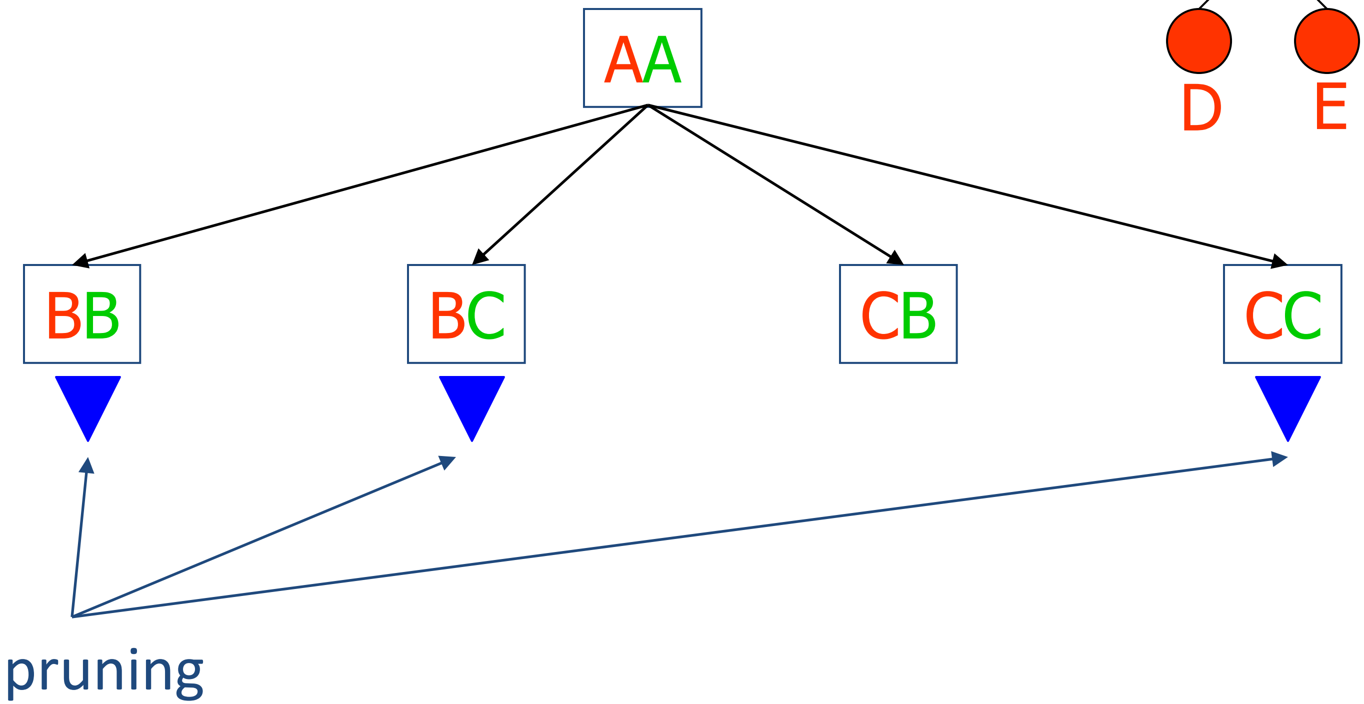




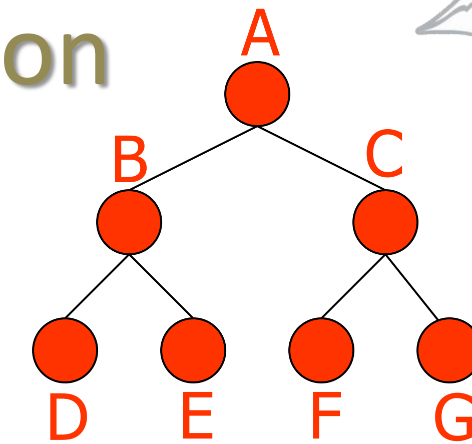
Collision Detection



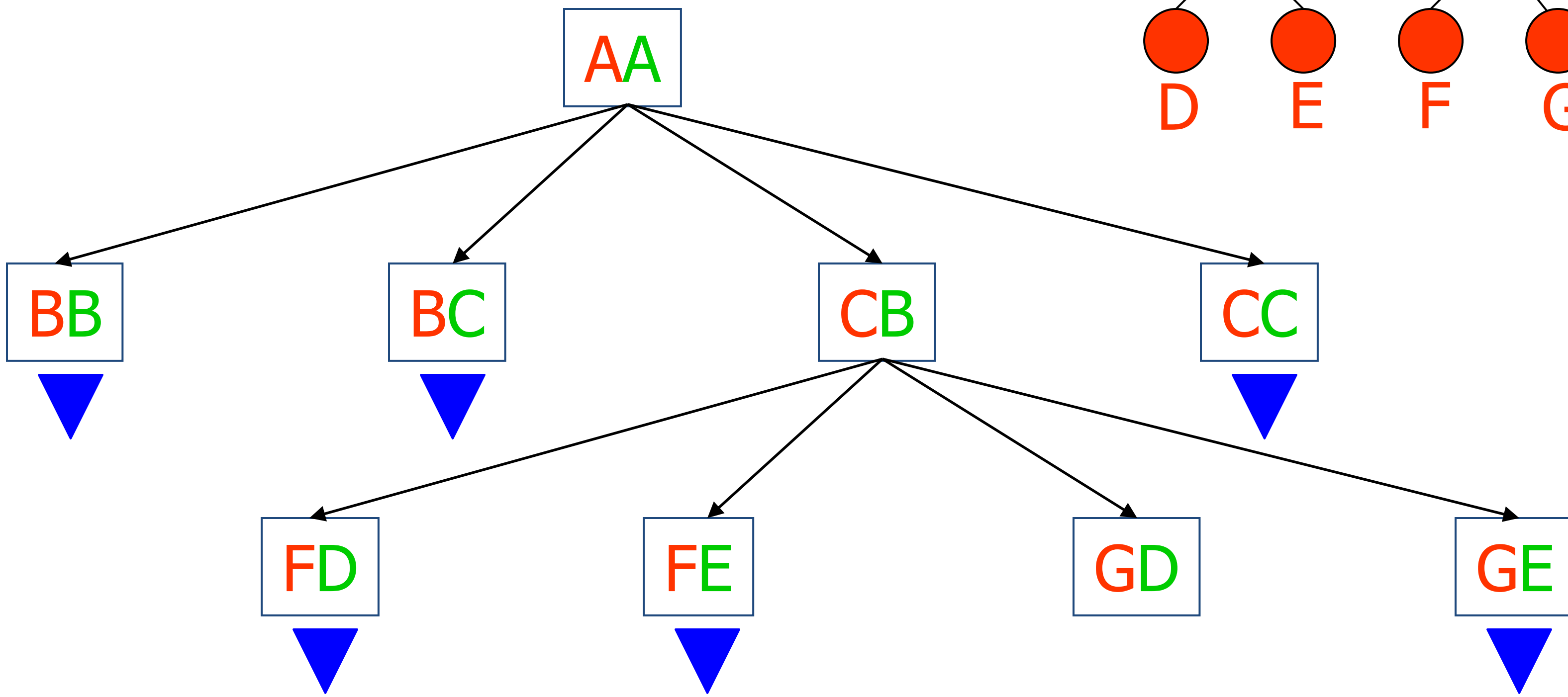
Search tree



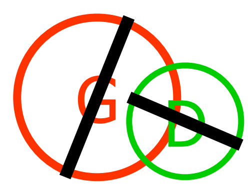
Collision Detection



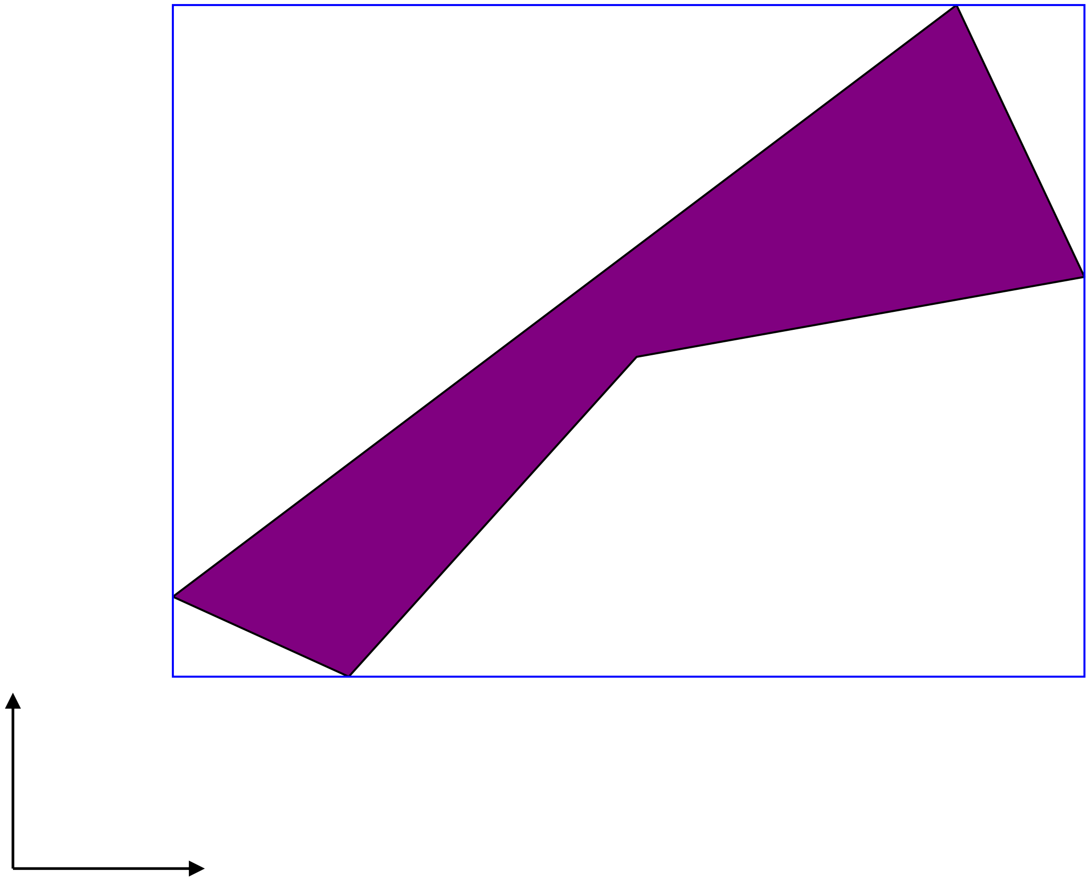
Search tree



If the pieces contained in G and D overlap → collision

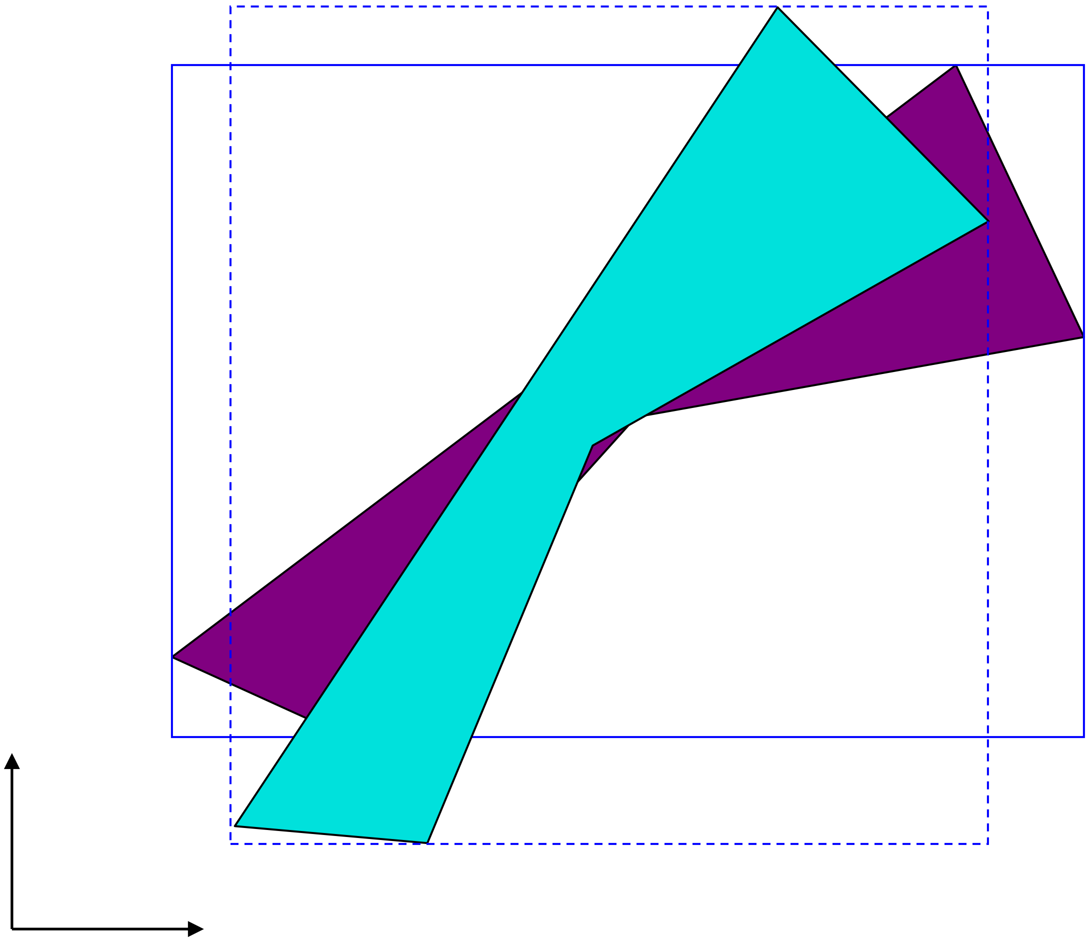


AABB

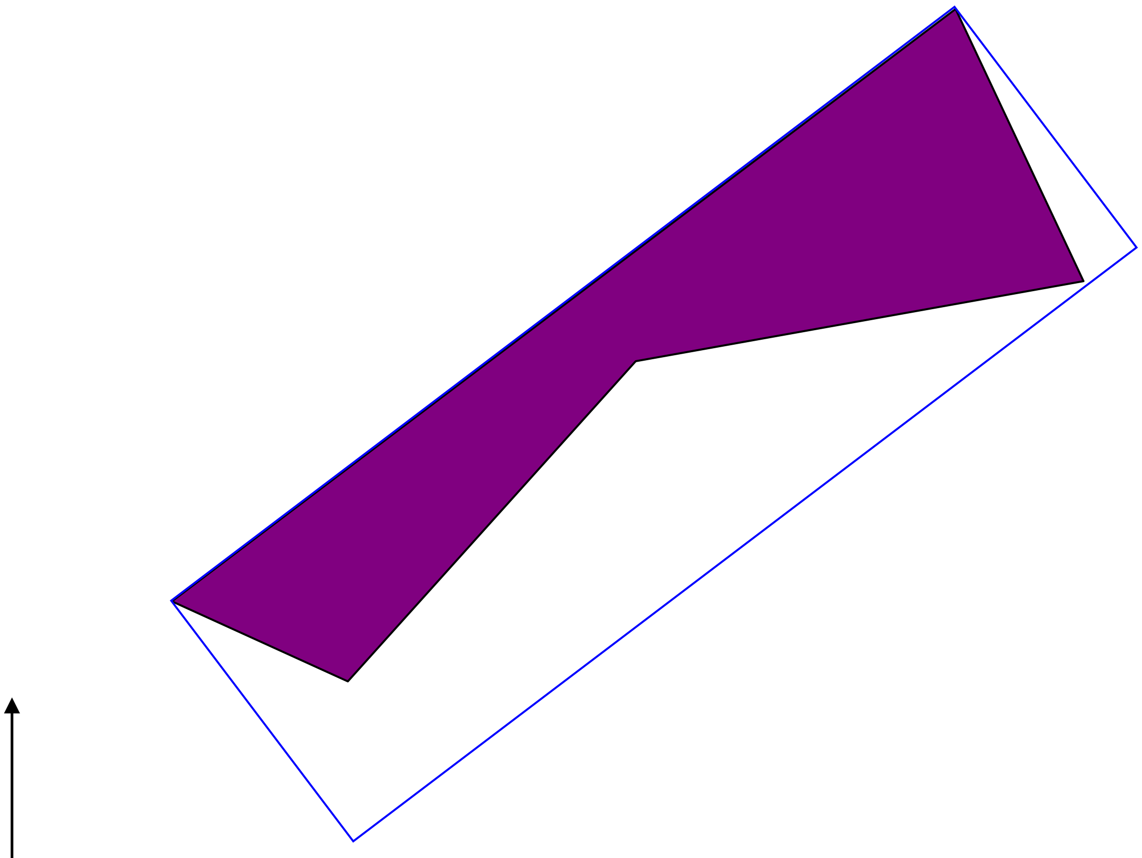


AABB

- Not invariant
- Efficient to test
- Not tight

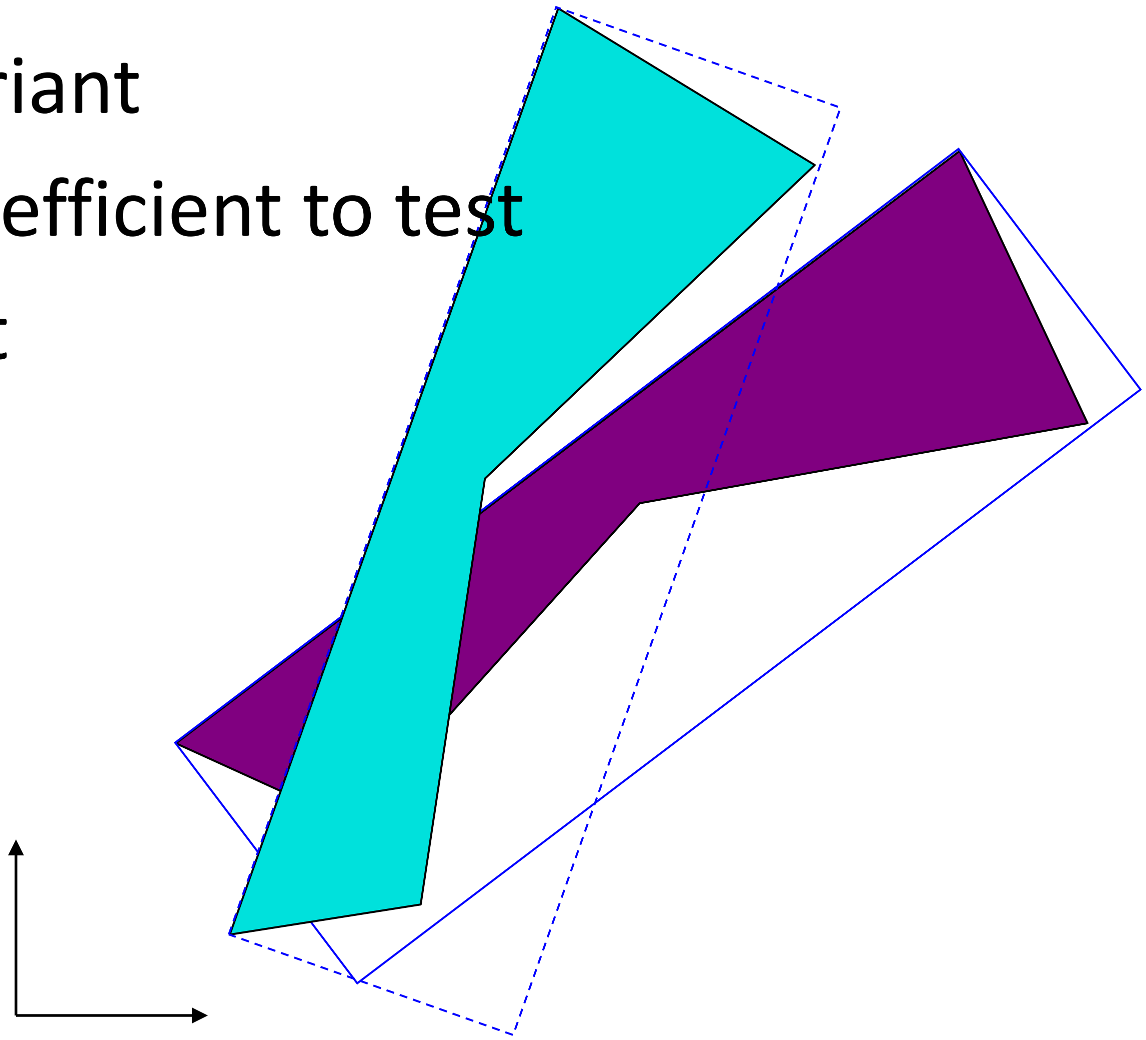


OBB



OBB

- Invariant
- Less efficient to test
- Tight



Comparison

	Sphere	AABB	OBB
Tightness	-	--	+
Testing	+	+	o
Invariance	yes	no	yes

No type of BV is optimal for all situations

BVH Exercises

- See board…

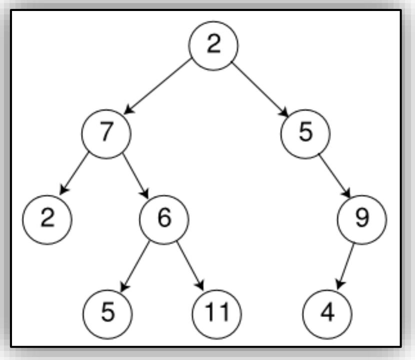
Space Subdivision

- Binary tree / Quadtree / Octree
- k-D tree
- Binary Space Partitioning (BSP) Tree

Binary Tree

- A directed edge refers to the link from the parent to the child (the arrows in the picture of the tree).
- The root node of a tree is the node with no parents; there is at most one root node in a rooted tree.
- A leaf is a node that has no children.
- The depth of a node is the length of the path from the root to the node. The root node is at depth zero.
- The height of a tree is the depth of its furthest leaf. A tree with only a root node has a height of zero.
- Siblings are nodes that share the same parent node

Size = 9
Height = 3
Root node = 2

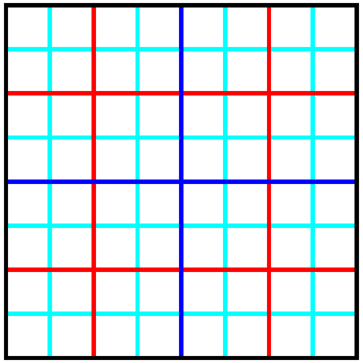


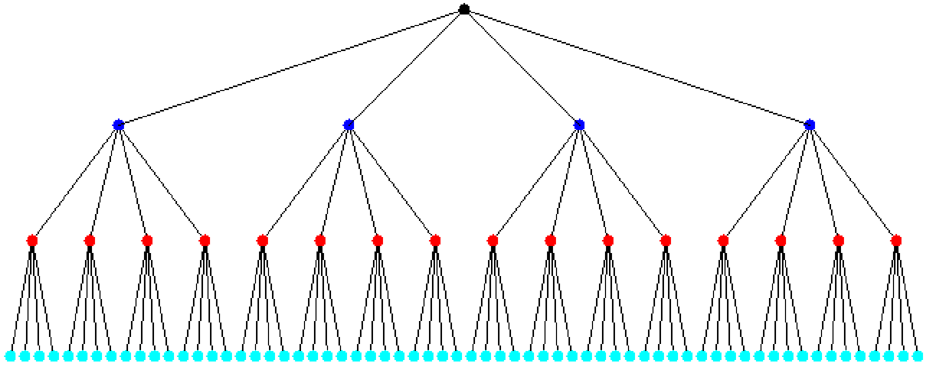
Binary Tree

- Operations
  - Search
  - Insert
  - Delete

Quadtree

- Similar to binary-tree, but have 4 children per node
- Each node corresponds to one of four rectangular regions of the current quad





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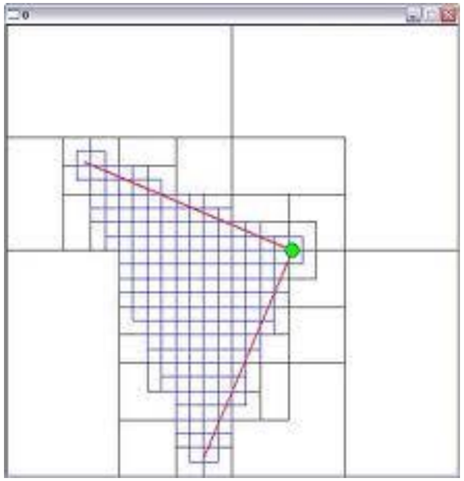
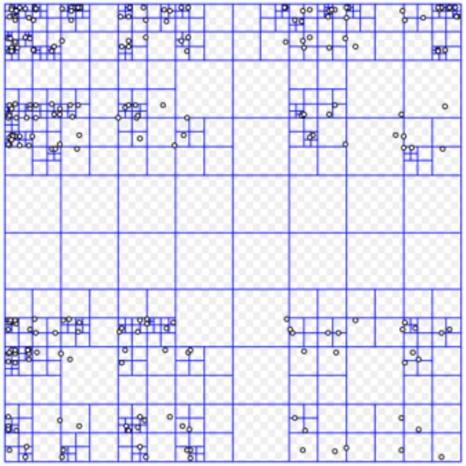


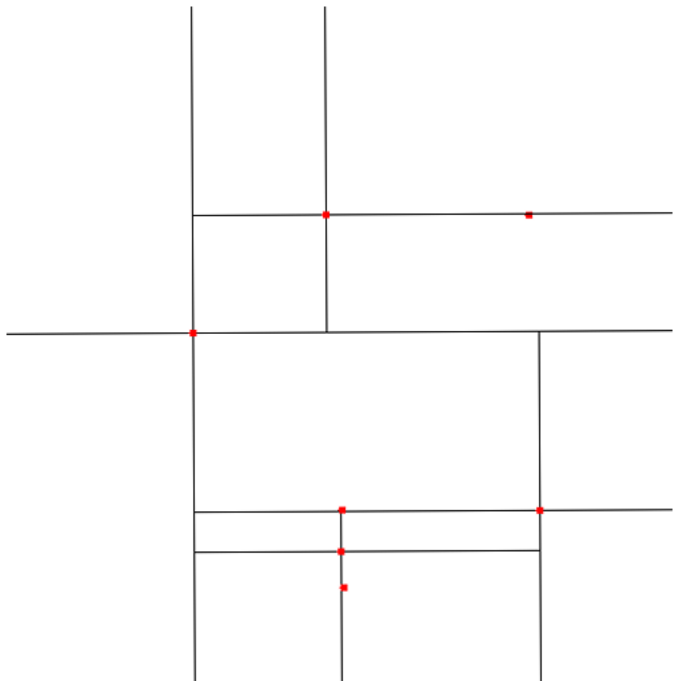


Quadtree

- Various types of quadtrees exist
- Questions/Applications:
  - Is point P in the dataset?
  - What points are near P?
  - Given an image, in which area/pixel is P?
  - What is the average feature value in an area A?

Quadtree

- Point quadtree
  - Partitions depend on the data
  - The quad is divided using the previous point within it
  - Point is stored in nodes

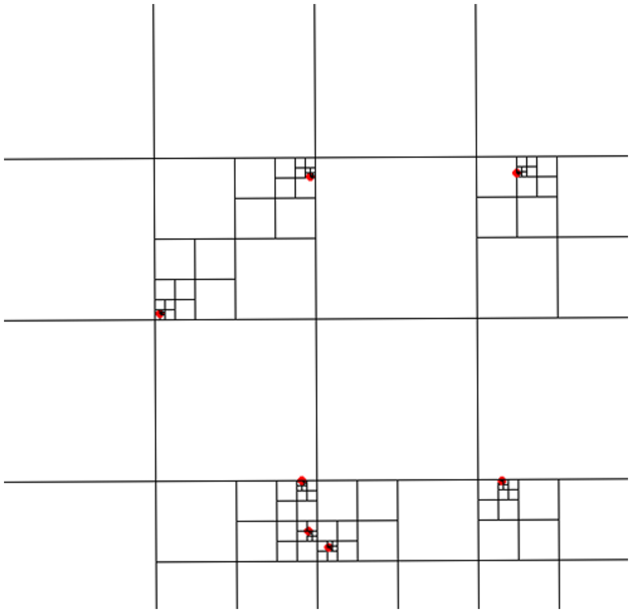


Quadtree

- Point quadtree
  - Partitions depend on the data
  - The quad is divided using the previous point within it
- Advantage
  - Data dependent subdivision reduces (unnecessary) number of quads
- Disadvantage
  - Quads do not tightly approximate region surrounding the point

Quadtree

- Matrix (MX) quadtree (or region quadtree)
  - Location of partition lines independent of the data
  - The occupied nodes are all subdivided until a tight fitting box
  - Point is stored in leaf

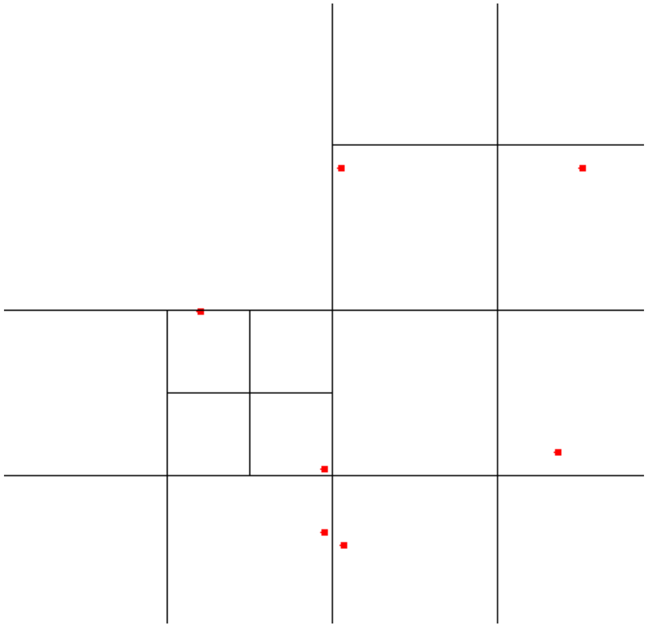


Quadtree

- MX quadtree
  - Location of partition lines independent of the data
  - The occupied nodes are all subdivided until a tight fitting box
- Advantage
  - Quads leaf nodes always tightly approximate region surrounding the point
  - Shape of tree independent of insertion order
- Disadvantage
  - Potentially lots of levels from root to a point

Quadtree

- Point Region (PR) quadtree
  - Location of partition lines independent of the data
  - The nodes are all subdivided until p or less points per node (e.g., p=1)

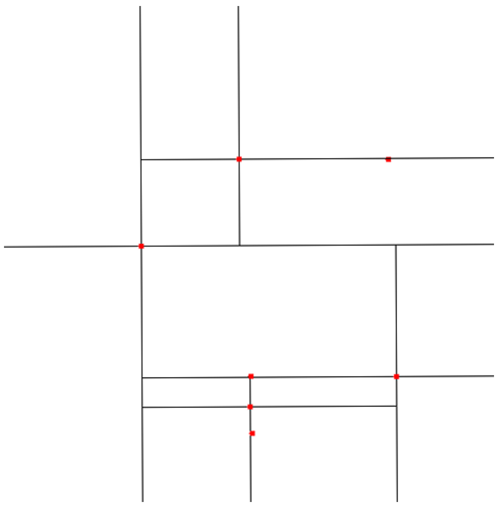


Quadtree

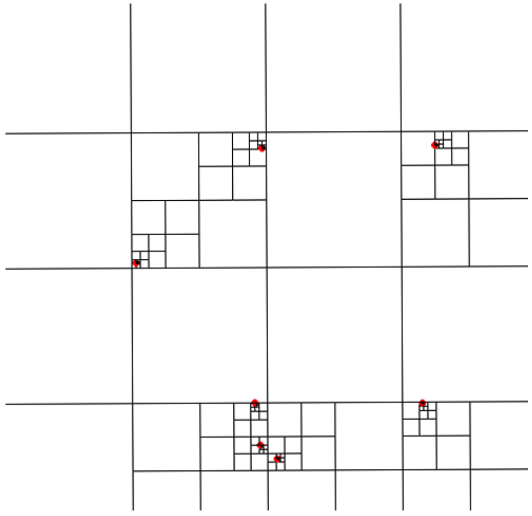
- PR quadtree
  - Location of partition lines independent of the data
  - The nodes are all subdivided until p or less points per node (e.g., p=1)
- Advantage
  - Partition lines are known and paths from root to point is only as long as needs to be
- Disadvantage
  - Quads do not tightly approximate region surrounding the point

Quadtree

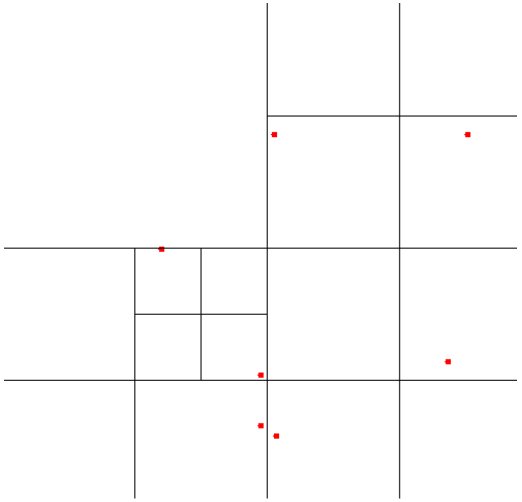
- Comparison



Point QT



MX QT



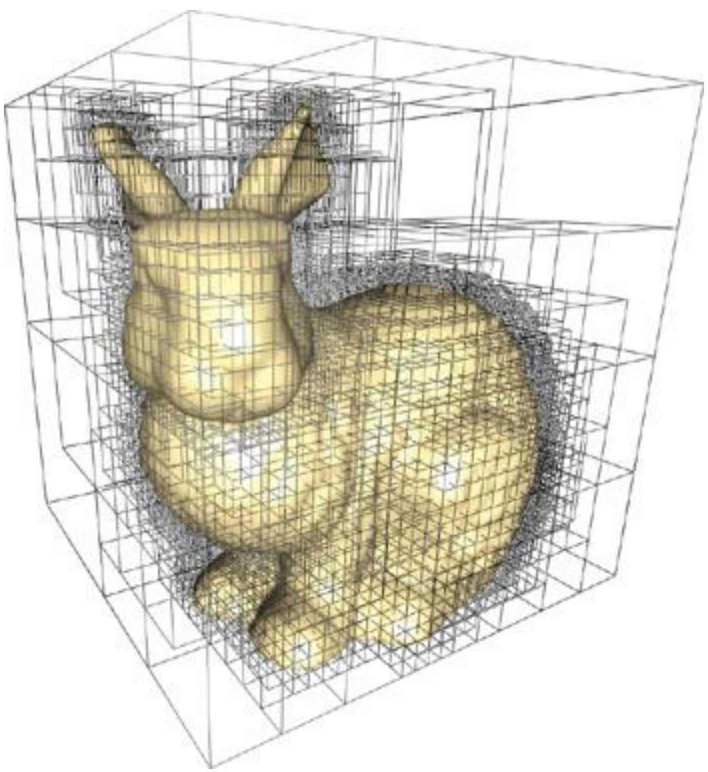
PR QT

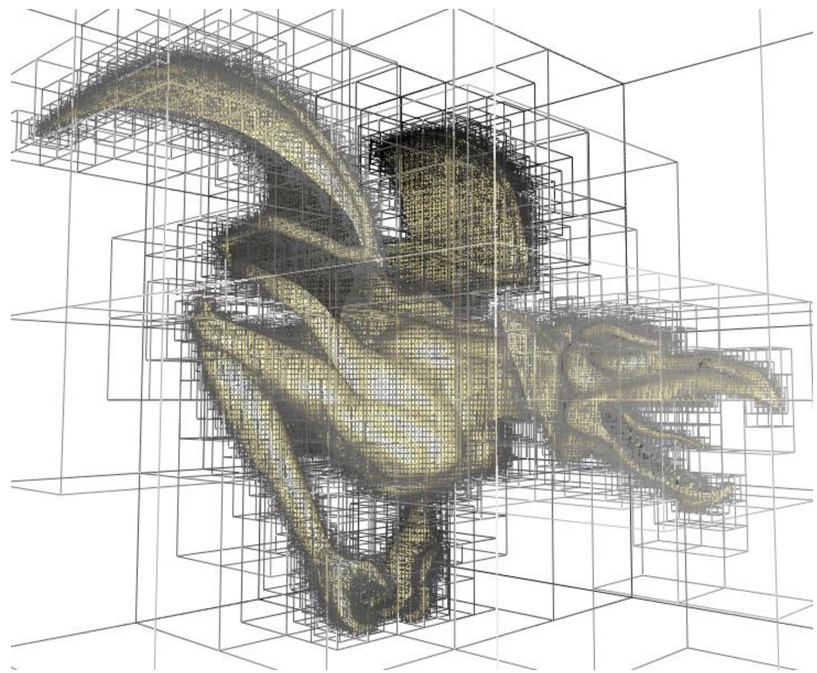
Demo

- http://donar.umiacs.umd.edu/quadtree/

Octree

- Analogous to Quadtree but extended to 3D
- Each node is divided into eight subboxes





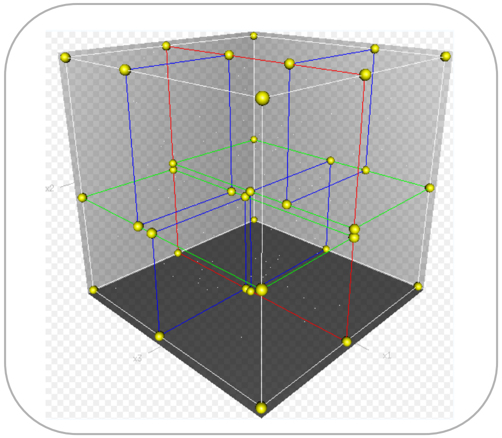
Octree

- Analogous to Quadtree but extended to 3D
- Each node is divided into eight subboxes
- Similar, there are
  - Point octrees
  - MX octrees
  - PR octrees

K-D tree

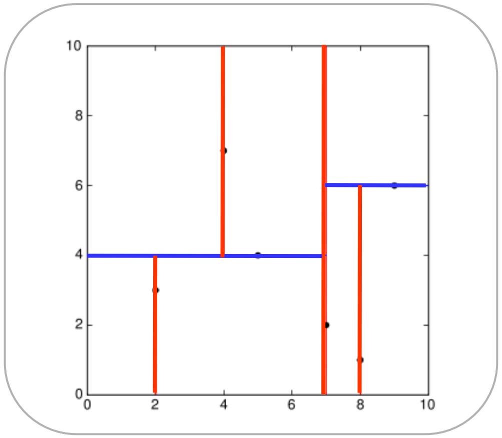
- Partition each dimension in a cyclical fashion
  - Thus, can be applied to 2D, 3D, or higher dimensions
- Each node stores a next partitioned “half-space” of data points (or of the data space)

k-D tree

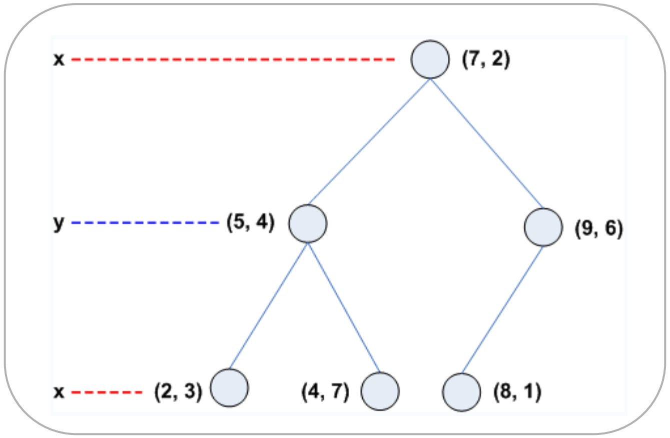


A 3-dimensional kd-tree

- The first split (red) cuts the root cell (white) into two
- Each of which is then split (green) into two subcells
- Each of those four is split (blue) into two subcells
- The final eight called leaf cells
- The yellow spheres represent the tree vertices



The resulting kd-tree decomposition



The resulting kd-tree

Demo

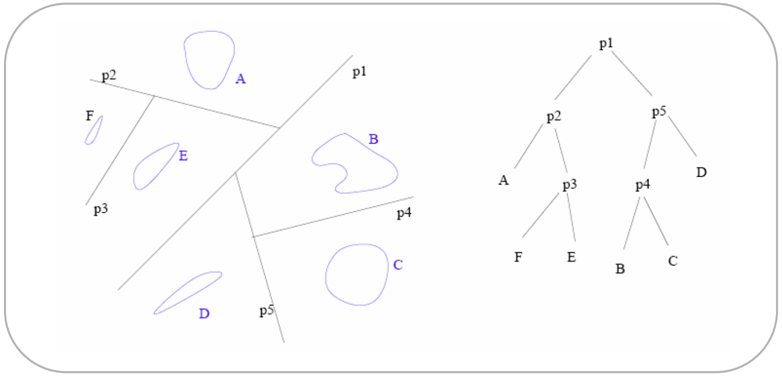
- http://donar.umiacs.umd.edu/quadtree/

Binary Space Partitioning (BSP)

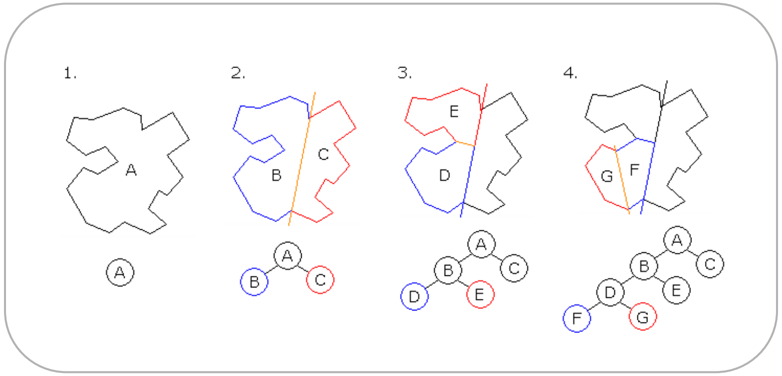
- Similar to k-D tree but splitting lines/planes are not necessarily axis-aligned
- Can adapt better to data
- Was algorithm used for visibility sorting…

Binary Space Partitioning (BSP)

- Suitable for any number of dimensions



Separating planes are shown in black and objects in blue)



BSP trees

Demo

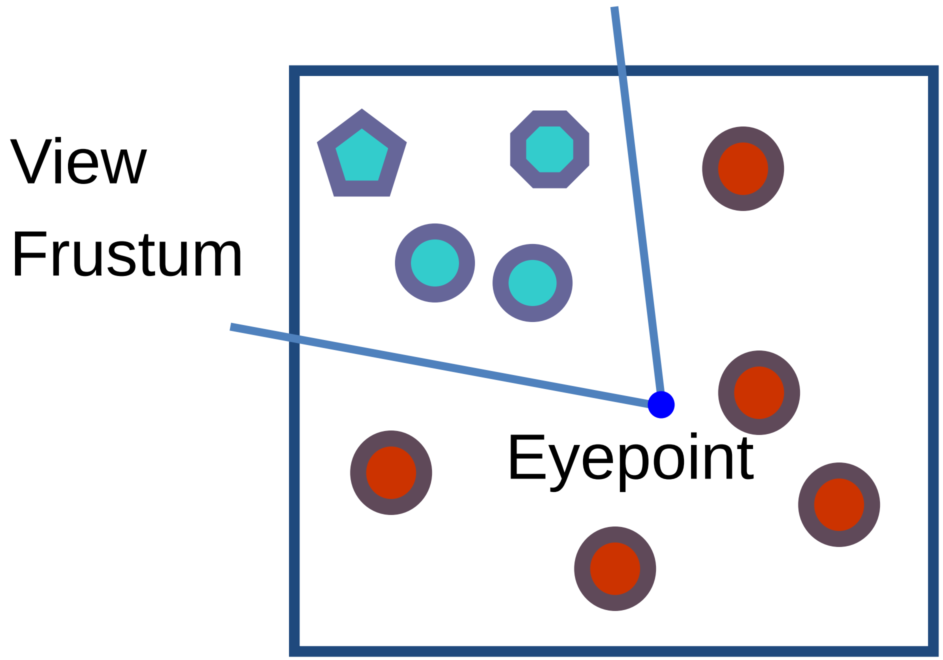
- More stuff at
  - http://donar.umiacs.umd.edu/quadtree
- See
  - H. Samet, *Foundations of Multidimensional and Metric Data Structures*, Morgan-Kaufmann, San Francisco, 2006

Example Uses of Spatial Data Structures

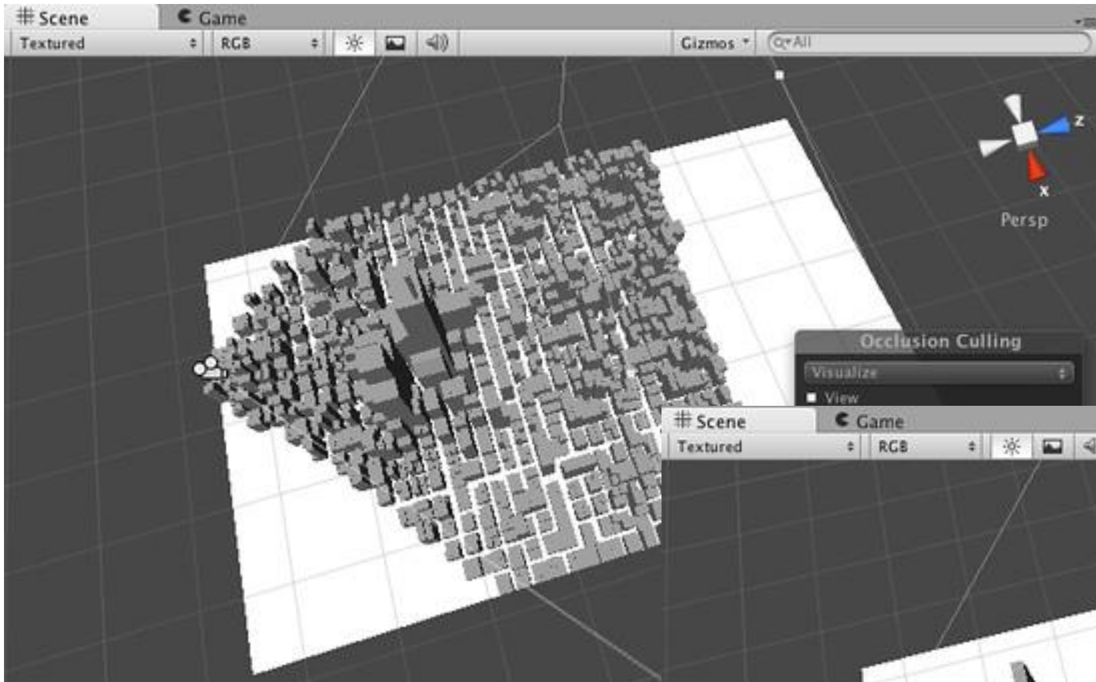
- View Frustum Culling
- Ray Tracing
- Collision Detection
- and more…

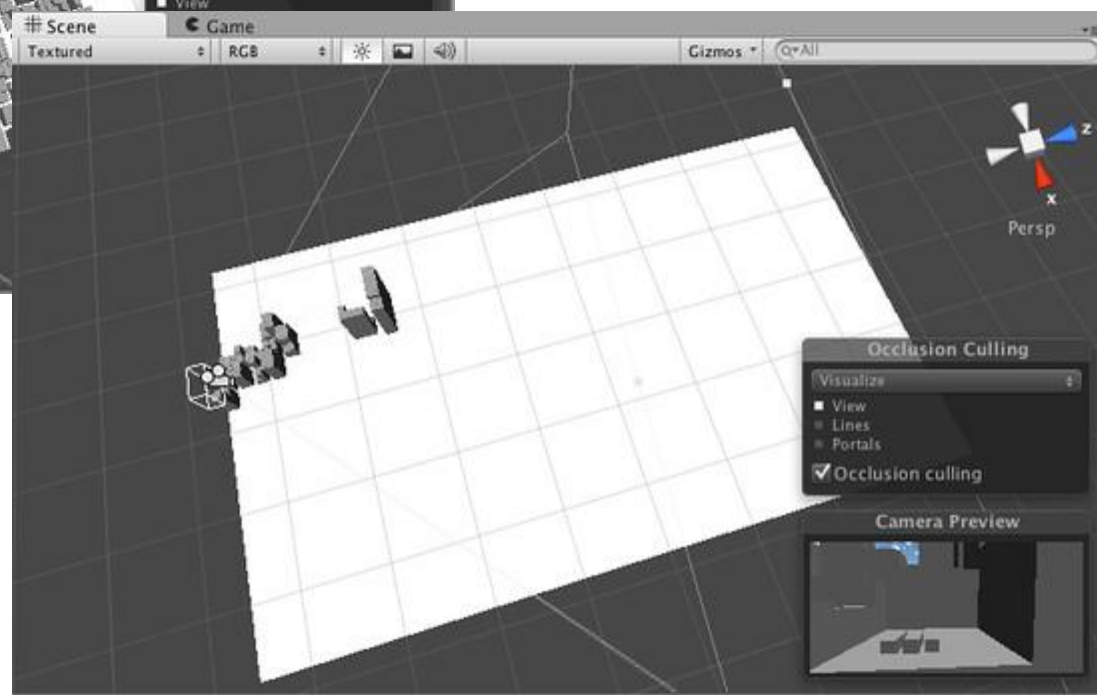
View Frustum Culling

- Omit rendering geometry outside the view frustum



View Frustum Culling

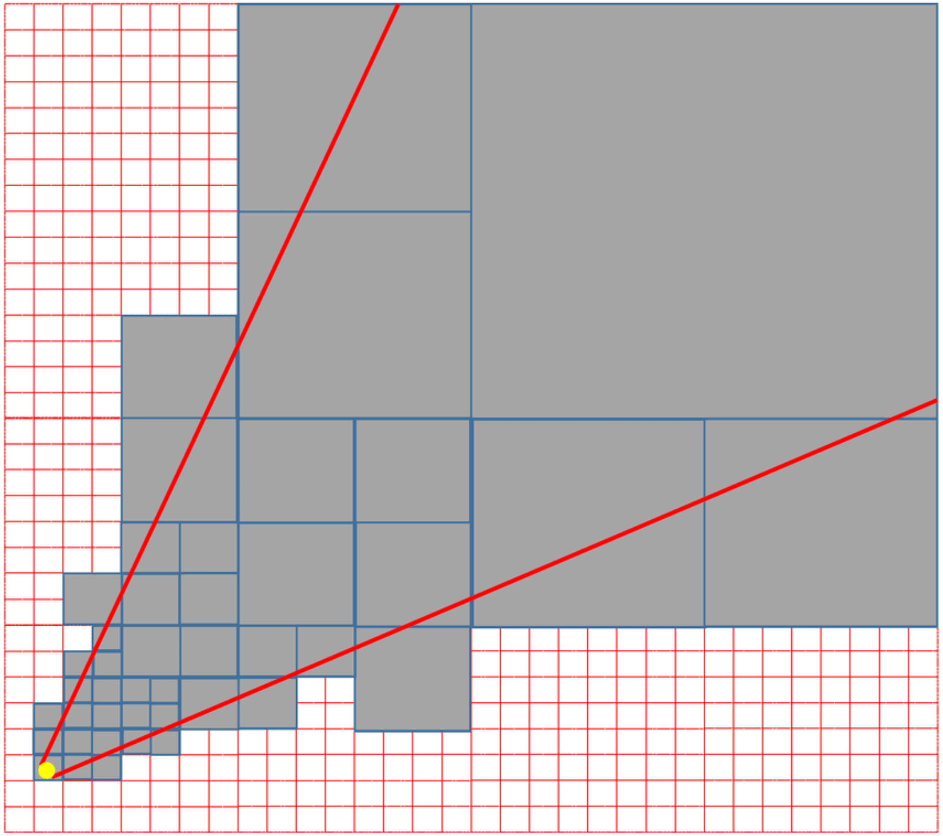




and occlusion culling…

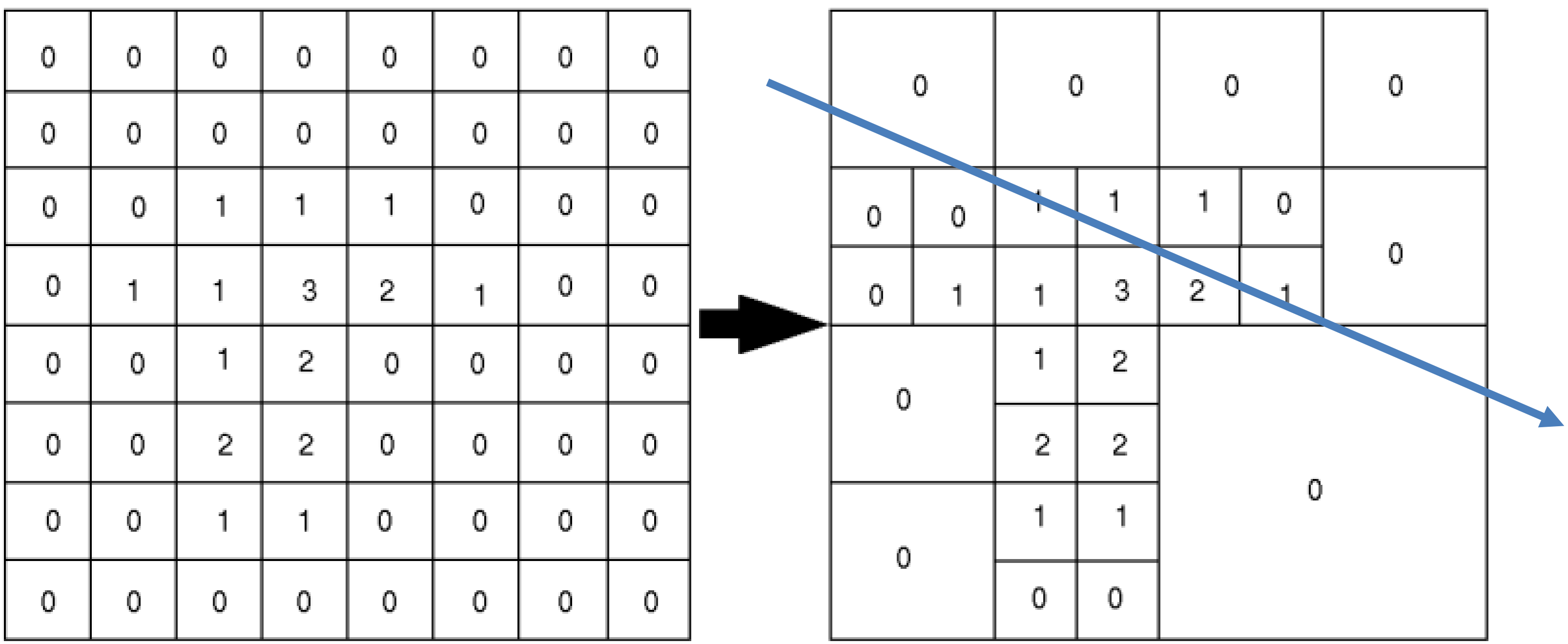
Hierarchical View Frustum Culling

- See board…



Ray Tracing: Octree (or Quadtree)

- See board…(construction, neighbor finding, etc)



SP Exercises

- See board…