



# Procedural Modeling

CS334 Fall 2023

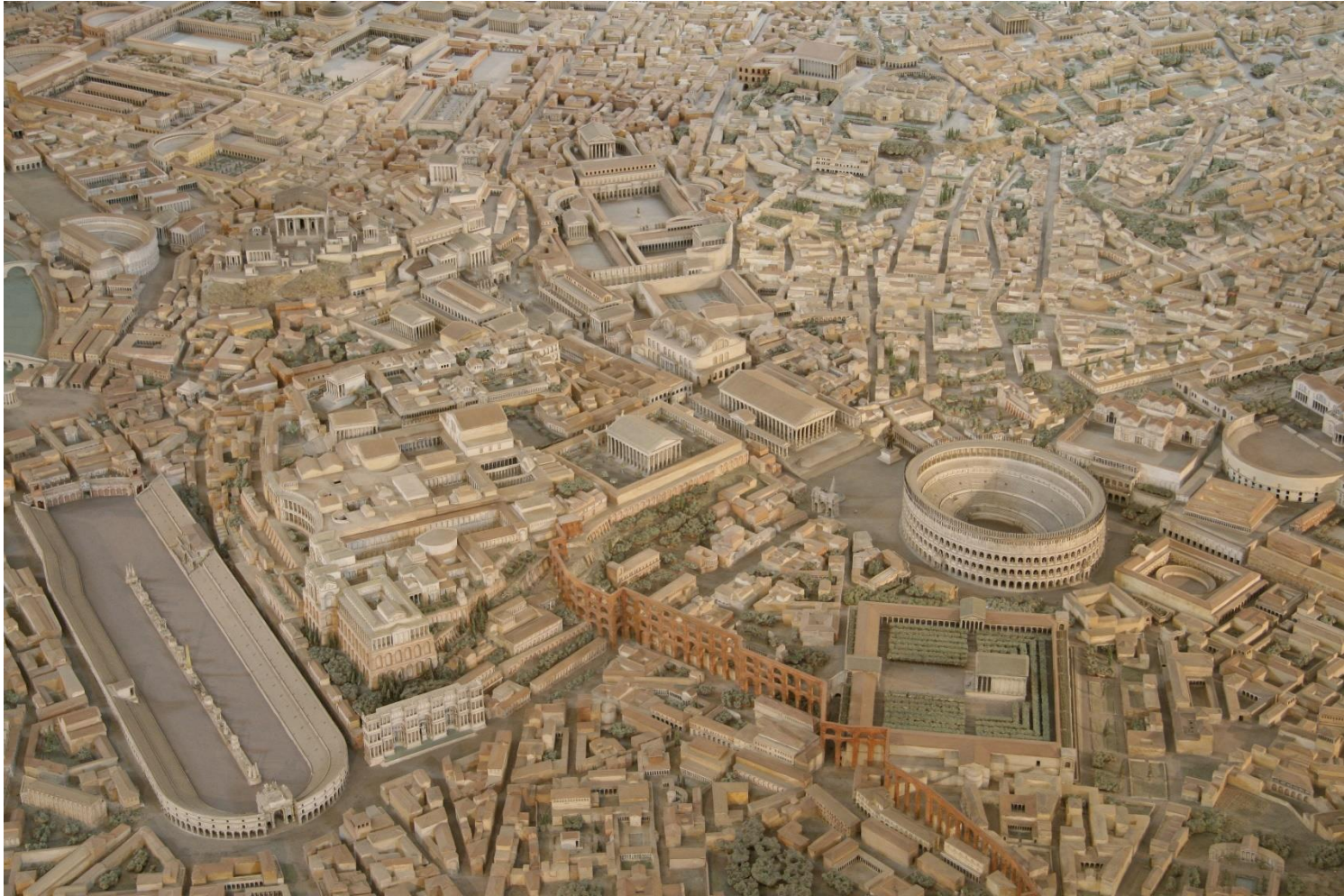
Daniel G. Aliaga  
Department of Computer Science  
Purdue University



# How to?



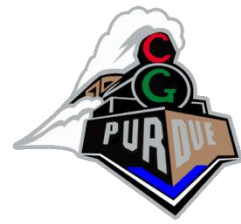
# How to?



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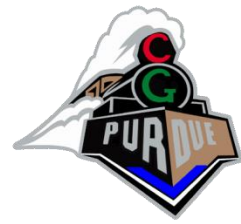
# Procedural Modeling

- Apply algorithms for producing objects and scenes
- The rules may either be embedded into the algorithm, configurable by parameters, or externally provided



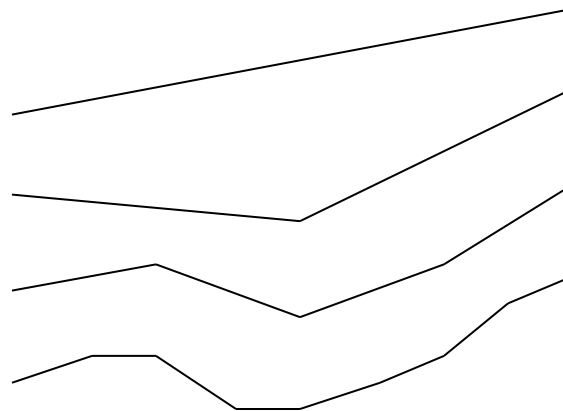
# Procedural Modeling

- Fractals
- Terrains
- Image-synthesis
  - Perlin Noise
  - Clouds
- Plants
- Cities
- And procedures in general...



# Linear Fractals

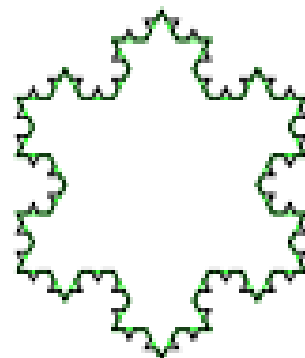
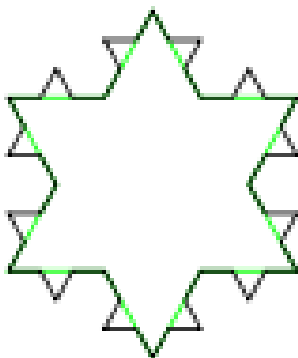
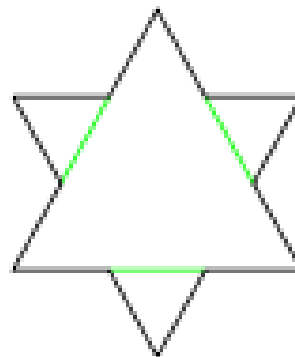
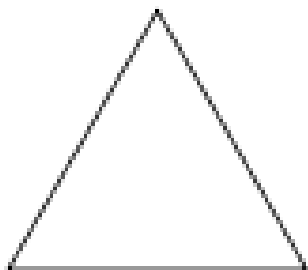
- Definition: a shape is repeated in different orientations/scales -- a never-ending pattern.
- Consider a simple line fractal
  - Split a line segment, randomize the height of the midpoint by some number in the  $[-r,r]$  range
  - Repeat and randomize by  $[-r/2,r/2]$
  - Continue until a desired number of steps, randomizing by half as much each step







# Koch Snowflake





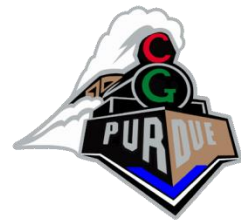
# Demo

- <http://nolandc.com/sandbox/fractals/>



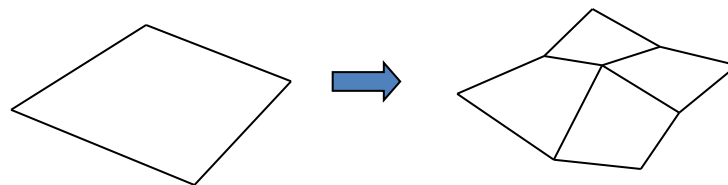
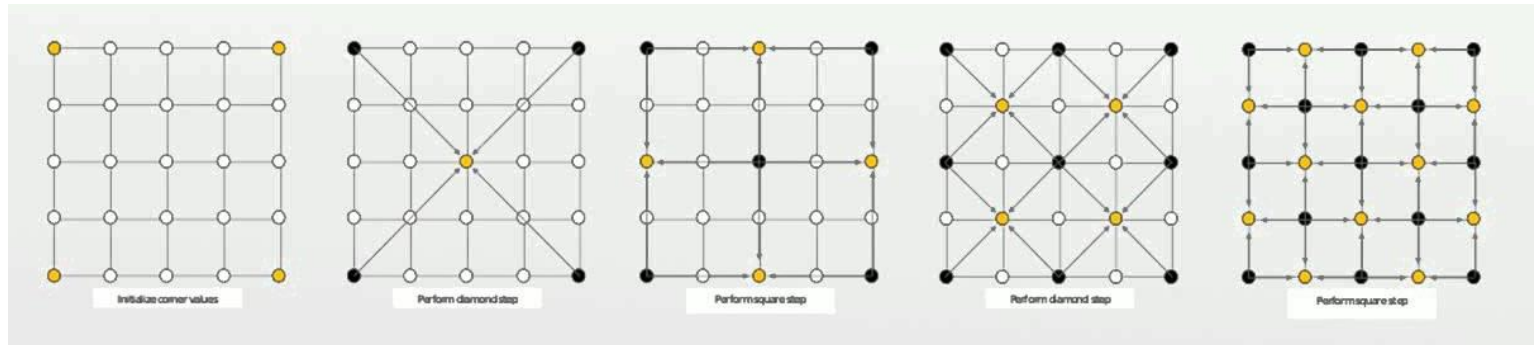
# Non-linear Fractals

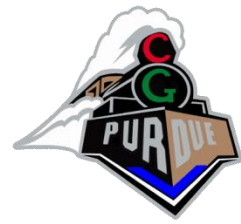
- Example: Mandelbrot Set
  - Iterations of “ $z_{n+1} = z_n^2 + C$ ” starting at  $z_0 = c_0$
  - <https://www.youtube.com/watch?v=pCpLWbHVNhk>



# Fractals and Terrains

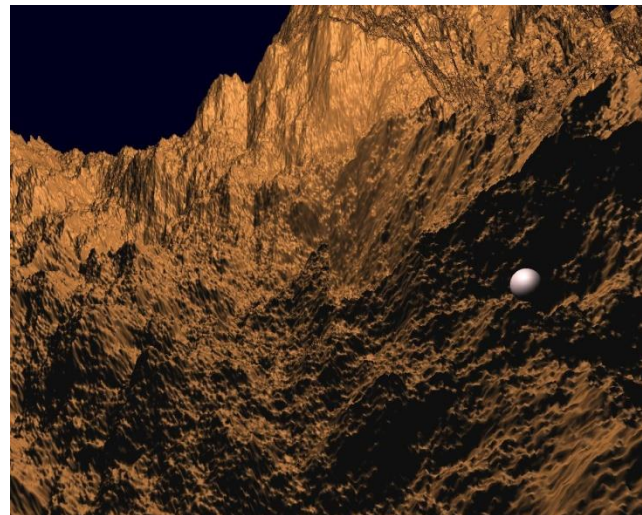
- A similar process can be applied to squares in the xz plane (Diamond-Square Algorithm):
  - At each step, an xz square is subdivided into 4 squares, and the y component of each new point is randomized
  - By repeating this process recursively, we can generate a mountain landscape





# Terrains

- A similar process can be applied to squares in the  $xz$  plane
  - At each step, an  $xz$  square is subdivided into 4 squares, and the  $y$  component of each new point is randomized
  - By repeating this process recursively, we can generate a mountain landscape





# Image Synthesis

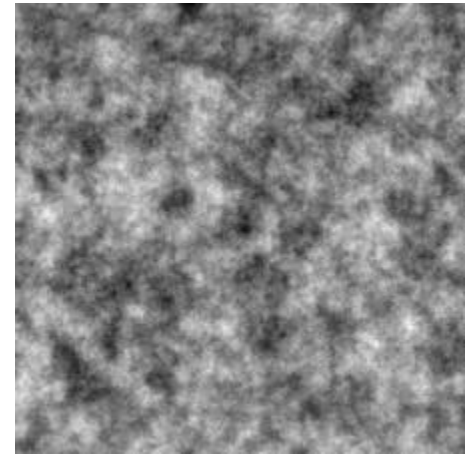
- Procedurally generate an image (pixels)





# Idea: Perlin Noise

- Procedurally generate noise
  - <http://js1k.com/demo/543>
- See other slides





# City Modeling

- Procedural Modeling of Cities  
(more on this later...)

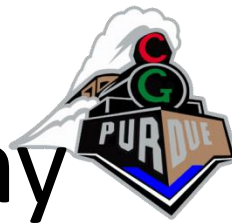




# Plant Modeling

- [The Algorithmic Beauty of Plants](#)

# Background: Chomsky Hierarchy

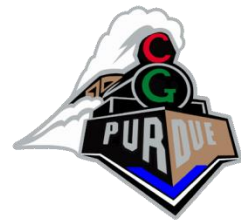


- Type 0 grammars
  - Unrestricted, recognized by Turing machine
- Type 1 grammars
  - Context-sensitive grammars
- Type 2 grammars
  - Context-free grammars
- Type 3 grammars
  - Regular grammars (e.g., regular expressions)

# Lindenmayer system (or L-system)

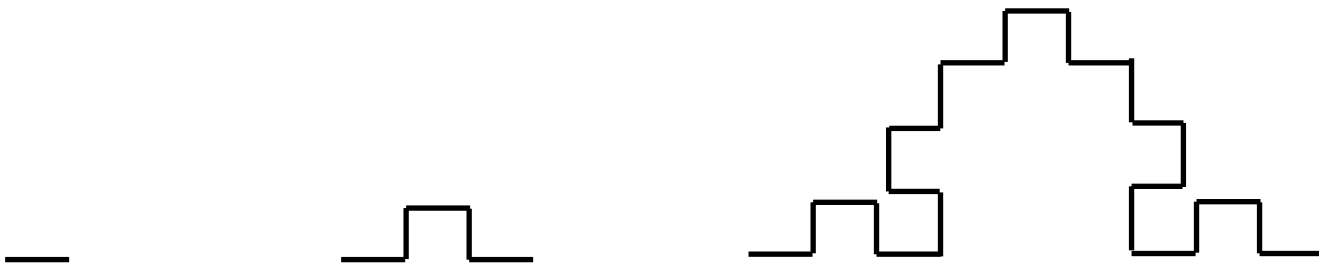


- A context-free or context-sensitive grammar
- All rules are applied in “every iteration” before jumping to the next level/iteration
- Can be deterministic or non-deterministic



# L-system

- Variables:  $a$
- Constants:  $+$ ,  $-$  (rotations of  $+$  or  $-$  90 degrees)
- Initial string (axiom):  $s=a$
- Rules:  $a \rightarrow a+a-a-a+a$



# What does this produce?



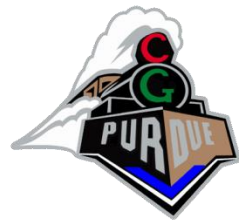
Initial string: | (=F)

Rotation: 25-degrees

Rule:  $F \rightarrow F[+F]F[-F]F$

Iterations: 1

# What does this produce?



Initial string: | (=F)

Rotation: 25-degrees

Rule:  $F \rightarrow F[+F]F[-F]F$

Iterations: 2

# What does this produce?

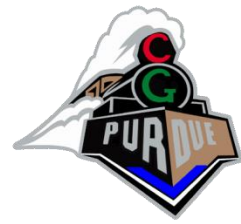


Initial string: | (=F)

Rotation: 25-degrees

Rule:  $F \rightarrow F[+F]F[-F]F$

Iterations: 3



# What does this produce?

Initial string: | (=F)

Rotation: 25-degrees

Rule:  $F \rightarrow F[+F]F[-F]F$

Iterations: 5



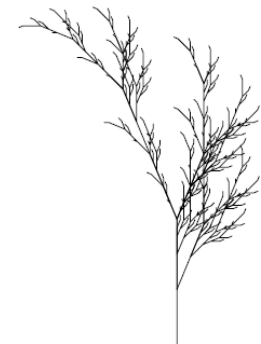
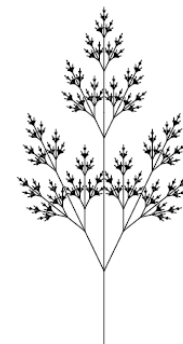
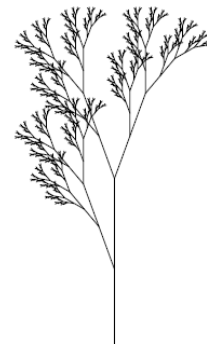
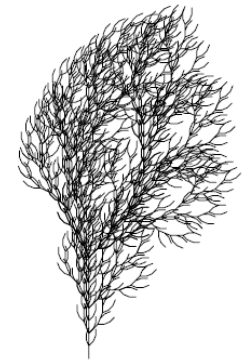
a  
 $n=5, \delta=25.7^\circ$   
F  
 $F \rightarrow F[+F]F[-F]F$





# Exercise!

- **You propose a L-system**
- Starting string:
- Rotation angle:
- Rule:
- Num iterations (about)



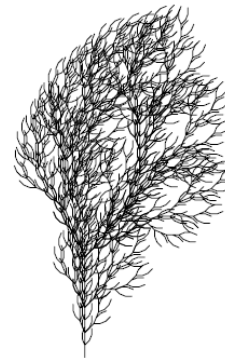
# (Context-Free) L-system for Plants



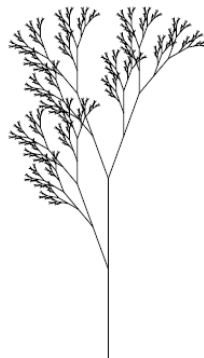
**a**  
 $n=5, \delta=25.7^\circ$   
 $F$   
 $F \rightarrow F[+F]F[-F]F$



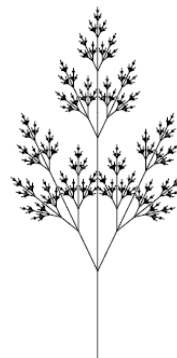
**b**  
 $n=5, \delta=20^\circ$   
 $F$   
 $F \rightarrow F[+F]F[-F][F]$



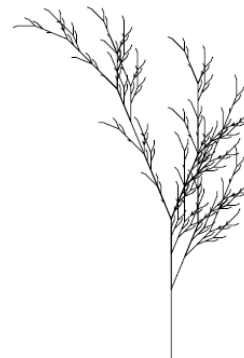
**c**  
 $n=4, \delta=22.5^\circ$   
 $F$   
 $F \rightarrow FF-[-F+F+F]+$   
 $[+F-F-F]$



**d**  
 $n=7, \delta=20^\circ$   
 $X$   
 $X \rightarrow F[+X]F[-X]+X$   
 $F \rightarrow FF$



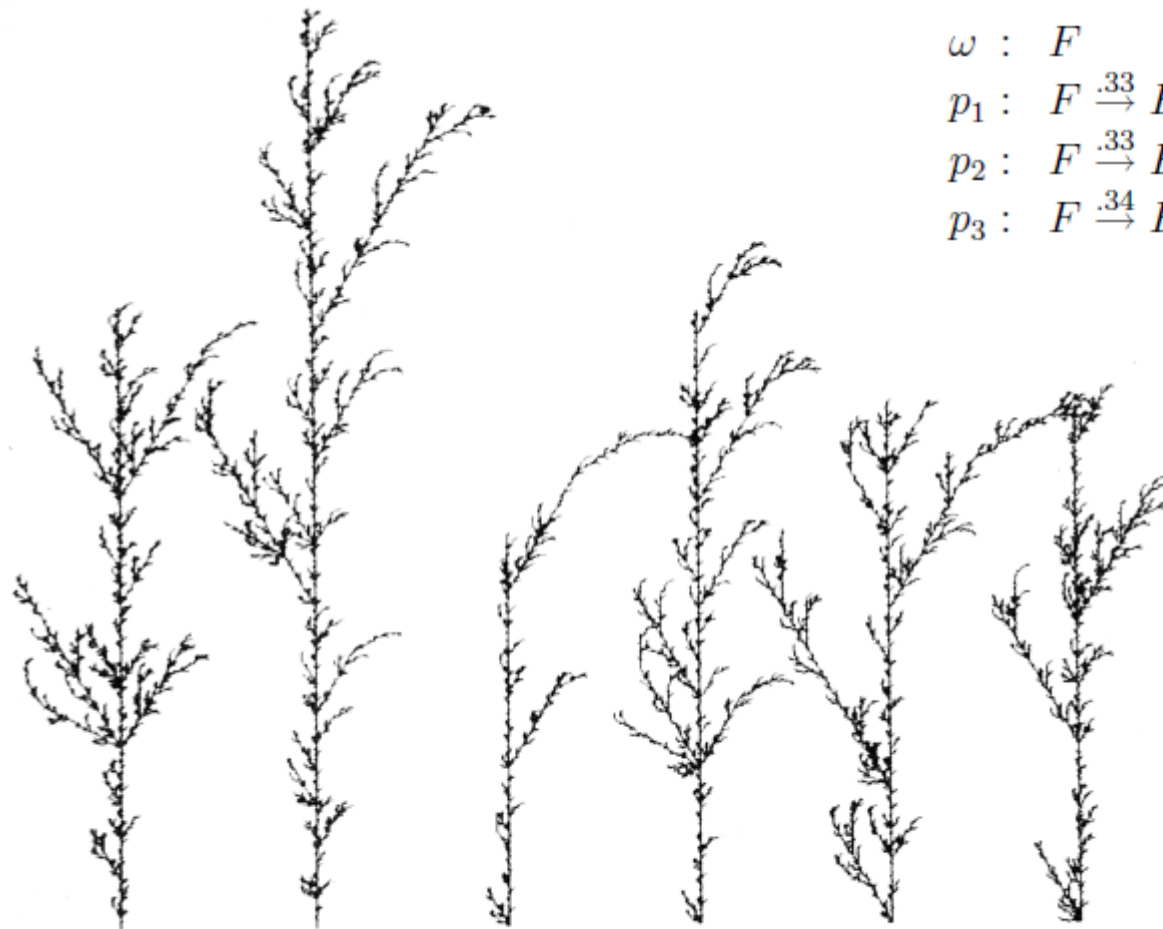
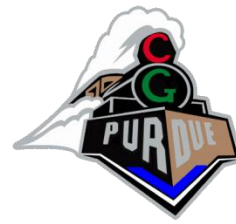
**e**  
 $n=7, \delta=25.7^\circ$   
 $X$   
 $X \rightarrow F[+X][-X]FX$   
 $F \rightarrow FF$



**f**  
 $n=5, \delta=22.5^\circ$   
 $X$   
 $X \rightarrow F-[[X]+X]+F[+FX]-X$   
 $F \rightarrow FF$

Figure 1.24: Examples of plant-like structures generated by bracketed OL-systems. L-systems (a), (b) and (c) are edge-rewriting, while (d), (e) and (f) are node-rewriting.

# L-system for Plants (stochastic)



$$\begin{aligned}\omega &: F \\ p_1 &: F \xrightarrow{.33} F[+F]F[-F]F \\ p_2 &: F \xrightarrow{.33} F[+F]F \\ p_3 &: F \xrightarrow{.34} F[-F]F\end{aligned}$$

Figure 1.27: Stochastic branching structures



# L-system for Plants (3D)



$n=5, \delta=18^\circ$

```

 $\omega$  : plant
 $p_1$  : plant  $\rightarrow$  internode + [ plant + flower] - - //
      [ - - leaf ] internode [ + + leaf ] -
      [ plant flower ] + + plant flower
 $p_2$  : internode  $\rightarrow$  F seg [ // & & leaf ] [ // ^ ^ leaf ] F seg
 $p_3$  : seg  $\rightarrow$  seg F seg
 $p_4$  : leaf  $\rightarrow$  [ ' { +f-ff-f+ | +f-ff-f } ]
 $p_5$  : flower  $\rightarrow$  [ & & & pedicel ' / wedge // // wedge // //
      wedge // // wedge // // wedge ]
 $p_6$  : pedicel  $\rightarrow$  FF
 $p_7$  : wedge  $\rightarrow$  [ ' ^ F ] [ { & & & -f+f | -f+f } ]
  
```

Figure 1.26: A plant generated by an L-system



Figure 1.28: Flower field



# Recent Result

- Growing Demo (Houdini)
  - <https://www.youtube.com/watch?v=-e39SktwmkU>
- SIGGRAPH Asia 2020
  - <https://www.youtube.com/watch?v=MU9E7xJzVGs>



# Shape Grammar

- Is used to generate geometric models from a set of shapes and rules

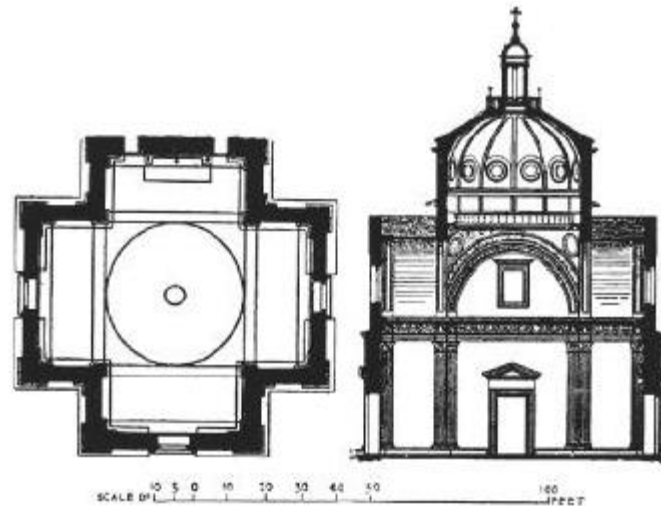
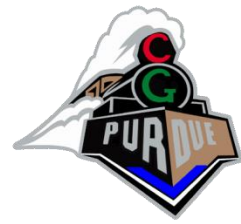


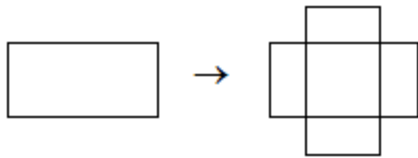
Illustration by Peter Murray, "the Architecture of the Italian Renaissance", Shoken Books Inc. 1963, Pp.96.

# Shape Grammar



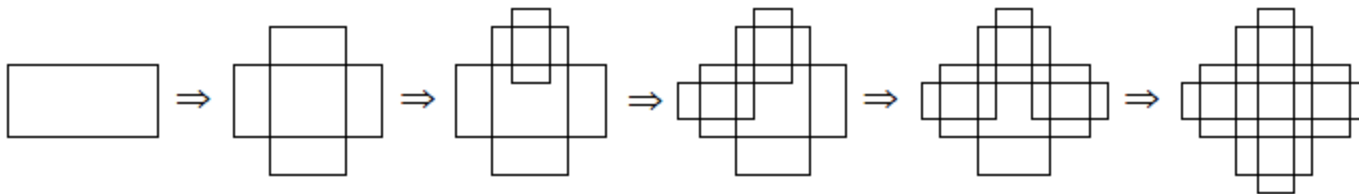


# Shape Grammar



rule

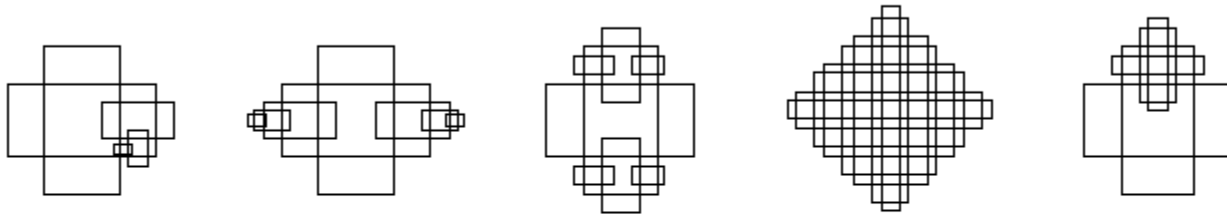
DERIVATION







# Shape Grammar



OTHER DESIGNS IN THE LANGUAGE

# Exercise: let's make some art!



- What is a shape grammar that makes this?



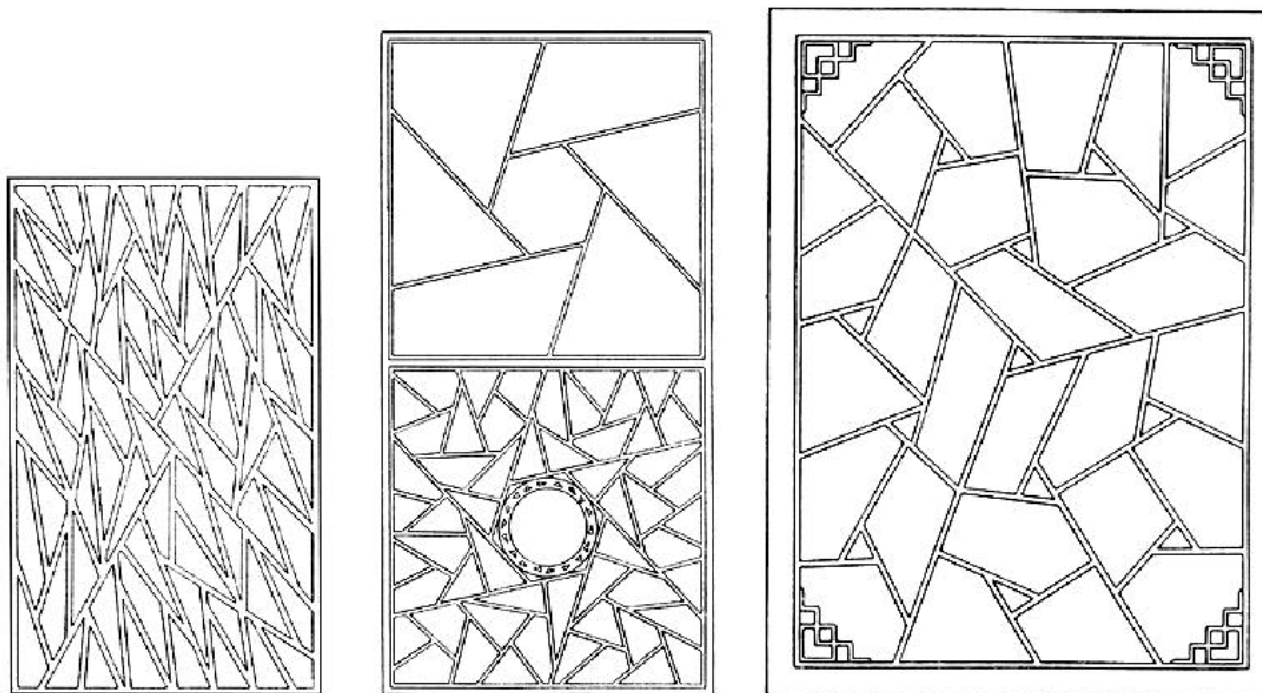
# Exercise: let's make some art!



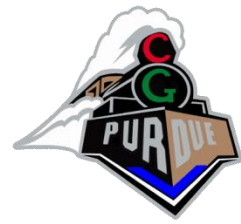
- Consult with your neighbor(s)
- What is a shape grammar that makes the art of the previous slide?
- Go!



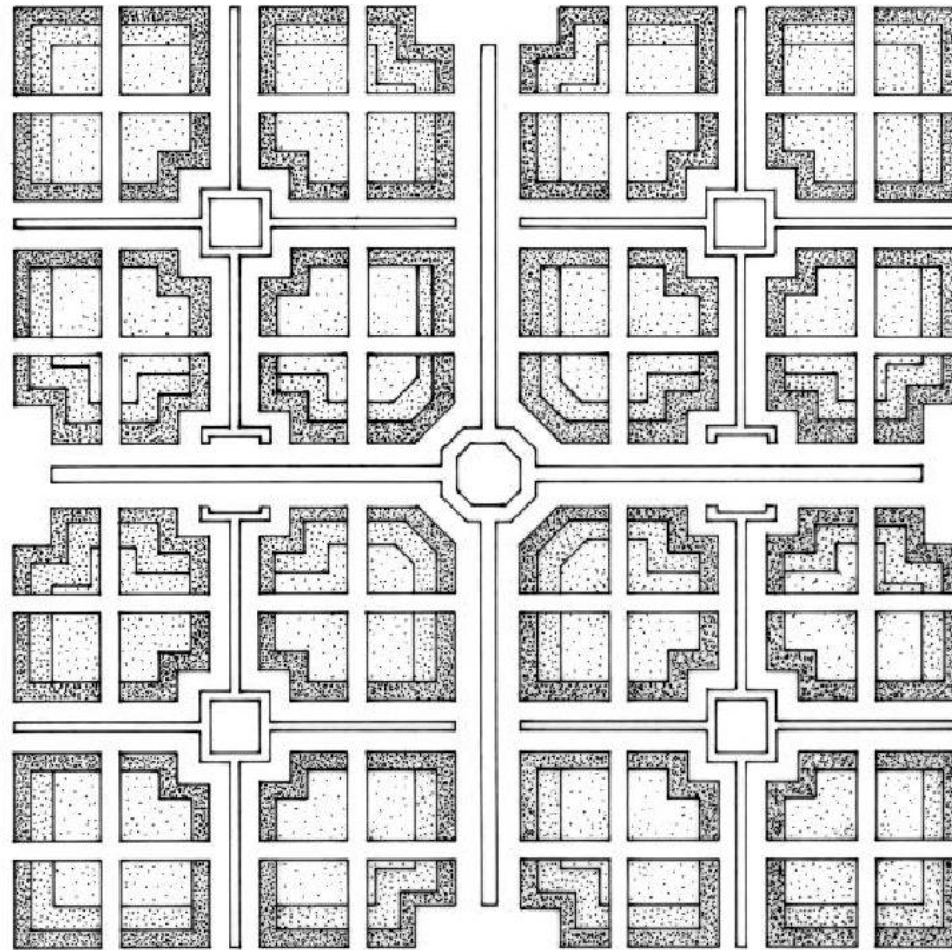
# Shape Grammar



Ice-ray grammar



# Shape Grammar



Mughul garden grammar



# Shape Grammar

- Style: Mediterranean



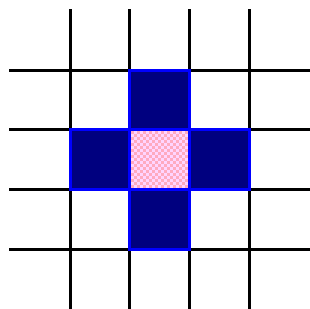


# Cellular Automata

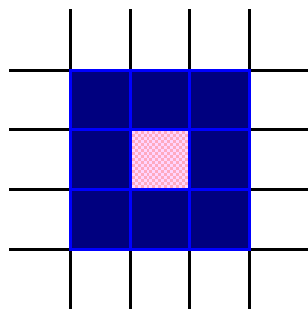
- A cellular automata (CA) is a spatial lattice of  $N$  cells, each of which is one of  $k$  states at time  $t$ .
- Each cell follows the same simple rule for updating its state.
- The cell's state  $s$  at time  $t+1$  depends on its own state and the states of some number of neighbouring cells at  $t$ .
- For one-dimensional CAs, the neighbourhood of a cell consists of the cell itself and  $r$  neighbours on either side. Hence,  $k$  and  $r$  are the parameters of the CA.
- CAs are often described as discrete dynamical systems with the capability to model various kinds of natural discrete or continuous dynamical systems



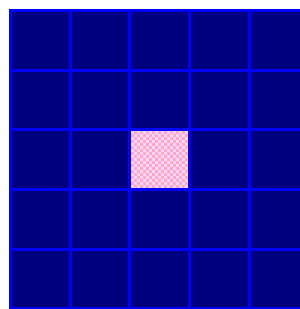
# Types of Neighborhoods



von Neumann  
neighbourhood



Moore  
Neighbourhood



Extended Moore  
Neighbourhood

Many more  
neighborhood  
techniques exist!



# Classes of cellular automata (Wolfram)

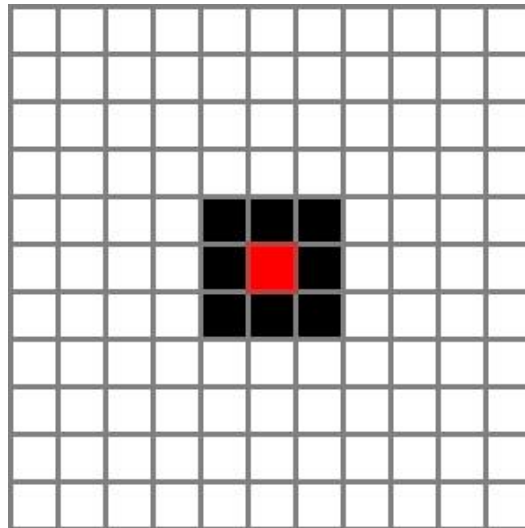


- Class 1: after a finite number of time steps, the CA tends to achieve a unique state from nearly all possible starting conditions (limit points)
- Class 2: the CA creates patterns that repeat periodically or are stable (limit cycles) – probably equivalent to a regular grammar/finite state automaton
- Class 3: from nearly all starting conditions, the CA leads to aperiodic-chaotic patterns, where the statistical properties of these patterns are almost identical (after a sufficient period of time) to the starting patterns (self-similar fractal curves) – computes ‘irregular problems’
- Class 4: after a finite number of steps, the CA usually dies, but there are a few stable (periodic) patterns possible (e.g. Game of Life) - Class 4 CA are believed to be capable of universal computation

# John Conway's Game of Life



- 2D cellular automata system.
- Each cell has 8 neighbors - 4 adjacent orthogonally, 4 adjacent diagonally.
- This is the Moore Neighborhood.



# John Conway's Game of Life

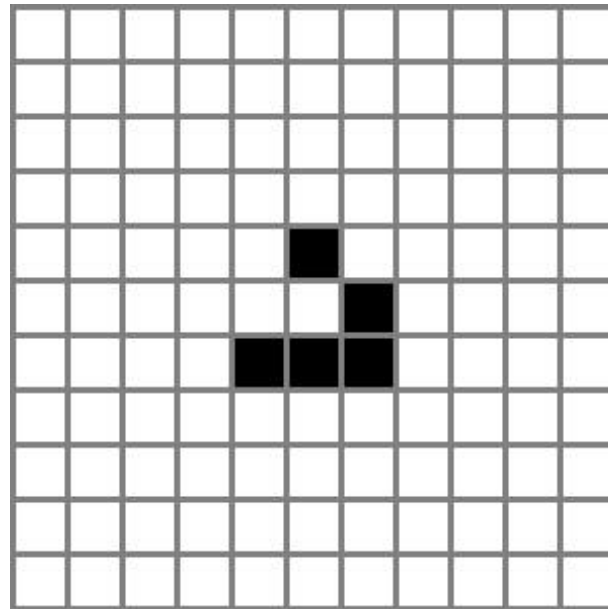


- A live cell with 2 or 3 live neighbors survives to the next round.
- A live cell with 4 or more neighbors dies of overpopulation.
- A live cell with 1 or 0 neighbors dies of isolation.
- An empty cell with exactly 3 neighbors becomes a live cell in the next round.



# Is it alive?

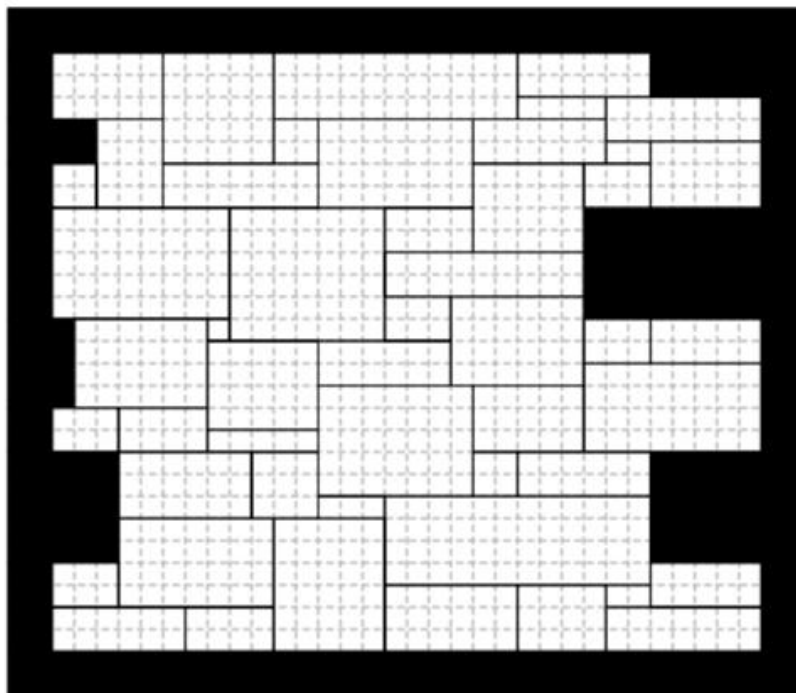
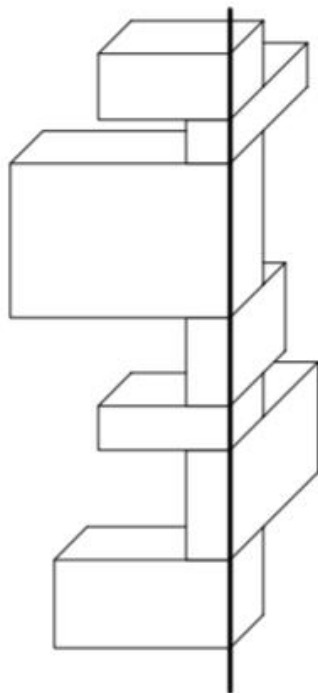
- <http://www.bitstorm.org/gameoflife/>
- Compare it to the definitions...

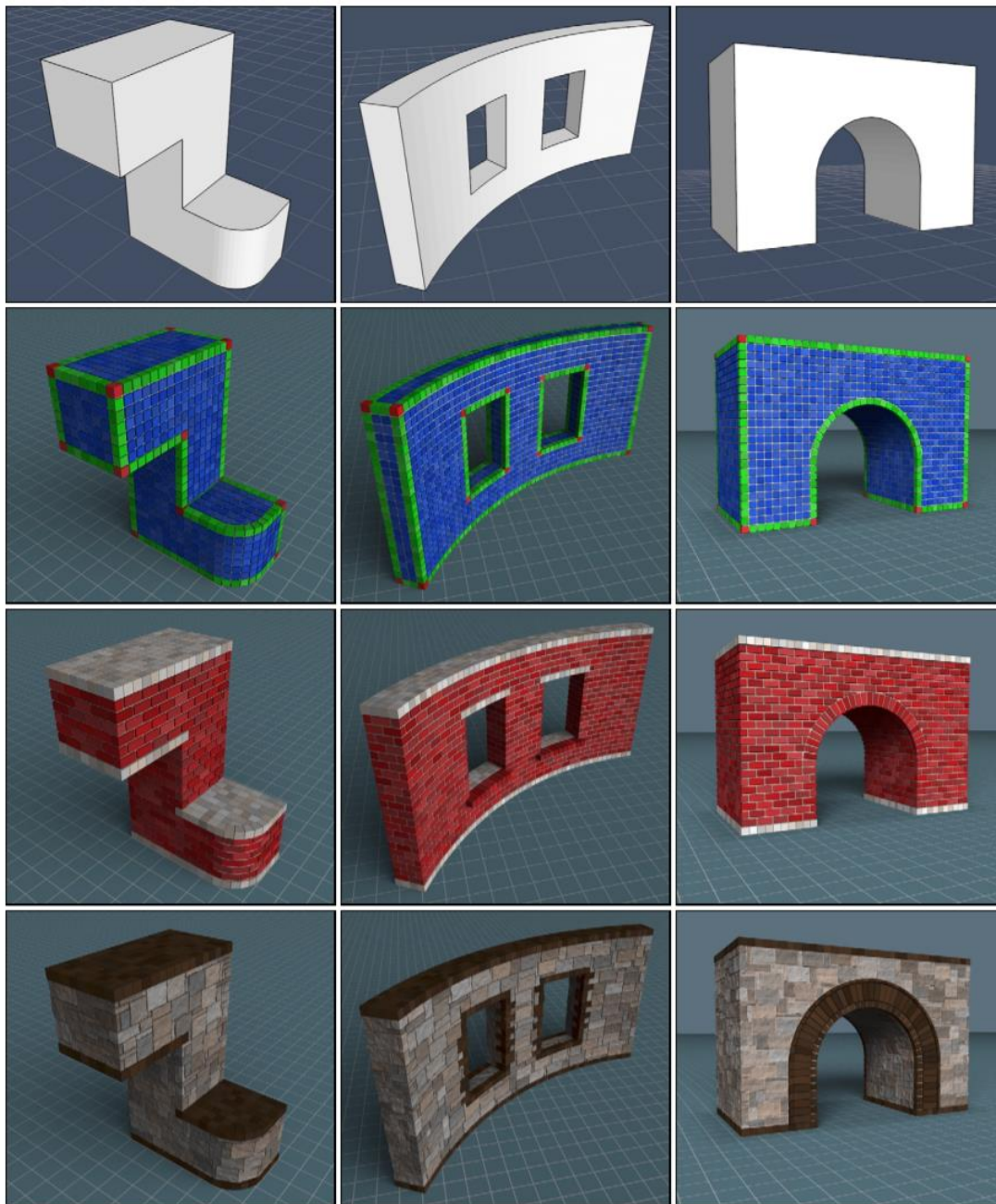




# Cellular Automata

- Used in computer graphics:
  - [Cellular Texturing](#)





# Urban Procedural Modeling



- Seminal paper:
  - [“Procedural Modeling of Cities”](#), Parish and Mueller, SIGGRAPH 2001

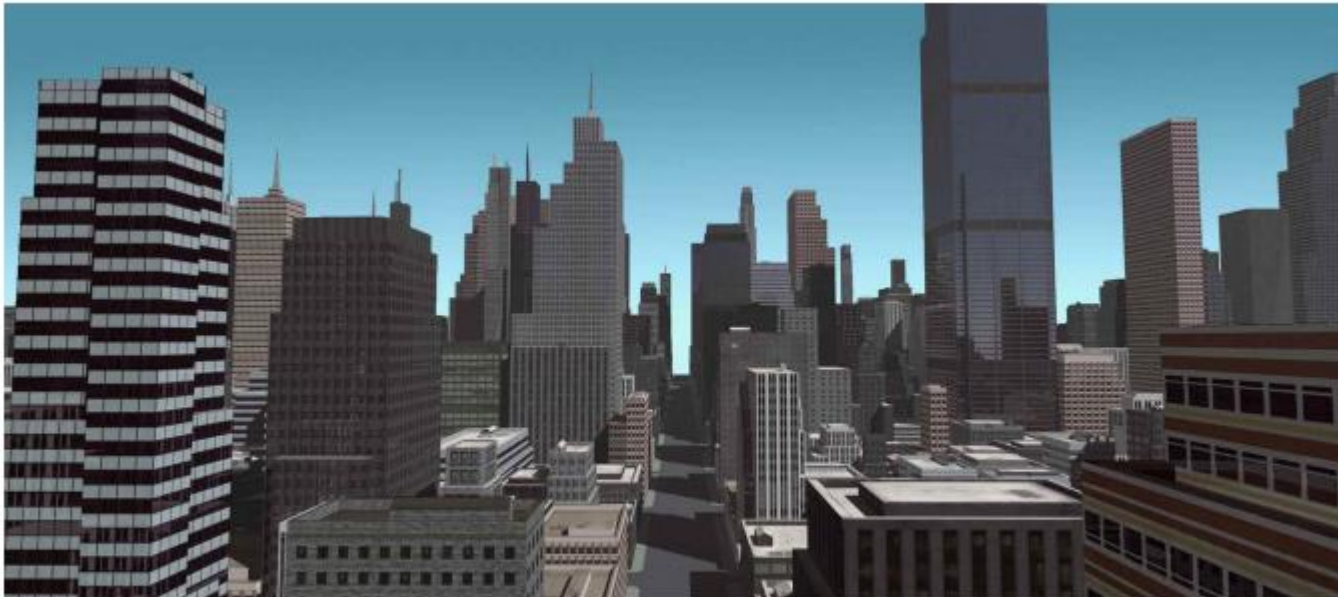


Figure 18. Somewhere in a virtual Manhattan.



# Split Grammars

## Instant Architecture

Peter Wonka<sup>\*,†</sup>

Michael Wimmer<sup>†</sup>

François Sillion<sup>‡</sup>

William Ribarsky<sup>\*</sup>

<sup>\*</sup>Georgia Institute of Technology    <sup>†</sup>Vienna University of Technology    <sup>‡</sup>INRIA



Figure 1: Left: This image shows several buildings generated with split grammars, a modeling tool introduced in this paper. Right: The terminal shapes of the grammar are rendered as little boxes. A scene of this complexity can be automatically generated within a few seconds.





# Split Grammars

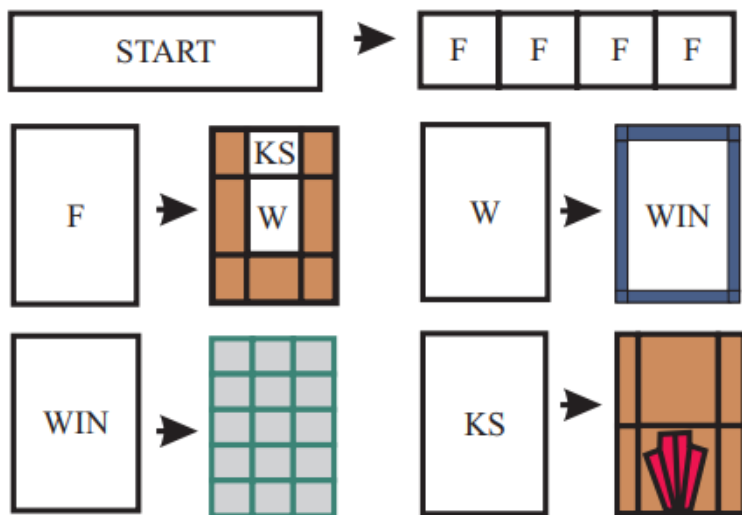


Figure 5: The rules for a simple example split grammar. The white areas (which contain symbols) represent the non-terminal shapes, colored elements are the terminal shapes of the split grammar. The start symbol is split into 4 façade elements, which are further split into a window element, a keystone element and some wall elements etc.

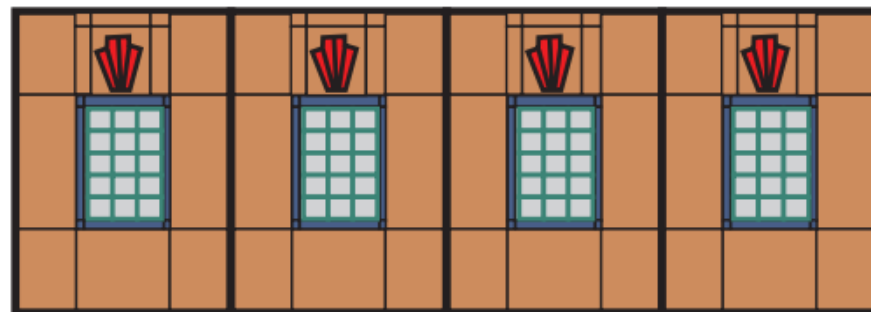
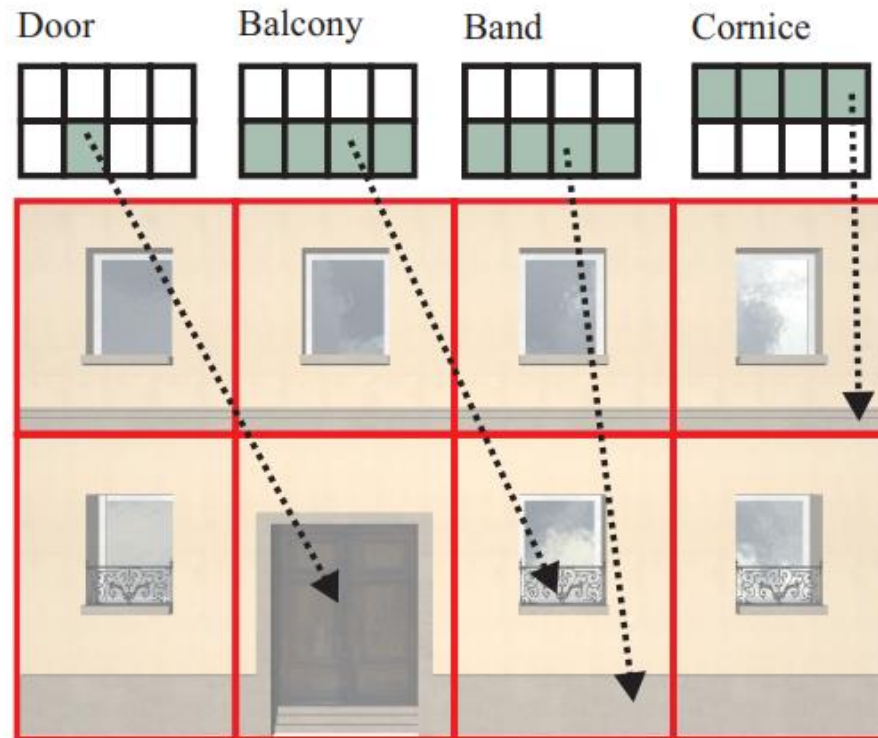


Figure 6: This figure shows the result of the derivation of the grammar in Figure 5.



# Split Grammars

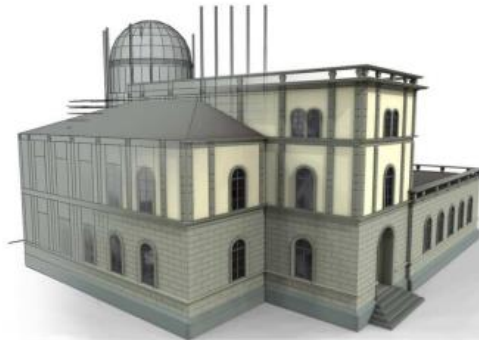


```
FACADE_CONTROL :- DOOR_PATTERN, RANDOM_PATTERN,  
                  RANDOM_PATTERN, RANDOM_PATTERN  
RANDOM_PATTERN  :- CORNICE | BAND | BALCONY  
                  | QUOIN | PILASTER  
DOOR_PATTERN   :- DOOR | GARAGE  
DOOR           :- <[0,1], door, 1> | <[0,2], door, 1>  
                  | <[0,MAX], door, 1>
```



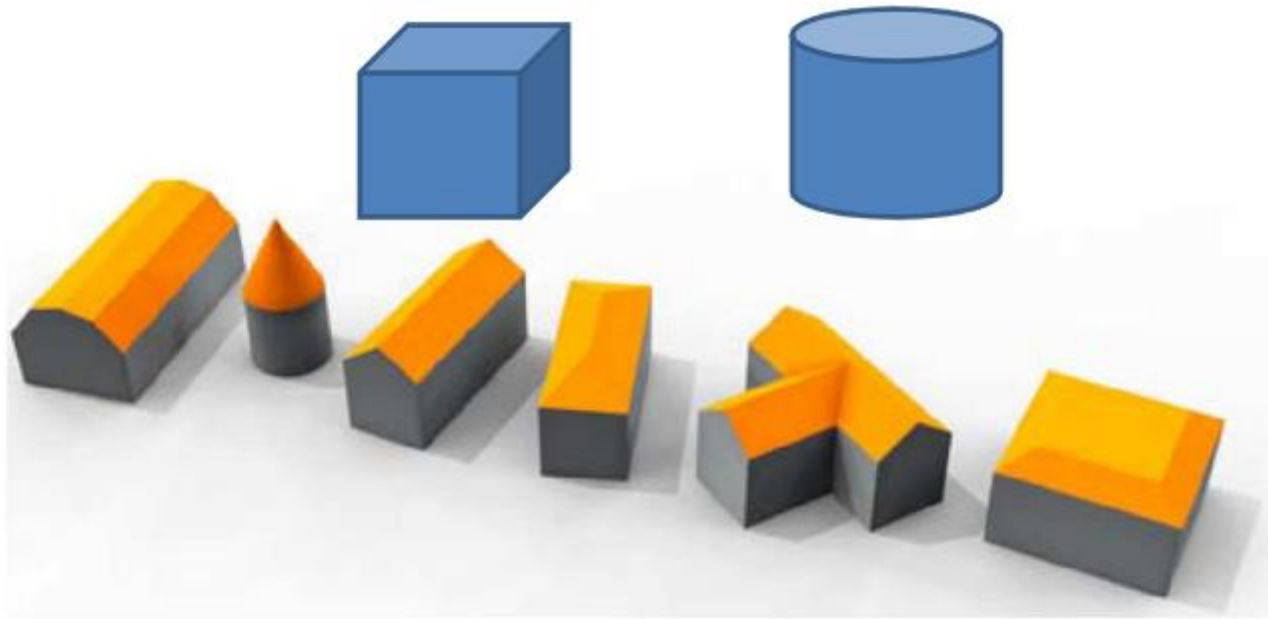
# CGA Shape Grammar

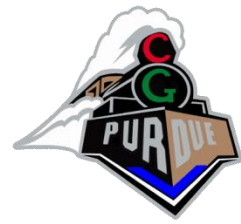
- P. Müller, P. Wonka, S. Haegler, A. Ulmer, L. Van Gool: Procedural modeling of buildings SIGGRAPH 2006





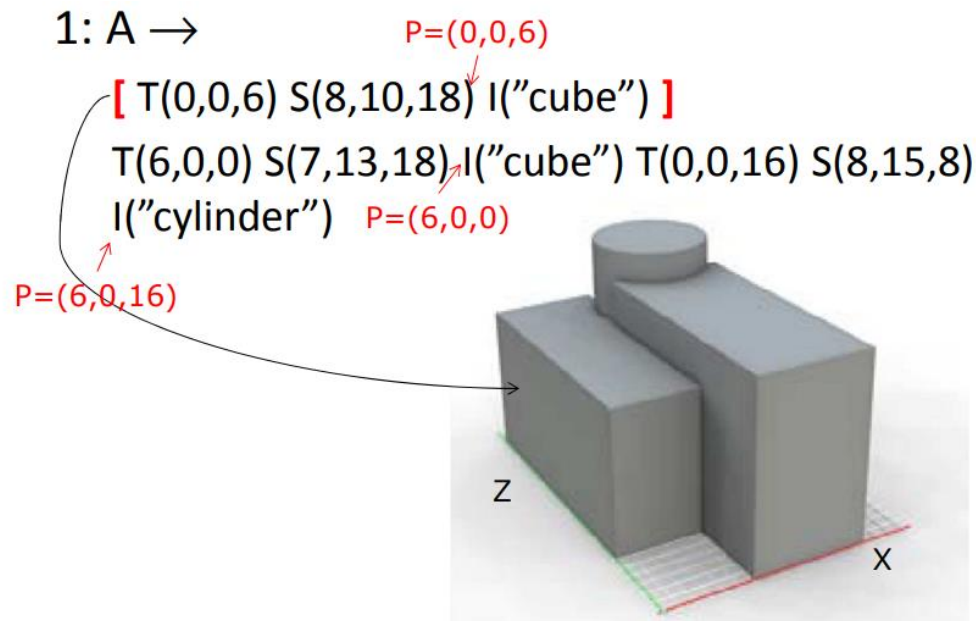
# Basic Shapes





# Rules and Operations

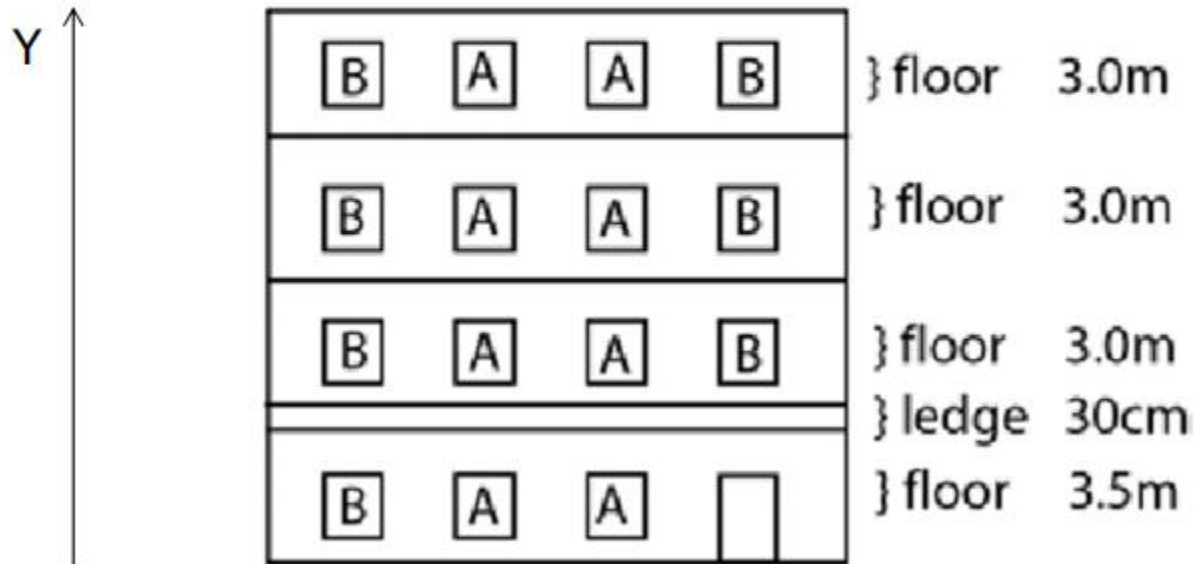
- $T(x,y,z)$  = translate by  $[x \ y \ z]$
- $S(a,b,c)$  = scale by  $[a \ b \ c]$
- Context (like  $[]$  in L-systems) =





# Subdivision

1: fac  $\rightsquigarrow$  Subdiv("Y",3.5,0.3,3,3,3){ floor | ledge | floor | floor | floor }



# Examples



PRIORITY 1:

- 1: lot  $\rightsquigarrow$  S(1r, *building\_height*, 1r)  
Subdiv("Z", Scope.sz\*rand(0.3,0.5), 1r){ facades | sidewings }
- 2: sidewings  $\rightsquigarrow$   
Subdiv("X", Scope.sx\*rand(0.2,0.6), 1r){ siding |  $\epsilon$  }  
Subdiv("X", 1r, Scope.sx\*rand(0.2,0.6)){  $\epsilon$  | siding }
- 3: siding  
 $\rightsquigarrow$  S(1r, 1r, Scope.sz\*rand(0.4,1.0)) facades : 0.5  
 $\rightsquigarrow$  S(1r, Scope.sy\*rand(0.2,0.9), Scope.sz\*rand(0.4,1.0))  
facades : 0.3  
 $\rightsquigarrow$   $\epsilon$  : 0.2
- 4: facades  $\rightsquigarrow$  Comp("sidefaces"){ facade }

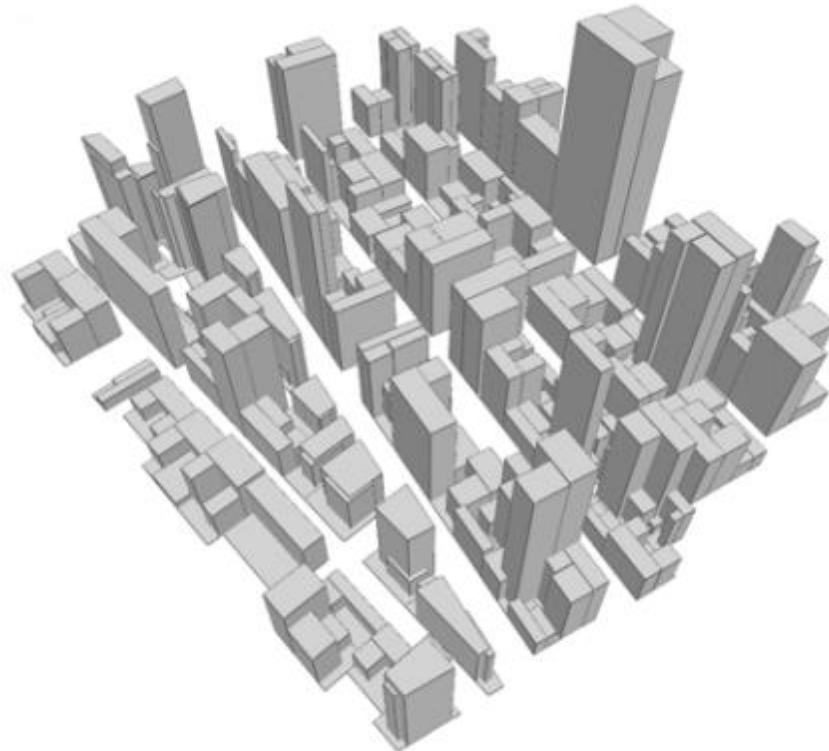


Figure 14: Stochastic variations of building mass models generated with only four rules (starting with the building lot as axiom).

# Examples



PRIORITY 2:

- ```
5: facade : Shape.visible("Street") == 0 ~>
  Subdiv("Y",ground_floor_height,1r,top_floor_height)
  { groundfloor | floors | topfloors } firescape
6: groundfloor ~> Subdiv("X",1r,entrance_width,1r){ groundtiles |
  entrance SnapLines("Y","entrancesnap") | groundtiles }
```

PRIORITY 3:

- ```
7: facade ~> floors
8: floors ~> Repeat("YS",floor_height){ floor Snap("XZ") }
9: floor ~> Repeat("XS",tile_width){ tile Snap("Y","tilesnap") }
```

...

- ```
15: wall : Shape.visible("Street") ~> I("frontwall.obj")
```

PRIORITY 4:

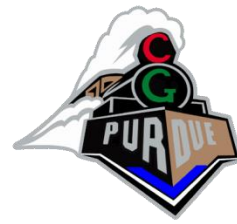
- ```
16: firescape ~> Subdiv("XS",1r,2*tile_width,7r,"tilesnap")
  { epsilon | escapestairs | ε }
17: escapestairs ~> S(1r,1r,firescape_depth)
  T(0,0,-firescape_depth) Subdiv("YS",ground_floor_height,1r)
  { ε | Repeat("YS",floor_height){ I("firescape.obj") } }
```



Figure 15: A procedurally generated building modeled with snap lines. Note the alignment of important lines and planes in the construction.



# Urban Procedural Modeling



- Cities
- Buildings
- CityEngine
  - CityEngine
  - <https://www.youtube.com/watch?v=xJCIIIE9pulk>
  - (for Unreal:  
<https://www.youtube.com/watch?v=faOdiVcxRG4>)



# Videos and more

- Procedural Modeling of Cities
  - <http://www.youtube.com/watch?v=khrWonALQiE>
- Procedural Modeling of Buildings
  - <http://www.youtube.com/watch?v=iDsSrMkW1uc>
- Procedural Modeling of Structurally Sound Masonry Buildings
  - <http://www.youtube.com/watch?v=zXBathLSxSQ>
- Image-based Procedural Modeling of Facades
  - <http://www.youtube.com/watch?v=SncibzYy0b4>
- Image-based Modeling
  - Facades: [http://www.youtube.com/watch?v=amD6\\_i3MVZM](http://www.youtube.com/watch?v=amD6_i3MVZM)