



Image/View Morphing and Warping

CS334

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Motivation – Rendering from Images



- Given
 - left image
 - right image
- Create intermediate images
 - simulates camera movement



Related Work

- Panoramas (e.g., QuicktimeVR, etc)
 - user can look in any direction at few given locations but camera translations are *not* allowed...



Topics

- Image morphing (2D)
- View morphing (2D+)
- Image warping (3D)



Topics

- Image morphing (2D)
- View morphing (2D+)
- Image warping (3D)

Image Morphing

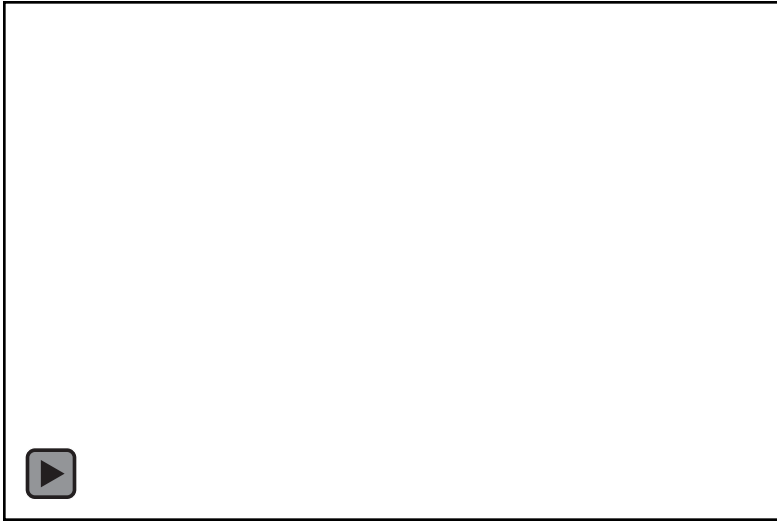




Image Morphing

- Identify correspondences between input/output image
- Produce a sequence of images that allow a smooth transition from the input image to the output image



Image Morphing

1. Correspondences





Image Morphing

1. Correspondences





Image Morphing

1. Correspondences





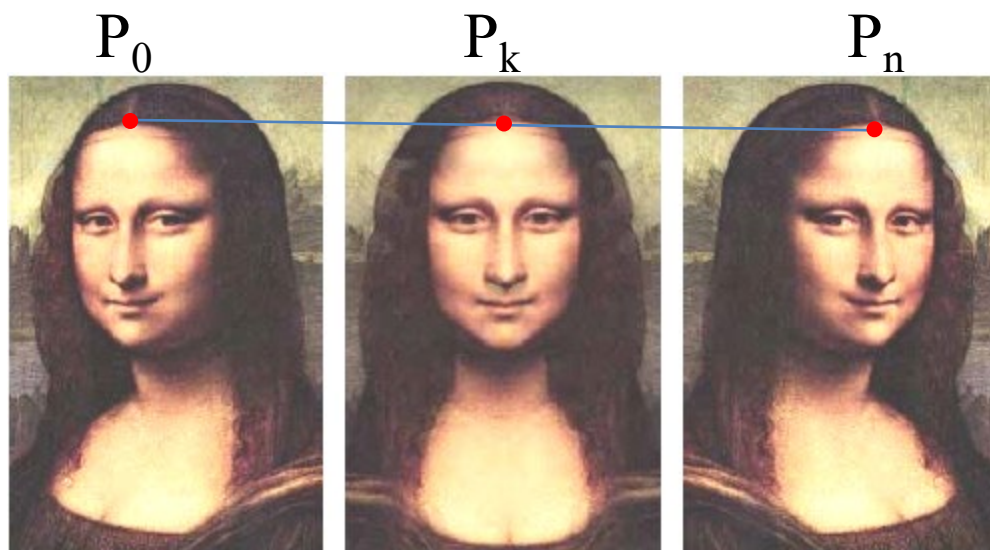
Image Morphing

1. Correspondences





Image Morphing



1. Correspondences
2. Linear interpolation

$$P_k = \left(1 - \frac{k}{n}\right)P_0 + \frac{k}{n}P_n$$

Image Morphing

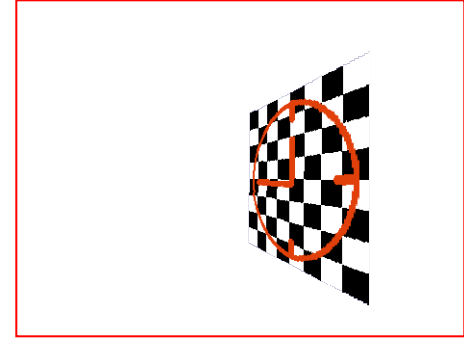
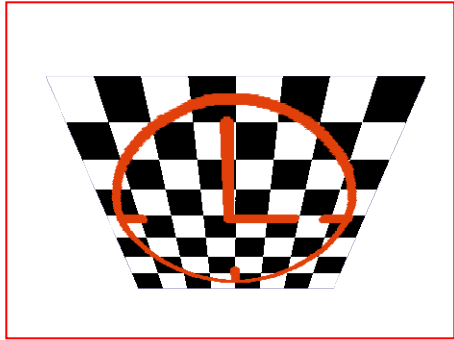
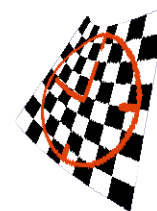
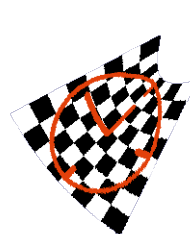
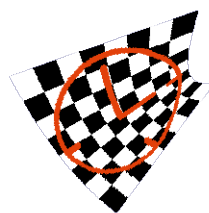
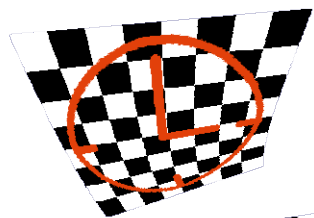
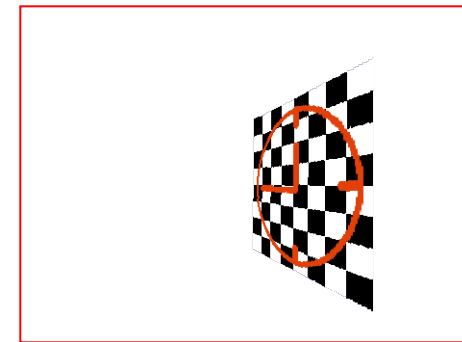




Image Morphing



Image morphing is not
shape preserving

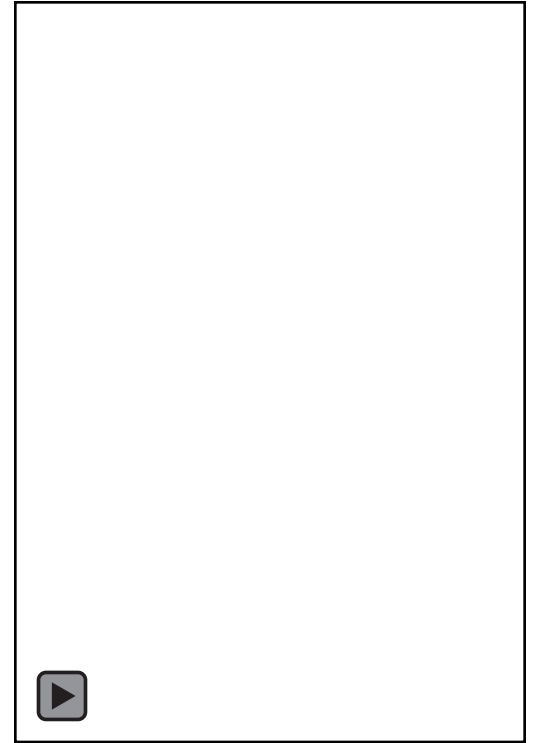
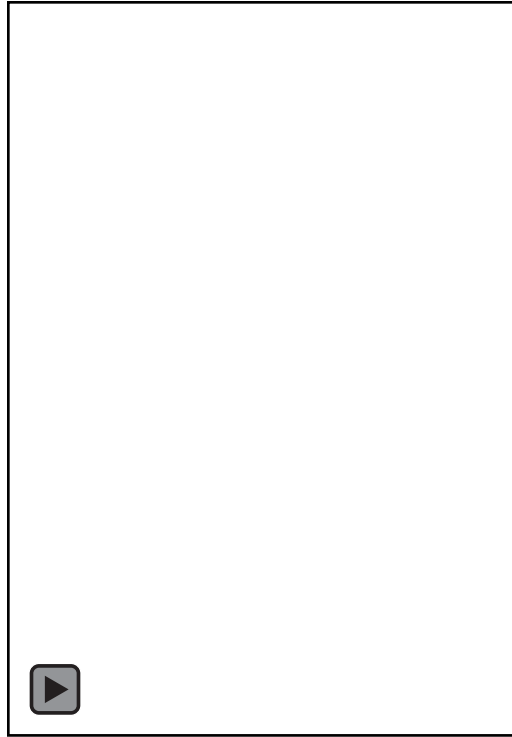
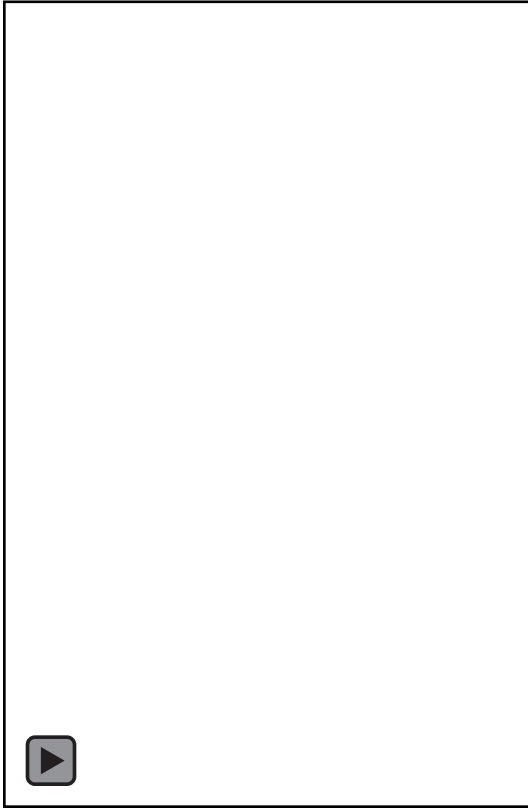




Topics

- Image morphing (2D)
- **View morphing (2D+)**
- Image warping (3D)

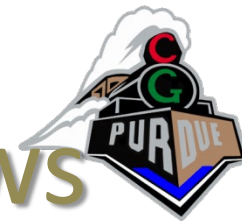
View Morphing



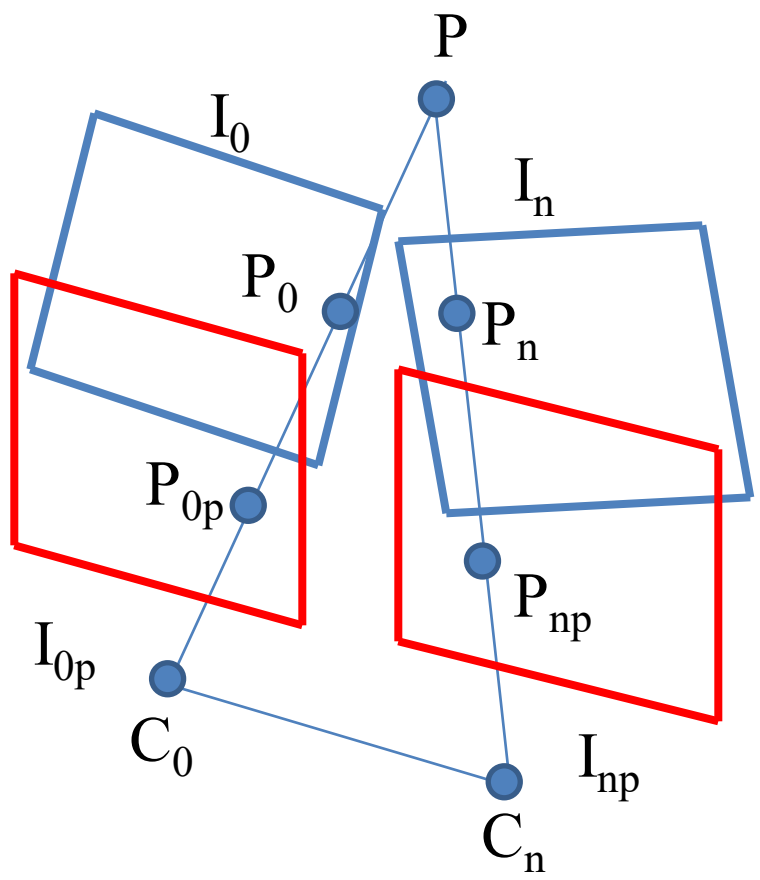


View Morphing

- Shape preserving morph
- Three step algorithm
 - Prewarp first and last images to parallel views
 - Image morph between prewarped images
 - Postwarp to interpolated view



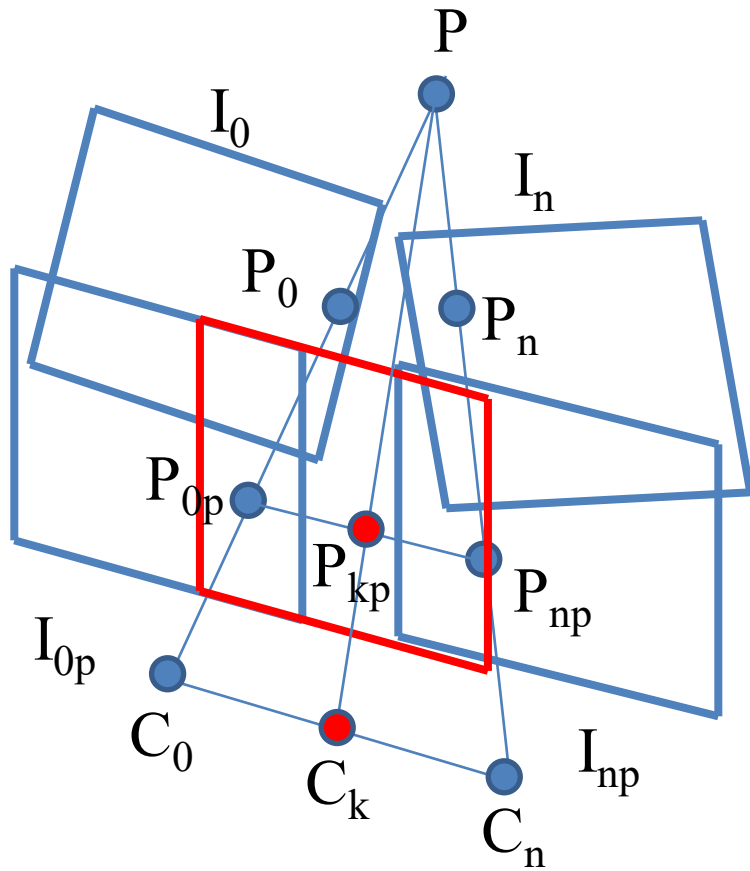
Step 1: prewarp to parallel views



- Parallel views
 - same image plane
 - image plane parallel to segment connecting the two centers of projection
- Prewarp
 - compute parallel views I_{0p} , I_{np}
 - rotate I_0 and I_n to parallel views
 - prewarp correspondence is $(P_0, P_n) \rightarrow (P_{0p}, P_{np})$



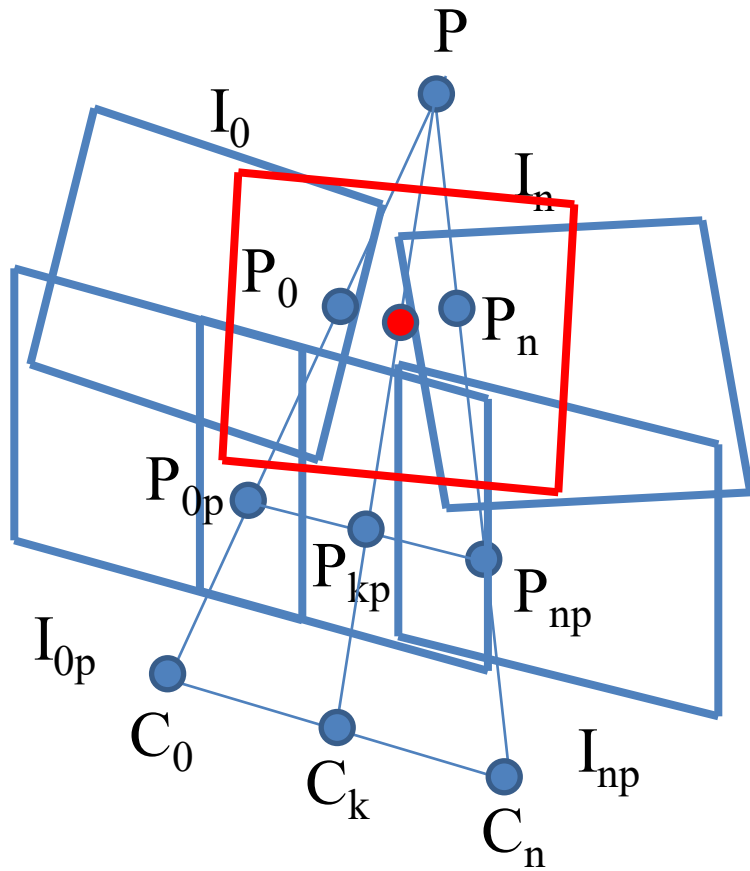
Step 2: morph parallel images



- Shape preserving
- Use prewarped correspondences
- Interpolate C_k from C_0 C_n



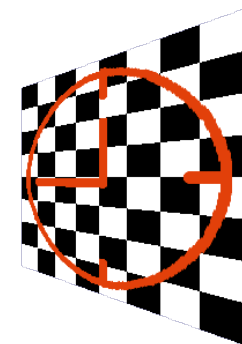
Step 3: postwarp image



- Postwarp morphed image
 - create intermediate view
 - C_k is known
 - interpolate view direction and tilt
 - rotate morphed image to intermediate view



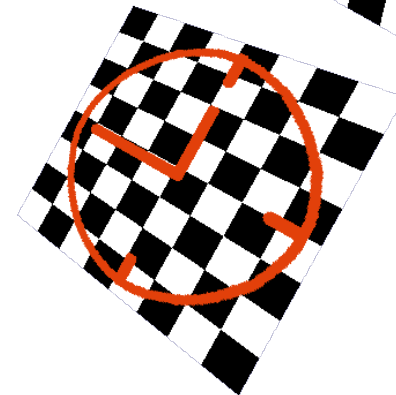
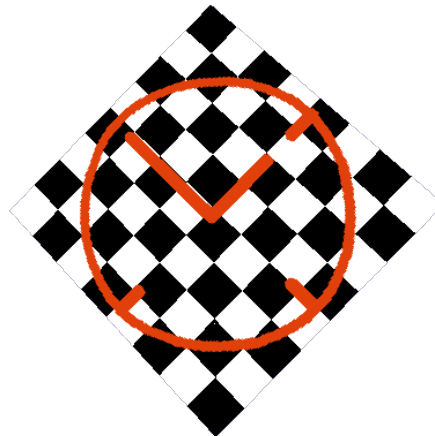
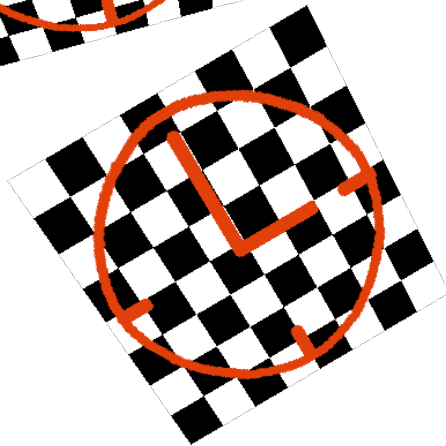
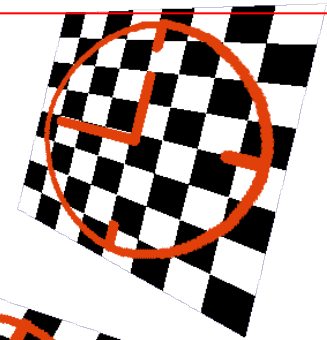
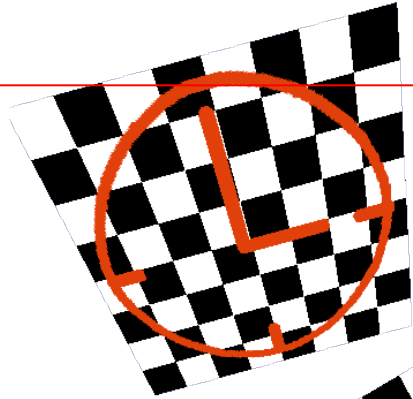
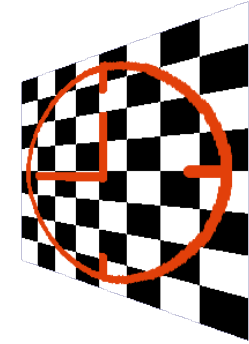
View morphing





View morphing

- View morphing is shape preserving





View Morphing Examples

- Using computer vision/stereo reconstruction techniques

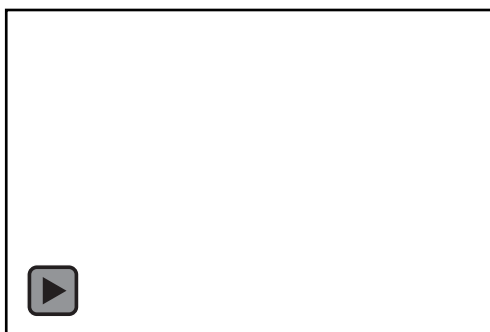
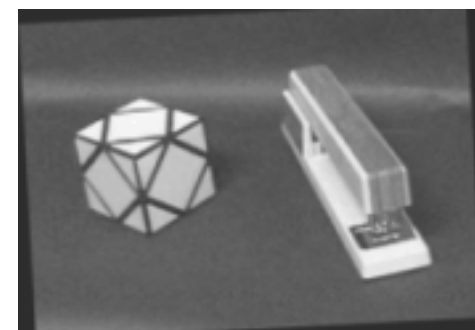
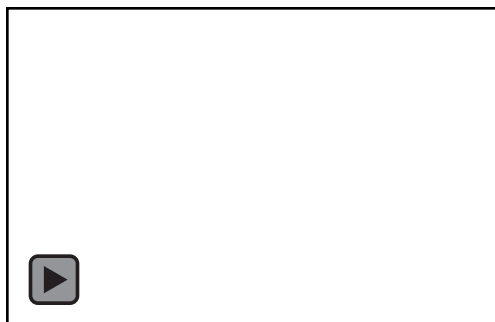
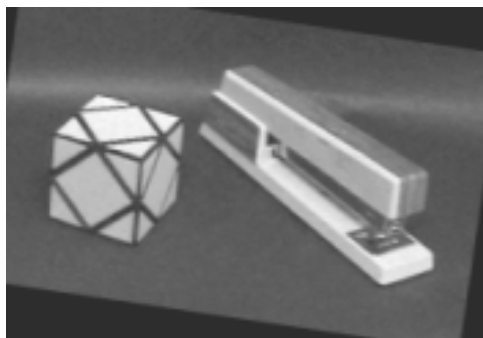




Image Transformations

- Intuitively, how do you compute the matrix M by which to transform P_0 to P_{0p} ?

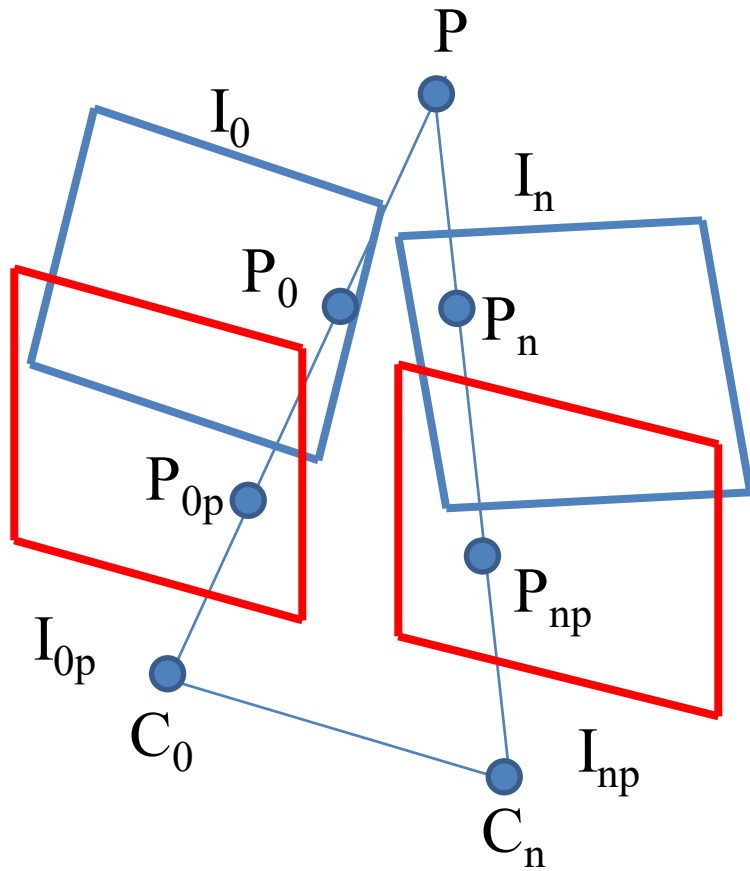




Image Transformations

- A geometric relationship between input (u,v) and output pixels (x,y)
 - Forward mapping:
$$(x,y) = (X(u,v), Y(u,v))$$
 - Inverse mapping:
$$(u,v) = (U(x,y), V(x,y))$$



Image Transformations

- General matrix form is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and operates in the “homogeneous coordinate system”.



Affine Transformations

- Matrix form is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and accommodates translations, rotations, scale, and shear.

- How many unknowns? How to create matrix?



Affine Transformations

- Transformation can be inferred from correspondences; e.g.,

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} \Leftrightarrow \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

- Given ≥ 3 correspondences can solve for T

Perspective/Projective Transformations



- Matrix form is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and it accommodates foreshortening of distant line
and convergence of lines to a vanishing point;
also, straight lines are maintained but not their
mutual angular relationships, and
only parallel lines parallel to the projection plane
remain parallel



Perspective/Projective Transformations

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- How many unknowns?
- How many correspondences are needed?



Direct Linear Transform

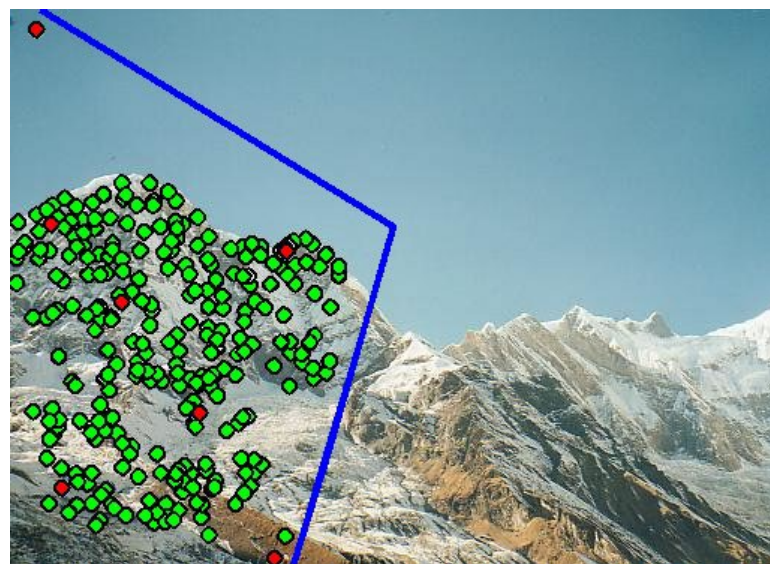
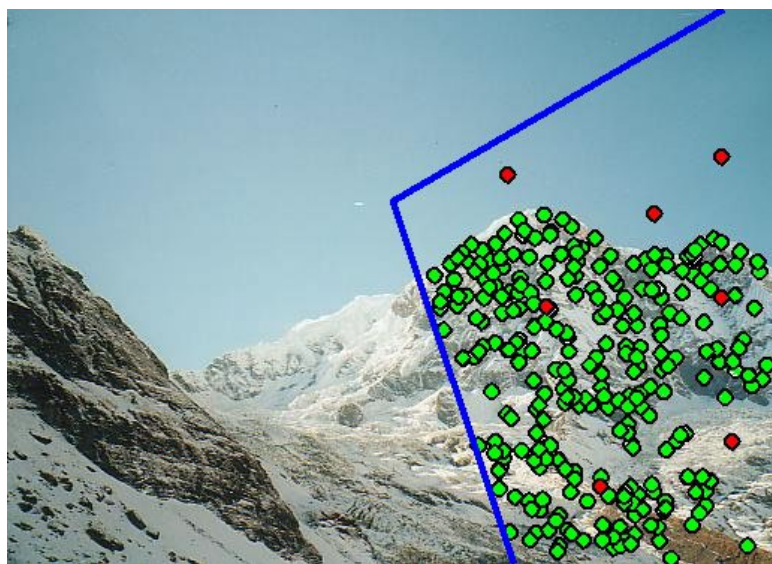
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- Set $w = 1$ and $z = 1$, then have

$$\alpha \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Divide line 1 and 2 by 3
- Rearrange terms to form...

Example





Example





“Image Stitching”

- A colloquial term for the same thing...





See blackboard...

$$\alpha(a_{11}u + a_{12}v + a_{13}) = x$$

$$\alpha(a_{21}u + a_{22}v + a_{23}) = y$$

$$\alpha(a_{31}u + a_{32}v + a_{33}) = 1$$

Divide 1st and 2nd line by 3rd line:

$$(a_{11}u + a_{12}v + a_{13}) = x(a_{31}u + a_{32}v + a_{33})$$

$$(a_{21}u + a_{22}v + a_{23}) = y(a_{31}u + a_{32}v + a_{33})$$

Rearrange terms:

$$a_{11}u + a_{12}v + a_{13} - a_{31}xu - a_{32}yv - a_{33}x = 0$$

$$a_{21}u + a_{22}v + a_{23} - a_{31}xu - a_{32}yv - a_{33}y = 0$$



See blackboard...

$$a_{11}u + a_{12}v + a_{13} - a_{31}xu - a_{32}yv - a_{33}x = 0$$

$$a_{21}u + a_{22}v + a_{23} - a_{31}xu - a_{32}yv - a_{33}y = 0$$

Assume $a_{33} = 1$,

$$a_{11}u + a_{12}v + a_{13} - a_{31}xu - a_{32}yv = x$$

$$a_{21}u + a_{22}v + a_{23} - a_{31}xu - a_{32}yv = y$$

Setup for 4+ points, yields 8 equations for 8 unknowns...



Perspective/Projective Transformations

- Solve Direct Linear Transform (DLT):

$$\begin{pmatrix} u_0 & v_0 & 1 & 0 & 0 & 0 & -u_0x_0 & -v_0x_0 \\ u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1x_1 & -v_1x_1 \\ u_2 & v_2 & 1 & 0 & 0 & 0 & -u_2x_2 & -v_2x_2 \\ u_3 & v_3 & 1 & 0 & 0 & 0 & -u_3x_3 & -v_3x_3 \\ 0 & 0 & 0 & u_0 & v_0 & 1 & -u_0y_0 & -v_0y_0 \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -u_1y_1 & -v_1y_1 \\ 0 & 0 & 0 & u_2 & v_2 & 1 & -u_2y_2 & -v_2y_2 \\ 0 & 0 & 0 & u_3 & v_3 & 1 & -u_3y_3 & -v_3y_3 \end{pmatrix}$$

$$A = b$$

where A is the vector of unknown coefficients a_{ij}



Topics

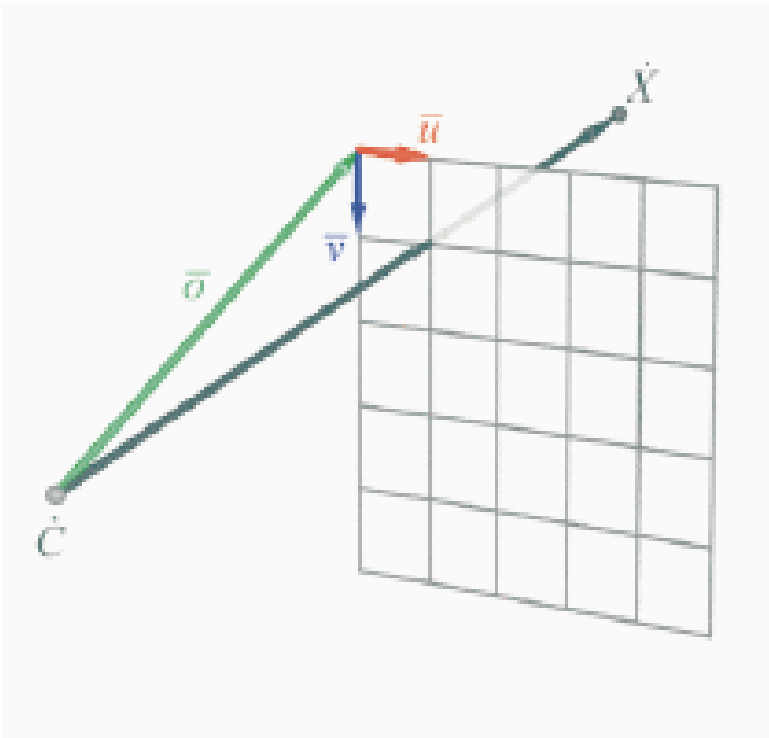
- Image morphing (2D)
- View morphing (2D+)
- Image warping (3D)



3D Image Warping

- Goal: “warp” the pixels of the image so that they appear in the correct place for a new viewpoint
- Advantage:
 - Don’t need a geometric model of the object/environment
 - Can be done in time proportional to screen size and (mostly) independent of object/environment complexity
- Disadvantage:
 - Limited resolution
 - Excessive warping reveals several visual artifacts
(see examples)

3D Image Warping Equations



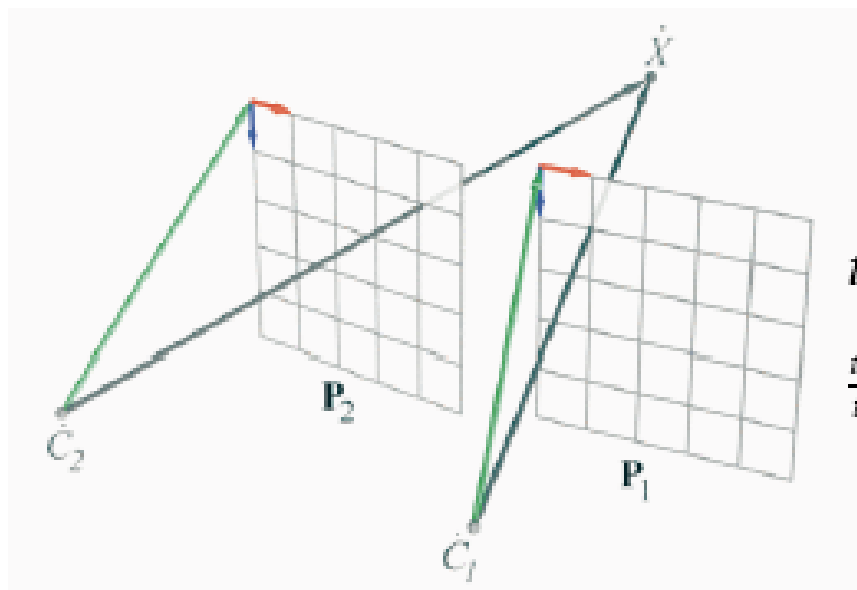
$$P = \begin{bmatrix} u_x & v_x & o_x \\ u_y & v_y & o_y \\ u_z & v_z & o_z \end{bmatrix}$$

$$\vec{X} = \vec{C} + t P \vec{x}$$

Some pictures courtesy of SIGGRAPH '99 course notes
(Leonard McMillan)



3D Image Warping Equations



$$\dot{C}_2 + t_2 P_2 \bar{x}_2 = \dot{C}_1 + t_1 P_1 \bar{x}_1$$

$$t_2 P_2 \bar{x}_2 = \dot{C}_1 - \dot{C}_2 + t_1 P_1 \bar{x}_1$$

$$t_2 \bar{x}_2 = P_2^{-1} (\dot{C}_1 - \dot{C}_2) + t_1 P_2^{-1} P_1 \bar{x}_1$$

$$\frac{t_2}{t_1} \bar{x}_2 = \frac{1}{t_1} P_2^{-1} (\dot{C}_1 - \dot{C}_2) + P_2^{-1} P_1 \bar{x}_1$$

$$\bar{x}_2 \doteq \underbrace{\frac{1}{\delta} P_2^{-1} (\dot{C}_1 - \dot{C}_2)}_{e_{21}} + \underbrace{P_2^{-1} P_1}_{H_{21}} \bar{x}_1$$

3D Image Warping Equations



McMillan & Bishop Warping Equation:

$$x_2 = \underbrace{\delta(x_1) P_2^{-1} (c_1 - c_2)}_{\text{Move pixels based on distance to eye}} + \underbrace{P_2^{-1} P_1 x_1}_{\sim \text{Texture mapping}}$$

*Move pixels based on
distance to eye*

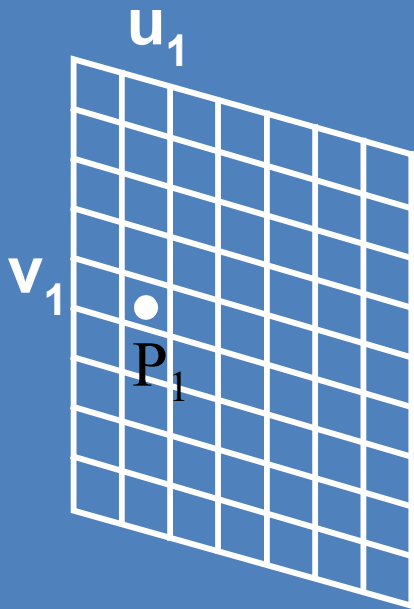
*~Texture
mapping*

- Per-pixel distance values are used to warp pixels to their correct location for the current eye position



3D Image Warping Equations

- Images enhanced with per-pixel depth
[McMillan95]

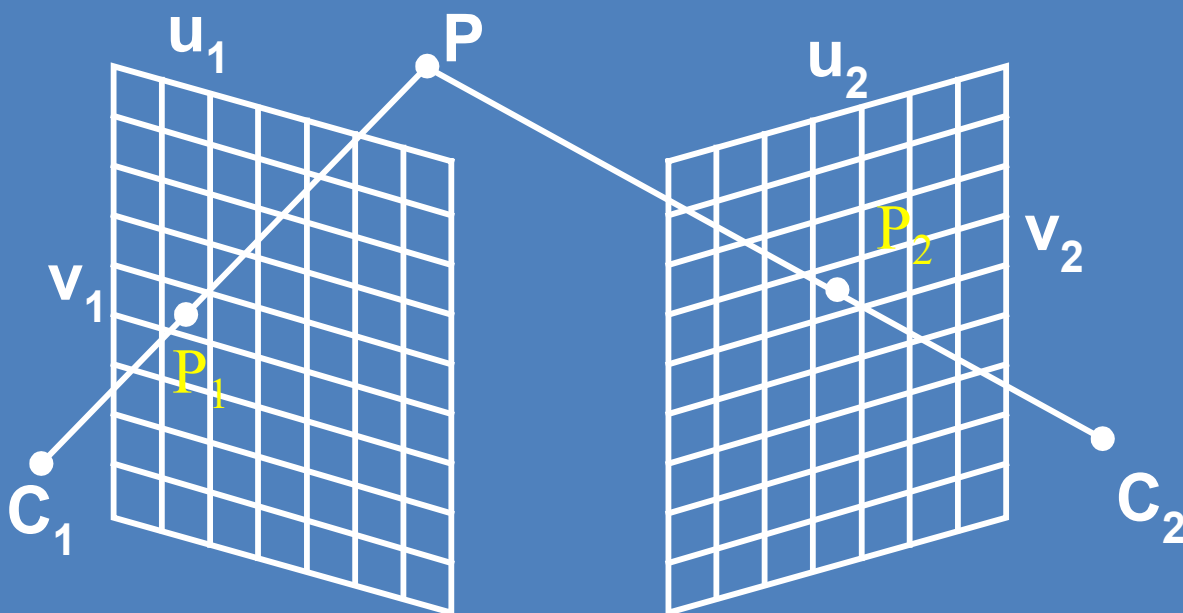




3D Image Warping Equations

$$\dot{P} = \dot{C}_1 + (\bar{c}_1 + u_1 \bar{a}_1 + v_1 \bar{b}_1) w_1$$

$$\dot{P} = \dot{C}_2 + (\bar{c}_2 + u_2 \bar{a}_2 + v_2 \bar{b}_2) w_2$$

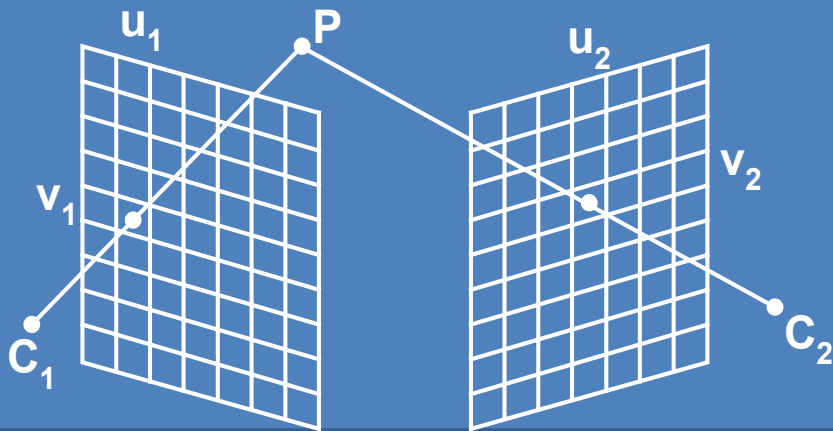


3D Image Warping Equations



$$u_2 = \frac{w_{11} + w_{12} \cdot u_1 + w_{13} \cdot v_1 + w_{14} \cdot \delta(u_1, v_1)}{w_{31} + w_{32} \cdot u_1 + w_{33} \cdot v_1 + w_{34} \cdot \delta(u_1, v_1)}$$

$$v_2 = \frac{w_{21} + w_{22} \cdot u_1 + w_{23} \cdot v_1 + w_{24} \cdot \delta(u_1, v_1)}{w_{31} + w_{32} \cdot u_1 + w_{33} \cdot v_1 + w_{34} \cdot \delta(u_1, v_1)}$$



3D Image Warping Example



3D Image Warping Example



- DeltaSphere
 - Lars Nyland *et al.*

3D Image Warping Example



3D Image Warping Example



3D Image Warping Example



3D Image Warping Example





Disocclusions

- Disocclusions (or exposure events) occur when unsampled surfaces become visible...



What can we do?



Disocclusions

- Bilinear patches: fill in the areas

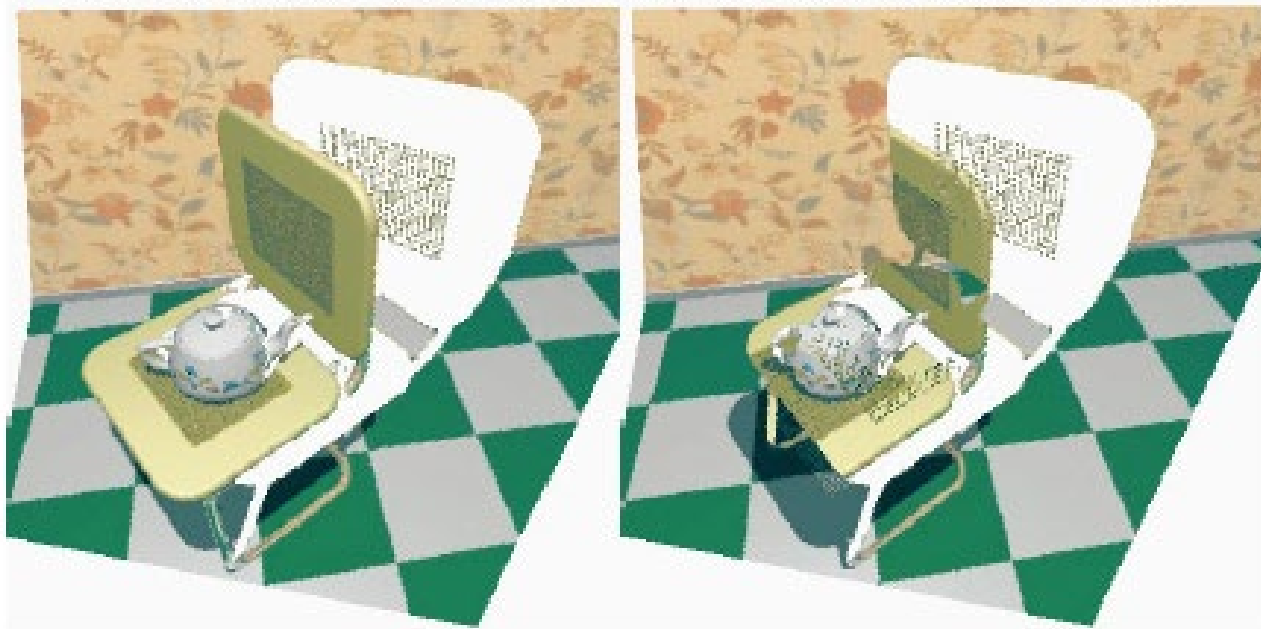


What else?



Rendering Order

- ✓ The warping equation determines where points go...

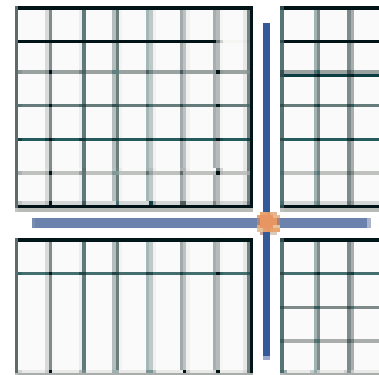
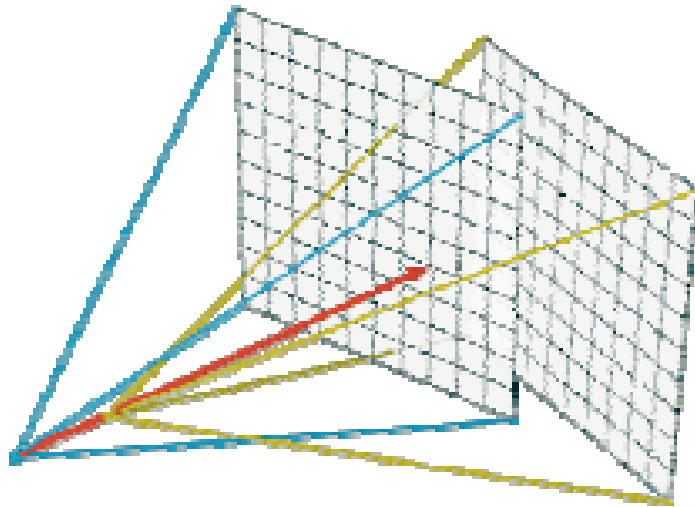


... but that is not sufficient

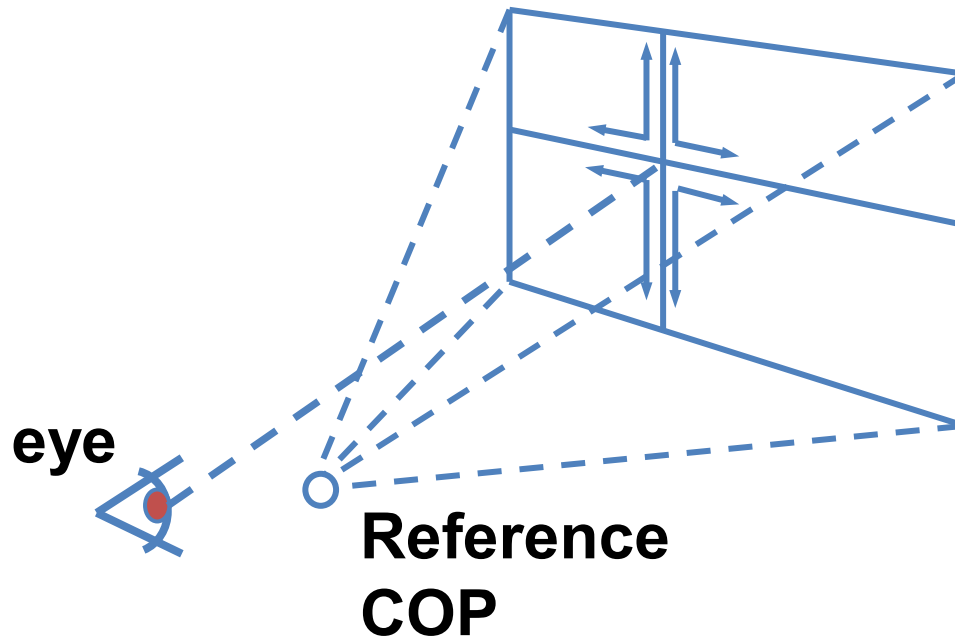


Occlusion Compatible Rendering Order

- Epipolar geometry
- Project the new viewpoint onto the original image and divide the image into 1, 2 or 4 “sheets”



Occlusion Compatible Rendering Order



- A raster scan of each sheet produces a back-to-front ordering of warped pixels



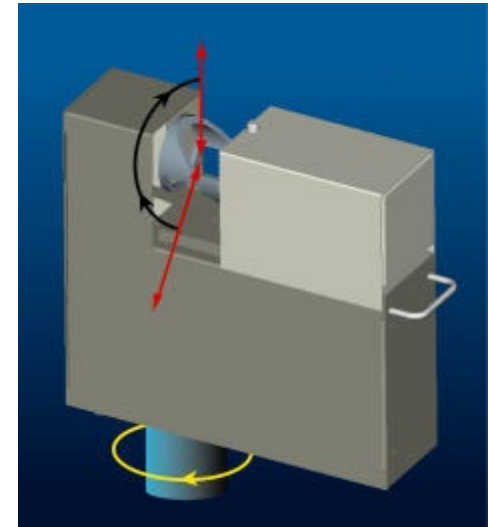
Splatting

- One pixel in the source image does not necessarily project to one pixel in the destination image
 - e.g., if you are walking towards something, the sample might get larger...
- A solution: estimate shape and size of footprint of warped samples
 - expensive to do accurately
 - square/rectangular approximations can be done quickly (3x3 or 5x5 splats)
 - occlusion-compatible rendering will take care of oversized splats
 - *BUT large splats can make the image seem blocky/low-res*

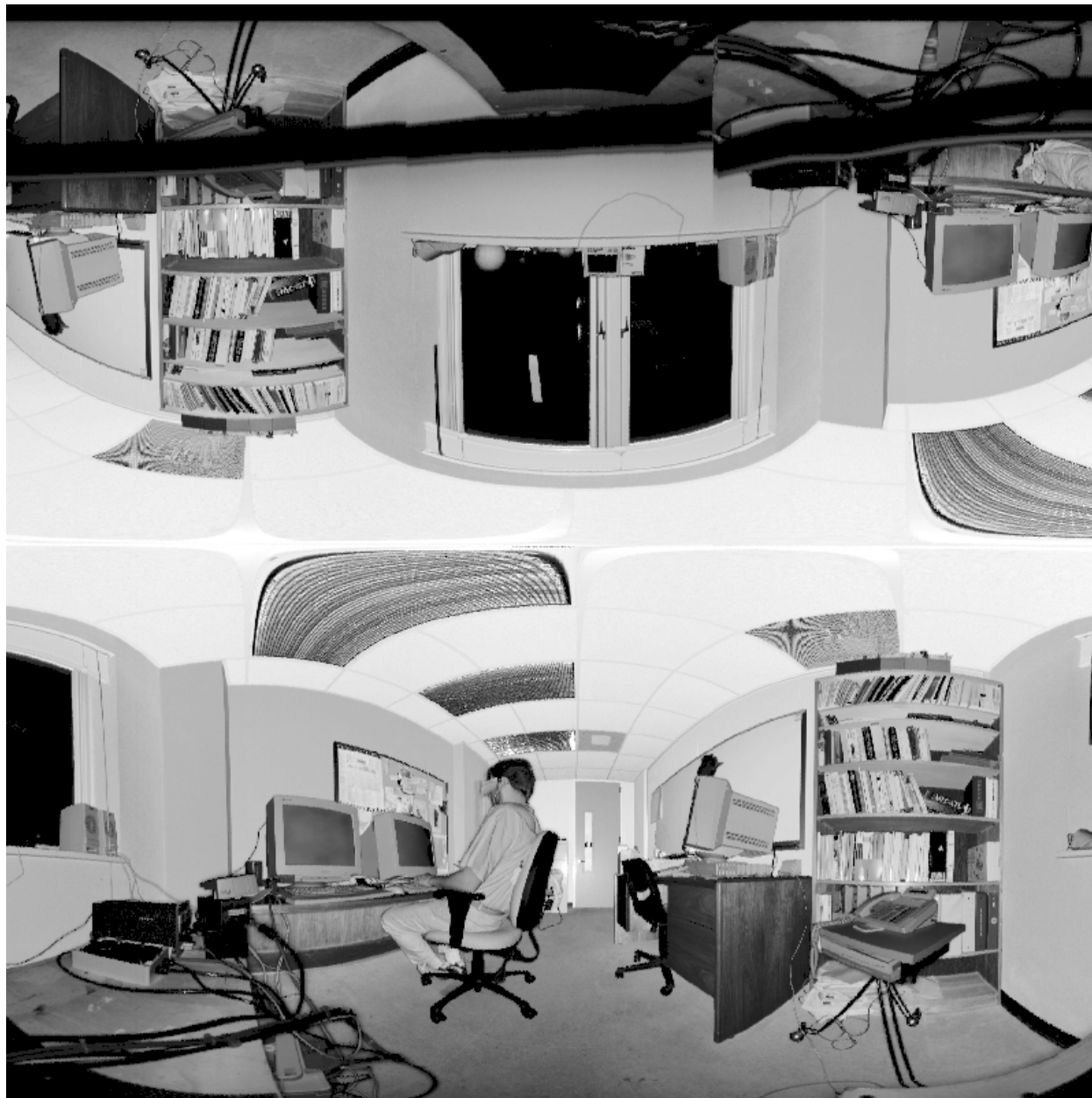


More Examples Using the DeltaSphere

- Lars Nyland *et al.*



courtesy 3rd Tech Inc.



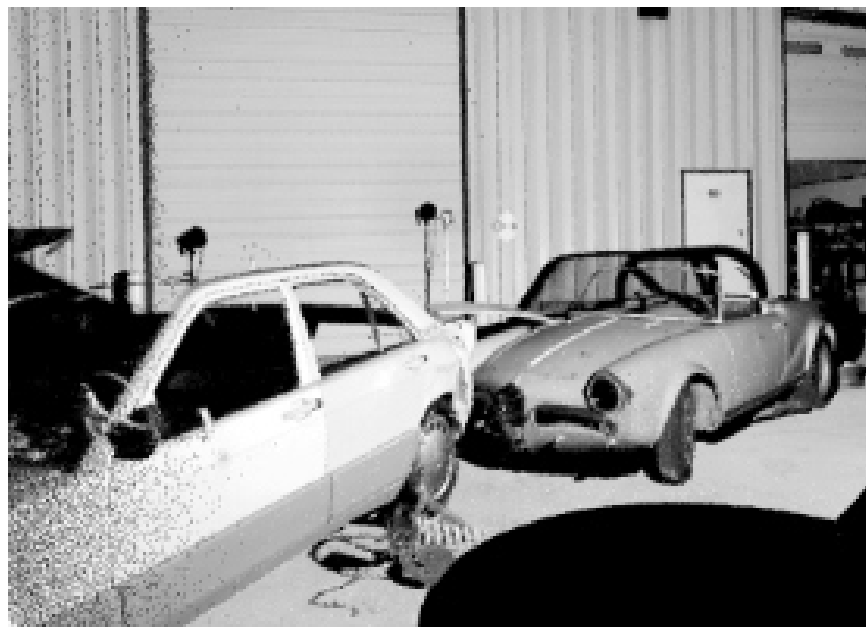
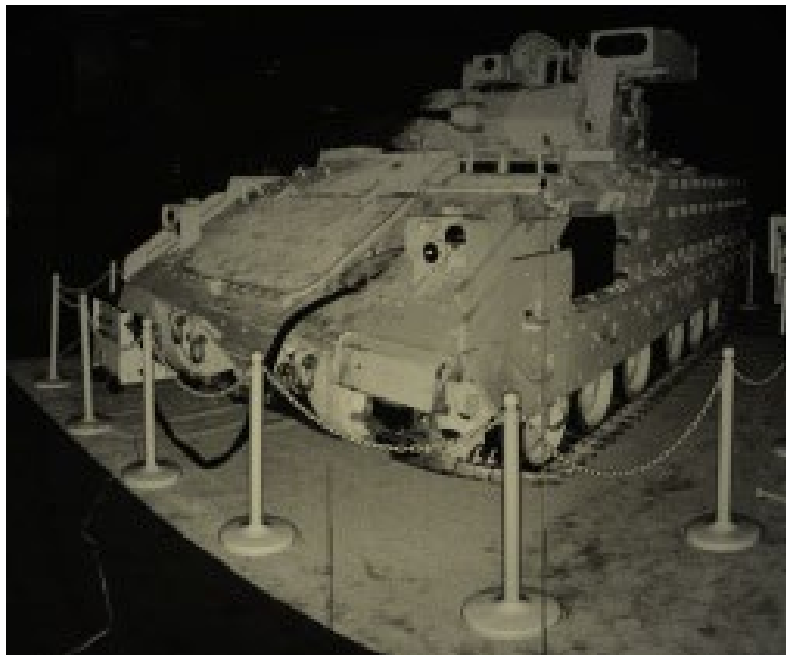
- 300° x 300° panorama
- this is the reflected light



- 300° x 300° panorama
- this is the range light



spherical range panoramas



planar re-projection

Courtesy 3rd Tech Inc.



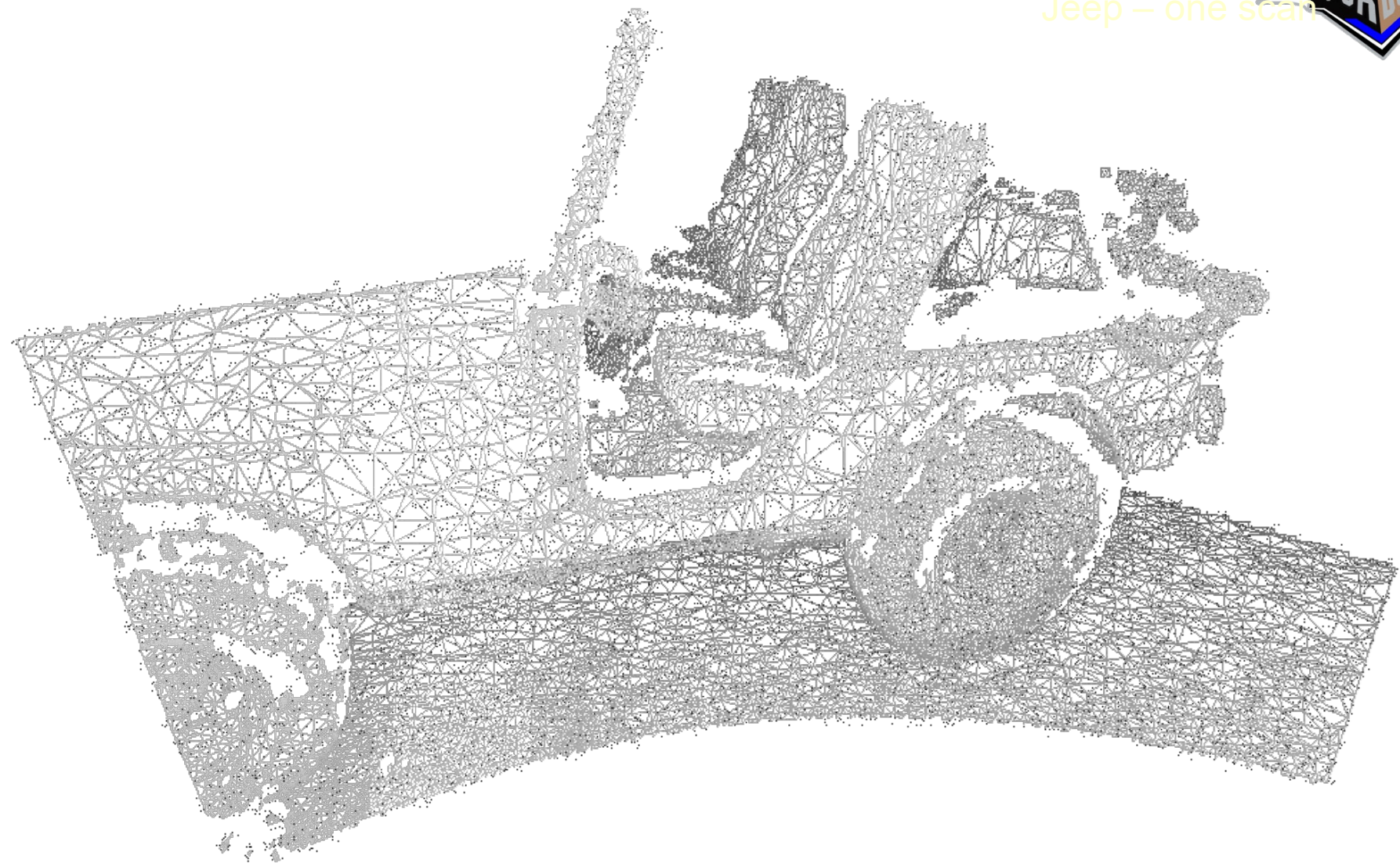
Jeep – one scan



Courtesy 3rd Tech Inc.



Jeep – one scan



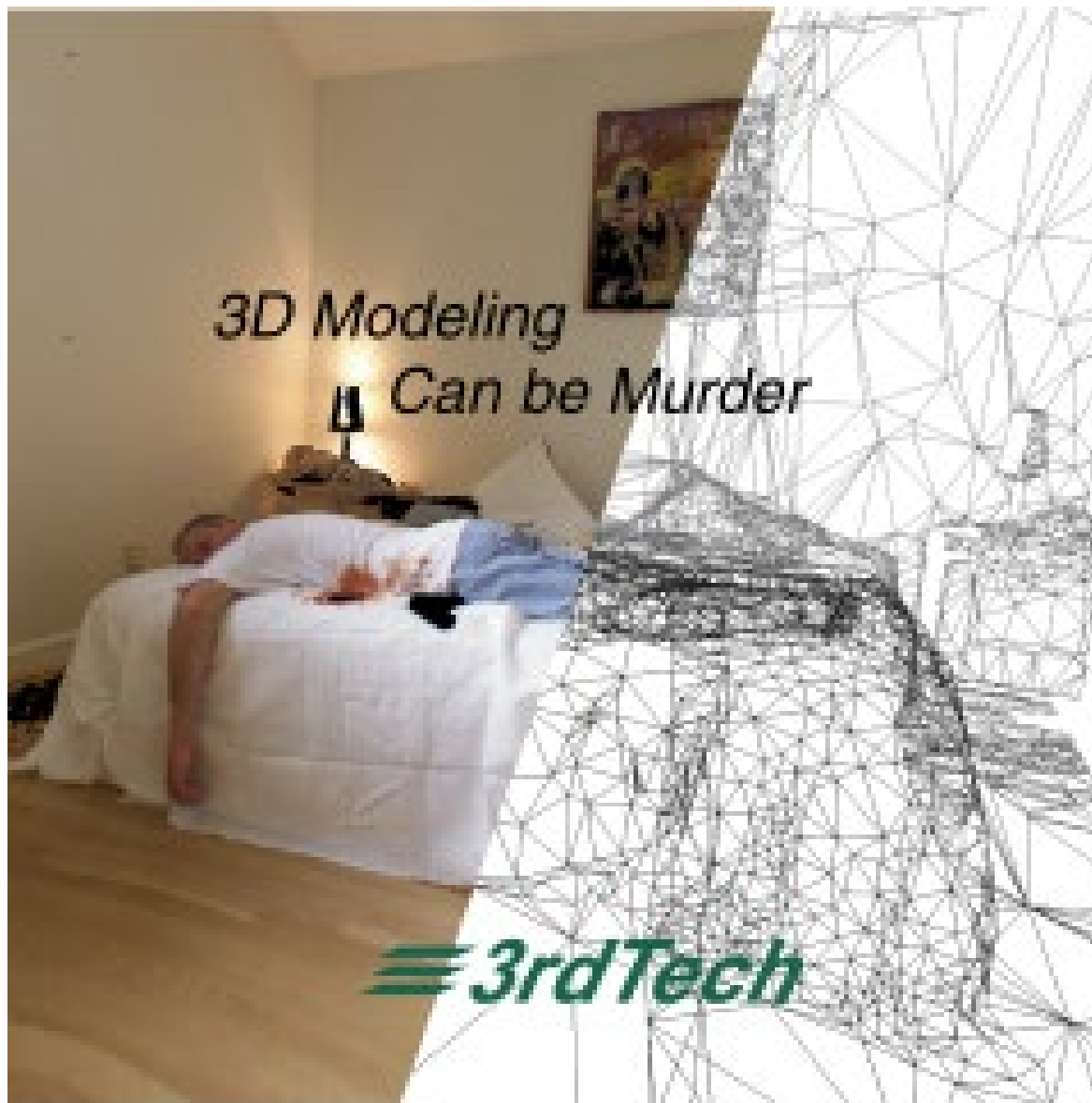
Courtesy 3rd Tech Inc.



Complete Jeep model



Courtesy 3rd Tech Inc.



*3D Modeling
Can be Murder*

≡ 3rdTech