Physically Based Simulations on the GPU (just briefly...)

CS334

Daniel G. Aliaga
Department of Computer Science
Purdue University
Simulating the world

• Floating point arithmetic on GPUs and their speed enable us to simulate a wide variety of phenomena using PDEs
Some Basics

• Operators (on images/lattices)
• Diffusion
• Bouyancy
Operators

• Given an image:
  – Gradient (vector)

\[ \nabla f(x, y) = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} \]

  – Laplacian (scalar)

\[ \nabla^2 f(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \]
Discrete Laplacian

\[ \nabla^2 f(x, y) = \]
\[ f(x - 1, y) + f(x + 1, y) + f(x, y - 1) + f(x, y + 1) - 4f(x, y) \]

• Matrix form K = ??
Discrete Laplacian

• $\nabla^2 f(x, y) =$

\[
f(x - 1, y) + f(x + 1, y) +
f(x, y - 1) + f(x, y + 1) -
4f(x, y)
\]

• Matrix form $K =$

\[
\begin{bmatrix}
0 & 1 & 0 \\
1 & -4 & 1 \\
0 & 1 & 0
\end{bmatrix}
\]
Image convolution

• Convolve an image with a kernel $K$: 

[source pixel]

Convolution

New pixel value (destination pixel)
(Image) Convolution

- Convolution
  - Define a kernel
  - "Convolve the image"
(Image) Convolution

- Kernel: \((1/16) \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}\)

- What if kernel is not normalized?

- Image:

  \[
  \begin{bmatrix}
  p_{11} & \cdots & p_{m1} \\
  \vdots & \ddots & \vdots \\
  p_{1n} & \cdots & p_{mn}
  \end{bmatrix}
  \]

- What if image is multi-channel?

- What if kernel falls off the side of the image?
Convolution
(Image) Convolution
(Image) Convolution
(Image) Convolution
Edge Detection

• What would you do? What kernel?
Edge Detection:
First Order Operator

• Roberts operator (1963) on image $A$:

- $G_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \ast A$, $G_y = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \ast A$

- $G = \sqrt{G_x^2 + G_y^2}$

- $\theta = \tan^{-1}\left(\frac{G_y}{G_x}\right)$
Edge Detection

• Sobel operator (1968) on image $A$:

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \ast A, \quad G_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \ast A$$

• $G = \sqrt{G_x^2 + G_y^2}$

• $\theta = \tan^{-1}\left(\frac{G_y}{G_x}\right)$
Edge Detection

• Prewitt operator (1970) on image $A$ (different spectral response as compared to Sobel):

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \ast A, \quad G_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \ast A$$

• $G = \sqrt{G_x^2 + G_y^2}$

• $\theta = \tan^{-1}\left(\frac{G_y}{G_x}\right)$
Edge Detection

• Canny Edges (1986)
  – Multi-stage algorithm, uses Sobel/Prewitt (or other) edge detector on a Gaussian filtered image and then has a process of non-maximal suppression
Edge Detection: Second-Order Operator

- Laplacian: highlights regions of rapid intensity change

\[ L_A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \]

(positive Laplacian takes out outward edges; negative Laplacian is possible too)
Edge Detection

• Online demo:
  – https://fiveko.com/online-tools/
Heat Equation

\[ \frac{\partial f}{\partial t} = \nabla^2 f \]
Demo

[See “Ready” Demo: Heat Equation]
Diffusion Equation

[Weisstein 1999]

\[ f'(x, y) = f(x, y) + \frac{c_d}{4} \nabla^2 f(x, y) \]

where \( c_d \) is the coefficient of diffusion...
(Anisotropic) Diffusion

(a) Original Image

(b) Time = 5

(c) Time = 10

(d) Time = 30
Buoyancy

• Used in convection, cloud formations, etc.
• Given a temperature state $T$:
  – a vertical buoyancy velocity is ‘upwards’ if a node is hotter than its neighbors’ and
  – a vertical buoyancy velocity is ‘downwards’ if a node is cooler than its neighbors
Buoyancy

\[ v(x, y)' = v(x, y) + \frac{c_b}{2} (2f(x, y) - f(x + 1, y) - f(x - 1, y)) \]

where \( c_b \) is the buoyancy strength
Bouyancy
(considering neighbors)

\[ f(x, y)' = f(x, y) - \frac{\sigma}{2} f(x, y) \]
\[ [\rho(f(x, y + 1)) - \rho(f(x, y - 1))] \]

where \( \rho(f) = \tanh(\alpha(f - f_c)) \) (an approx. of density relative to temperature \( f \)) and \( \sigma \) is buoyancy strength and \( \alpha \) and \( f_c \) are constants.
Euler Method (for ODE)

- Given:
  \[ y'(t) = f(t, y(t)) \text{ with } y(t_0) = y_0 \]

- Do:
  \[ y_{n+1} = y_n + hf(t_n, y_n) \]
Classical Runge Kutta Method

• Given:

\[ y'(t) = f(t, y(t)) \text{ with } y(t_0) = y_0 \]

• Do:

\[ y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \]
\[ t_{n+1} = t_n + h \]

where

\[ k_1 = f(t_n, y_n), \]
\[ k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1), \]
\[ k_3 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2), \]
\[ k_4 = f(t_n + h, y_n + hk_3). \]
Example: (Water) Boiling

• Based on [Harris et al. 2002]
• State = Temperature
• Three operations:
  – Diffusion, buoyancy, & latent heat
• 3D Simulation
  – Stack of 2D texture slices
Wave Equation

• Remember heat equation:
  – Rate of change of value proportional to Laplacian

• Wave equation:
  – Rate of change of the rate of change is also proportional to the Laplacian
Wave Equation

\[ \frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u \]

where \( u \) models the displacement and \( c \) is the propagation speed
Water Simulation: Wave Equation

\[ U = \text{value}, \ V = \text{rate of change} \]

\[ \frac{\partial U}{\partial t} = \frac{b}{k} + d\nabla^2 U \]

\[ \frac{\partial V}{\partial t} = k\nabla^2 U \]
Water Simulation: Wave Equation

- Demo...
Demo

[See “Ready” Demo: Wave Equation]

• Also:
  – [https://www.ibiblio.org/e-notes/webgl/gpu/contents.htm](https://www.ibiblio.org/e-notes/webgl/gpu/contents.htm)
“Alan Turing in 1952 describing the way in which non-uniformity (stripes, spots, spirals, etc.) may arise naturally out of a homogeneous, uniform state. The theory (which can be called a reaction–diffusion theory of morphogenesis), has served as a basic model in theoretical biology, and is seen by some as the very beginning of chaos theory.”

\[
\frac{\partial U}{\partial t} = D_U \nabla^2 U - k(UV - 16) \\
\frac{\partial V}{\partial t} = D_V \nabla^2 V + k(UV - 12 - V)
\]
Gray-Scott Reaction-Diffusion

- State = two scalar chemical concentrations
- Simple:
  - just Diffusion and Reaction ops

\[
\frac{\partial U}{\partial t} = D_u \nabla^2 U - UV^2 + F (1 - U), \\
\frac{\partial V}{\partial t} = D_v \nabla^2 V + UV^2 - (F + k)V
\]

*U, V are chemical concentrations, F, k, D_u, D_v are constants*
Some research...

Demo

[See “Ready” Demo: Gray-Scott Equation]

• Also:
  – https://www.ibiblio.org/e-notes/webgl/gpu/contents.htm
Fluid Simulations
Thermodynamics

• Temperature affected by
  – Heat sources
  – Advection
  – Latent heat released / absorbed during condensation / evaporation

• $\Delta$ temperature = advection + latent heat release
  + temperature input
Cloud Dynamics

- 3 components
  - 7 unknowns

- Fluid dynamics
  - Motion of the air

- Thermodynamics
  - Temperature changes

- Water continuity
  - Evaporation, condensation

Velocity: \( \mathbf{u} = (u, v, w) \)
Pressure: \( p \)
Potential temperature: \( \theta \)
(see dissertation)
Water vapor mixing ratio: \( q_v \)
Liquid water mixing ratio: \( q_c \)
Cloud Dynamics
Water Simulation: Sine Waves

$$Asin(\omega x + t)$$
Water Simulation: 
Sine Waves

\[ A_1 \sin(\omega_1 x + t_1) + A_2 \sin(\omega_2 x + t_2) + \cdots \]
Water Simulation: Sine Waves

• Using sine-wave summations:

\[ H(x, y, t) = \sum A_i \sin(D_i \cdot (x, y) \omega_i + t \phi_i) \]

[use \(H\) as height or a pixel intensity]

• Pixel values over time are:

\[ P(x, y, t) = (x, y, H(x, y, t)) \]
Water Simulation: Sine Waves

(here, pixel normals are computed as well for reflections)
Water: Surface Normals

• Use binormal and tangent:

\[ B(x, y, t) = \left( \frac{dx}{dx}, \frac{dy}{dx}, \frac{dH(x, y, t)}{dx} \right) = (1, 0, \frac{dH(x, y, t)}{dx}) \]

\[ T(x, y, t) = \ldots = \left( 0, 1, \frac{dH(x, y, t)}{dy} \right) \]

• Normal is:

\[ N(x, y, t) = B \times T \]

\[ N(x, y, t) = \left( -\frac{dH(x, y, t)}{dx}, -\frac{dH(x, y, t)}{dy}, 1 \right) \]
Water Simulation: Gerstner Waves

• These waves also change the $x, y$ of the wave imitating how points at top of wave are squished together and points at bottom are separated
Water Simulation:
Gerstner Waves

\[ P(x, y, t) = \left[ x + \sum Q_i A_i D_i \cdot x \cos(\omega_i D_i \cdot (x, y) + \phi_i t) \right. \]

\[ = y + \sum Q_i A_i D_i \cdot y \cos(\omega_i D_i \cdot (x, y) + \phi_i t) \]

\[ \sum A_i \sin(\omega_i D_i \cdot (x, y) + \phi_i t) \]

where \( Q_i = \) sharpness
Water Simulation: Gerstner Waves
Video

- https://www.youtube.com/watch?v=lqPa389vi4s
- https://www.youtube.com/watch?v=8DxL-ErCRVo